

# PV211: Introduction to Information Retrieval

<https://www.fi.muni.cz/~sojka/PV211>

## IIR 3: Dictionaries and tolerant retrieval Handout version

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# Overview

- 1 Dictionaries
- 2 Wildcard queries
- 3 Edit distance
- 4 Spelling correction
- 5 Soundex

# Take-away

- **Tolerant retrieval:** What to do if there is no exact match between query term and document term
- Wildcard queries
- Spelling correction

# Inverted index

For each term  $t$ , we store a list of all documents that contain  $t$ .

BRUTUS → 1 | 2 | 4 | 11 | 31 | 45 | 173 | 174

CAESAR → 1 | 2 | 4 | 5 | 6 | 16 | 57 | 132 | ...

CALPURNIA → 2 | 31 | 54 | 101

⋮

**dictionary**

**postings**

# Dictionaries

- The dictionary is the data structure for storing the term vocabulary.
- **Term vocabulary**: the **data**
- **Dictionary**: the **data structure** for storing the term vocabulary

# Dictionary as array of fixed-width entries

- For each term, we need to store a couple of items:
  - document frequency
  - pointer to postings list
  - ...
- Assume for the time being that we can store this information in a fixed-length entry.
- Assume that we store these entries in an array.

# Dictionary as array of fixed-width entries

| term   | document<br>frequency | pointer to<br>postings list |
|--------|-----------------------|-----------------------------|
| a      | 656,265               | →                           |
| aachen | 65                    | →                           |
| ...    | ...                   | ...                         |
| zulu   | 221                   | →                           |

space needed:    20 bytes    4 bytes    4 bytes

How do we look up a query term  $q_i$  in this array at query time?  
 That is: which data structure do we use to locate the entry (row)  
 in the array where  $q_i$  is stored?

# Data structures for looking up term

- Two main classes of data structures: hashes and trees
- Some IR systems use hashes, some use trees.
- Criteria for when to use hashes vs. trees:
  - Is there a fixed number of terms or will it keep growing?
  - What are the relative frequencies with which various keys will be accessed?
  - How many terms are we likely to have?



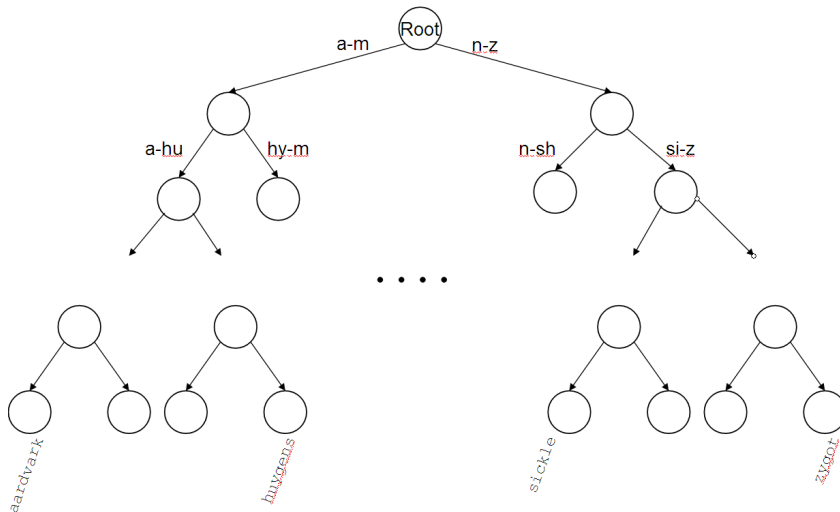
# Hashes

- Each vocabulary term is hashed into an integer, its row number in the array.
- At query time: hash query term, locate entry in fixed-width array.
- Pros: Lookup in a hash is faster than lookup in a tree.
  - Lookup time is constant.
- Cons
  - no way to find minor variants (*resume* vs. *résumé*)
  - no prefix search (all terms starting with *automat*)
  - need to rehash everything periodically if vocabulary keeps growing

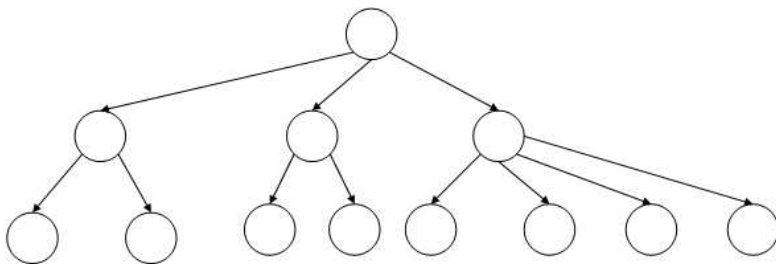
# Trees

- Trees solve the prefix problem (find all terms starting with *automat*).
- Simplest tree: binary tree.
- Search is slightly slower than in hashes:  $O(\log M)$ , where  $M$  is the size of the vocabulary.
- $O(\log M)$  only holds for **balanced** trees.
- Rebalancing binary trees is expensive.
- **B-trees** mitigate the rebalancing problem.
- B-tree definition: every internal node has a number of children in the interval  $[a, b]$  where  $a, b$  are appropriate positive integers, e.g.,  $[2, 4]$ .

# Binary tree



# B-tree



# Wildcard queries

- $mon^*$ : find all docs containing any term beginning with *mon*
- Easy with B-tree dictionary: retrieve all terms  $t$  in the range:  $mon \leq t < moo$
- $*mon$ : find all docs containing any term ending with *mon*
  - Maintain an additional tree for terms *backwards*.
  - Then retrieve all terms  $t$  in the range:  $nom \leq t < non$
- Result: A set of terms that are matches for wildcard query.
- Then retrieve documents that contain any of these terms.

# Query processing

- At this point, we have an enumeration of all terms in the dictionary that match the wildcard query.
- We still have to look up the postings for each enumerated term.

# How to handle \* in the middle of a term

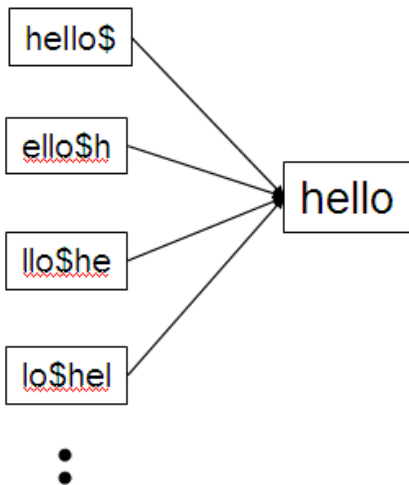
- Example: m\*nchen
- We could look up m\* and \*nchen in the B-tree and intersect the two term sets.
- Expensive
- Alternative: [permuterm](#) index
- Basic idea: Rotate every wildcard query, so that the \* occurs at the end.
- Store each of these rotations in the dictionary, say, in a B-tree

# Permuterm index

- For term HELLO: add *hello\$*, *ello\$h*, *llo\$he*, *lo\$hel*, *o\$hell*, and *\$hello* to the B-tree where \$ is a special symbol



# Permuterm $\rightarrow$ term mapping



# Permuterm index

- For HELLO, we've stored: *hello\$, ello\$h, llo\$he, lo\$hel, o\$hell, \$hello*
- Queries
  - For X, look up X\$
  - For X\*, look up \$X\*
  - For \*X, look up X\$\*
  - For \*X\*, look up X\*
  - For X\*Y, look up Y\$X\*
  - Example: For hel\*o, look up o\$hel\*
- Permuterm index would better be called a permuterm [tree](#).
- But permuterm index is the more common name.

# Processing a lookup in the permuterm index

- Rotate query wildcard to the right
- Use B-tree lookup as before
- Problem: Permuterm more than **quadruples** the size of the dictionary compared to a regular B-tree. (empirical number)

# $k$ -gram indexes

- More space-efficient than permuterm index
- Enumerate all character  $k$ -grams (sequence of  $k$  characters) occurring in a term
- 2-grams are called **bigrams**.
- Example: from *April is the cruelest month* we get the bigrams:  
*\$a ap pr ri il l\$ \$i is s\$ \$t th he e\$ \$c cr ru ue el le es st t\$ \$m mo on nt th h\$*
- \$ is a special word boundary symbol, as before.
- Maintain an inverted index from bigrams to the terms that contain the bigram

# Postings list in a 3-gram inverted index



## $k$ -gram (bigram, trigram, ...) indexes

- Note that we now have two different types of inverted indexes
- The term-document inverted index for finding documents based on a query consisting of terms
- The  $k$ -gram index for finding terms based on a query consisting of  $k$ -grams

# Processing wildcarded terms in a bigram index

- Query `mon*` can now be run as:  
`$m AND mo AND on`
- Gets us all terms with the prefix *mon* . . .
- . . . but also many “false positives” like `MOON`.
- We must postfilter these terms against query.
- Surviving terms are then looked up in the term-document inverted index.
- *k*-gram index vs. permuterm index
  - *k*-gram index is more space efficient.
  - Permuterm index doesn't require postfiltering.

# Exercise

- Google has very limited support for wildcard queries.
- For example, this query doesn't work very well on Google: [gen\* universit\*]
  - Intention: you are looking for the University of Geneva, but don't know which accents to use for the French words for university and Geneva.
- According to Google search basics, 2010-04-29: "Note that the \* operator works only on whole words, not parts of words."
- But this is not entirely true. Try [pythag\*] and [m\*nchen]
- Exercise: Why doesn't Google fully support wildcard queries?



# Processing wildcard queries in the term-document index

- Problem 1: we must potentially execute a large number of Boolean queries.
- Most straightforward semantics: Conjunction of disjunctions
- For [gen\* universit\*]: geneva university OR geneva université OR genève university OR genève université OR general universities OR ...
- Very expensive
- Problem 2: Users hate to type.
- If abbreviated queries like [pyth\* theo\*] for [pythagoras' theorem] are allowed, users will use them a lot.
- This would significantly increase the cost of answering queries.
- Somewhat alleviated by Google Suggest

# Spelling correction

- Two principal uses
  - Correcting documents being indexed
  - Correcting user queries
- Two different methods for spelling correction
- **Isolated word** spelling correction
  - Check each word on its own for misspelling
  - Will not catch typos resulting in correctly spelled words, e.g.,  
*an asteroid that fell **form** the sky*
- **Context-sensitive** spelling correction
  - Look at surrounding words
  - Can correct *form/from* error above

# Correcting documents

- We are not interested in interactive spelling correction of documents (e.g., MS Word) in this class.
- In IR, we use document correction primarily for OCR'ed documents. (OCR = optical character recognition)
- The general philosophy in IR is: do **not** change the documents.

# Correcting queries

- First: isolated word spelling correction
- Premise 1: There is a list of “correct words” from which the correct spellings come.
- Premise 2: We have a way of computing the **distance** between a misspelled word and a correct word.
- Simple spelling correction algorithm: return the “correct” word that has the smallest distance to the misspelled word.
- Example: *informaton* → *information*
- For the list of correct words, we can use the vocabulary of all words that occur in our collection.
- **Why is this problematic?**

# Alternatives to using the term vocabulary

- A standard dictionary (Webster's, OED, etc.)
- An industry-specific dictionary (for specialized IR systems)
- The term vocabulary of the collection, appropriately weighted

# Distance between misspelled word and “correct” word

- We will study several alternatives.
- Edit distance and Levenshtein distance
- Weighted edit distance
- $k$ -gram overlap

# Edit distance

- The edit distance between string  $s_1$  and string  $s_2$  is the minimum number of basic operations that convert  $s_1$  to  $s_2$ .
- Levenshtein distance: The admissible basic operations are insert, delete, and replace
- Levenshtein distance *dog-do*: 1
- Levenshtein distance *cat-cart*: 1
- Levenshtein distance *cat-cut*: 1
- Levenshtein distance *cat-act*: 2
- Damerau-Levenshtein distance *cat-act*: 1
- Damerau-Levenshtein includes transposition as a fourth possible operation.

# Levenshtein distance: Computation

|   |   |   |   |   |   |
|---|---|---|---|---|---|
|   |   | f | a | s | t |
|   | 0 | 1 | 2 | 3 | 4 |
| c | 1 | 1 | 2 | 3 | 4 |
| a | 2 | 2 | 1 | 2 | 3 |
| t | 3 | 3 | 2 | 2 | 2 |
| s | 4 | 4 | 3 | 2 | 3 |



# Levenshtein distance: Algorithm

LEVENSHTEINDISTANCE( $s_1, s_2$ )

```
1  for  $i \leftarrow 0$  to  $|s_1|$ 
2  do  $m[i, 0] = i$ 
3  for  $j \leftarrow 0$  to  $|s_2|$ 
4  do  $m[0, j] = j$ 
5  for  $i \leftarrow 1$  to  $|s_1|$ 
6  do for  $j \leftarrow 1$  to  $|s_2|$ 
7      do if  $s_1[i] = s_2[j]$ 
8          then  $m[i, j] = \min\{m[i-1, j]+1, m[i, j-1]+1, m[i-1, j-1]\}$ 
9          else  $m[i, j] = \min\{m[i-1, j]+1, m[i, j-1]+1, m[i-1, j-1]+1\}$ 
10 return  $m[|s_1|, |s_2|]$ 
```

Operations: insert (cost 1), delete (cost 1), replace (cost 1), copy (cost 0)

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```

Operations: insert, delete, replace, copy

## Levenshtein distance: Example

|   |  | f                    | a                        | s                        | t                        |                          |
|---|--|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   |  | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| c |  | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 5</u><br><u>4 4</u> |
| a |  | <u>2</u><br><u>2</u> | <u>2 2</u><br><u>3 2</u> | <u>1 3</u><br><u>3 1</u> | <u>3 4</u><br><u>2 2</u> | <u>4 5</u><br><u>3 3</u> |
| t |  | <u>3</u><br><u>3</u> | <u>3 3</u><br><u>4 3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>2 4</u><br><u>3 2</u> |
| s |  | <u>4</u><br><u>4</u> | <u>4 4</u><br><u>5 4</u> | <u>4 3</u><br><u>5 3</u> | <u>2 3</u><br><u>4 2</u> | <u>3 3</u><br><u>3 3</u> |



# Each cell of Levenshtein matrix

|  |   |
|--|---|
| cost of getting here from my upper left neighbor (copy or replace) | cost of getting here from my upper neighbor (delete)                            |
| cost of getting here from my left neighbor (insert)                | the minimum of the three possible “movements”; the cheapest way of getting here |

## Levenshtein distance: Example

|   |  | f                    | a                        | s                        | t                        |                          |
|---|--|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   |  | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| c |  | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 5</u><br><u>4 4</u> |
| a |  | <u>2</u><br><u>2</u> | <u>2 2</u><br><u>3 2</u> | <u>1 3</u><br><u>3 1</u> | <u>3 4</u><br><u>2 2</u> | <u>4 5</u><br><u>3 3</u> |
| t |  | <u>3</u><br><u>3</u> | <u>3 3</u><br><u>4 3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>2 4</u><br><u>3 2</u> |
| s |  | <u>4</u><br><u>4</u> | <u>4 4</u><br><u>5 4</u> | <u>4 3</u><br><u>5 3</u> | <u>2 3</u><br><u>4 2</u> | <u>3 3</u><br><u>3 3</u> |

# Dynamic programming (Cormen et al.)

- Optimal substructure: The optimal solution to the problem contains within it **subsolutions**, i.e., optimal solutions to subproblems.
- Overlapping subsolutions: The subsolutions overlap. These subsolutions are computed over and over again when computing the global optimal solution in a brute-force algorithm.
- Subproblem in the case of edit distance: what is the edit distance of two prefixes
- Overlapping subsolutions: We need most distances of prefixes 3 times – this corresponds to moving right, diagonally, down.

# Weighted edit distance

- As above, but weight of an operation depends on the characters involved.
- Meant to capture keyboard errors, e.g.,  $m$  more likely to be mistyped as  $n$  than as  $q$ .
- Therefore, replacing  $m$  by  $n$  is a smaller edit distance than by  $q$ .
- We now require a weight matrix as input.
- Modify dynamic programming to handle weights

# Using edit distance for spelling correction

- Given query, first enumerate all character sequences within a preset (possibly weighted) edit distance
- Intersect this set with our list of “correct” words
- Then suggest terms in the intersection to the user.
- → exercise in a few slides

# Exercise

- 1 Compute Levenshtein distance matrix for OSLO – SNOW
- 2 What are the Levenshtein editing operations that transform *cat* into *catcat*?

|   |                      | s          | n          | o          | w          |
|---|----------------------|------------|------------|------------|------------|
|   | <u>0</u>             | <u>1 1</u> | <u>2 2</u> | <u>3 3</u> | <u>4 4</u> |
| o | <u>1</u><br><u>1</u> |            |            |            |            |
| s | <u>2</u><br><u>2</u> |            |            |            |            |
| l | <u>3</u><br><u>3</u> |            |            |            |            |
| o | <u>4</u><br><u>4</u> |            |            |            |            |

|   |                      | s                        | n          | o          | w          |
|---|----------------------|--------------------------|------------|------------|------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u> | <u>3 3</u> | <u>4 4</u> |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 ?</u> |            |            |            |
| s | <u>2</u><br><u>2</u> |                          |            |            |            |
| l | <u>3</u><br><u>3</u> |                          |            |            |            |
| o | <u>4</u><br><u>4</u> |                          |            |            |            |



|   |                      | s                        | n          | o          | w          |
|---|----------------------|--------------------------|------------|------------|------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u> | <u>3 3</u> | <u>4 4</u> |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> |            |            |            |
| s | <u>2</u><br><u>2</u> |                          |            |            |            |
| l | <u>3</u><br><u>3</u> |                          |            |            |            |
| o | <u>4</u><br><u>4</u> |                          |            |            |            |

|   |                      | s                        | n                        | o          | w          |
|---|----------------------|--------------------------|--------------------------|------------|------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u> | <u>4 4</u> |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 ?</u> |            |            |
| s | <u>2</u><br><u>2</u> |                          |                          |            |            |
| l | <u>3</u><br><u>3</u> |                          |                          |            |            |
| o | <u>4</u><br><u>4</u> |                          |                          |            |            |

|   |                      | s                        | n                        | o          | w          |
|---|----------------------|--------------------------|--------------------------|------------|------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u> | <u>4 4</u> |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> |            |            |
| s | <u>2</u><br><u>2</u> |                          |                          |            |            |
| l | <u>3</u><br><u>3</u> |                          |                          |            |            |
| o | <u>4</u><br><u>4</u> |                          |                          |            |            |

|   |                      | s                        | n                        | o                        | w          |
|---|----------------------|--------------------------|--------------------------|--------------------------|------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u> |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 ?</u> |            |
| s | <u>2</u><br><u>2</u> |                          |                          |                          |            |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |            |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |            |

|   |                      | s                        | n                        | o                        | w          |
|---|----------------------|--------------------------|--------------------------|--------------------------|------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u> |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> |            |
| s | <u>2</u><br><u>2</u> |                          |                          |                          |            |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |            |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |            |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 ?</u> |
| s | <u>2</u><br><u>2</u> |                          |                          |                          |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> |                          |                          |                          |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 ?</u> |                          |                          |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |



|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> |                          |                          |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 ?</u> |                          |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> |                          |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 ?</u> |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> |                          |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 ?</u> |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> |                          |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 ?</u> |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |



|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> |                          |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 ?</u> |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> |                          |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 ?</u> |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> |                          |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 ?</u> |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> |                          |                          |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 ?</u> |                          |                          |                          |



|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> |                          |                          |                          |

|   |  | s        |          | n        |          | o        |          | w        |          |          |
|---|--|----------|----------|----------|----------|----------|----------|----------|----------|----------|
|   |  | <u>0</u> | <u>1</u> | <u>1</u> | <u>2</u> | <u>2</u> | <u>3</u> | <u>3</u> | <u>4</u> | <u>4</u> |
| o |  | <u>1</u> | <u>1</u> | <u>2</u> | <u>2</u> | <u>3</u> | <u>2</u> | <u>4</u> | <u>4</u> | <u>5</u> |
|   |  | <u>1</u> | <u>2</u> | <u>1</u> | <u>2</u> | <u>2</u> | <u>3</u> | <u>2</u> | <u>3</u> | <u>3</u> |
| s |  | <u>2</u> | <u>1</u> | <u>2</u> | <u>2</u> | <u>3</u> | <u>3</u> | <u>3</u> | <u>3</u> | <u>4</u> |
|   |  | <u>2</u> | <u>3</u> | <u>1</u> | <u>2</u> | <u>2</u> | <u>3</u> | <u>3</u> | <u>4</u> | <u>3</u> |
| l |  | <u>3</u> | <u>3</u> | <u>2</u> | <u>2</u> | <u>3</u> | <u>4</u> | <u>4</u> | <u>4</u> | <u>4</u> |
|   |  | <u>3</u> | <u>4</u> | <u>2</u> | <u>3</u> | <u>2</u> | <u>3</u> | <u>3</u> | <u>4</u> | <u>4</u> |
| o |  | <u>4</u> | <u>4</u> | <u>3</u> | <u>3</u> |          |          |          |          |          |
|   |  | <u>4</u> | <u>5</u> | <u>3</u> | <u>4</u> | <u>3</u> |          |          |          |          |
|   |  |          |          | <u>3</u> | <u>?</u> |          |          |          |          |          |
|   |  |          |          |          |          |          |          |          |          |          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> |                          |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 ?</u> |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 2</u> |                          |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 2</u> | <u>4 5</u><br><u>3 ?</u> |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 2</u> | <u>4 5</u><br><u>3 3</u> |

|   |                   | s                              | n                              | o                              | w  |
|---|-------------------|--------------------------------|--------------------------------|--------------------------------|--|
|   | $\frac{\quad}{0}$ | $\frac{1}{1}$                  | $\frac{2}{2}$                  | $\frac{3}{3}$                  | $\frac{4}{4}$                              |
| o | $\frac{1}{1}$     | $\frac{1}{2}$<br>$\frac{2}{1}$ | $\frac{2}{2}$<br>$\frac{3}{2}$ | $\frac{2}{3}$<br>$\frac{4}{2}$ | $\frac{4}{3}$<br>$\frac{5}{3}$             |
| s | $\frac{2}{2}$     | $\frac{1}{3}$<br>$\frac{2}{1}$ | $\frac{2}{2}$<br>$\frac{3}{2}$ | $\frac{3}{3}$<br>$\frac{3}{3}$ | $\frac{3}{4}$<br>$\frac{4}{3}$             |
| l | $\frac{3}{3}$     | $\frac{3}{4}$<br>$\frac{2}{2}$ | $\frac{2}{3}$<br>$\frac{3}{2}$ | $\frac{3}{3}$<br>$\frac{4}{3}$ | $\frac{4}{4}$<br>$\frac{4}{4}$             |
| o | $\frac{4}{4}$     | $\frac{4}{5}$<br>$\frac{3}{3}$ | $\frac{3}{4}$<br>$\frac{3}{3}$ | $\frac{2}{4}$<br>$\frac{4}{2}$ | $\frac{4}{3}$<br>$\frac{5}{3}$<br><b>3</b> |



|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 2</u> | <u>4 5</u><br><u>3 3</u> |

How do I read out the editing operations that transform OSLO into SNOW?

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 2</u> | <u>4 5</u><br><u>3 3</u> |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 1    | insert    | *     | w      |

|   |        | s          | n          | o          | w          |
|---|--------|------------|------------|------------|------------|
|   | 0      | 1 1        | 2 2        | 3 3        | 4 4        |
| o | 1<br>1 | 1 2<br>2 1 | 2 3<br>2 2 | 2 4<br>3 2 | 4 5<br>3 3 |
| s | 2<br>2 | 1 2<br>3 1 | 2 3<br>2 2 | 3 3<br>3 3 | 3 4<br>4 3 |
| l | 3<br>3 | 3 2<br>4 2 | 2 3<br>3 2 | 3 4<br>3 3 | 4 4<br>4 4 |
| o | 4<br>4 | 4 3<br>5 3 | 3 3<br>4 3 | 2 4<br>4 2 | 4 5<br>3 3 |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 0    | (copy)    | o     | o      |
| 1    | insert    | *     | w      |

|   |                      | s                        | n                        | o                        | w                        |
|---|----------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>             | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1</u><br><u>1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2</u><br><u>2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3</u><br><u>3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4</u><br><u>4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 2</u> | <u>4 5</u><br><u>3 3</u> |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 1    | replace   | l     | n      |
| 0    | (copy)    | o     | o      |
| 1    | insert    | *     | w      |

|   |            | s                        | n                        | o                        | w                        |
|---|------------|--------------------------|--------------------------|--------------------------|--------------------------|
|   | <u>0</u>   | <u>1 1</u>               | <u>2 2</u>               | <u>3 3</u>               | <u>4 4</u>               |
| o | <u>1 1</u> | <u>1 2</u><br><u>2 1</u> | <u>2 3</u><br><u>2 2</u> | <u>2 4</u><br><u>3 2</u> | <u>4 5</u><br><u>3 3</u> |
| s | <u>2 2</u> | <u>1 2</u><br><u>3 1</u> | <u>2 3</u><br><u>2 2</u> | <u>3 3</u><br><u>3 3</u> | <u>3 4</u><br><u>4 3</u> |
| l | <u>3 3</u> | <u>3 2</u><br><u>4 2</u> | <u>2 3</u><br><u>3 2</u> | <u>3 4</u><br><u>3 3</u> | <u>4 4</u><br><u>4 4</u> |
| o | <u>4 4</u> | <u>4 3</u><br><u>5 3</u> | <u>3 3</u><br><u>4 3</u> | <u>2 4</u><br><u>4 2</u> | <u>4 5</u><br><u>3 3</u> |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 0    | (copy)    | s     | s      |
| 1    | replace   | l     | n      |
| 0    | (copy)    | o     | o      |
| 1    | insert    | *     | w      |

|   |     | s          | n          | o          | w          |
|---|-----|------------|------------|------------|------------|
|   | 0   | 1 1        | 2 2        | 3 3        | 4 4        |
| o | 1 1 | 1 2<br>2 1 | 2 3<br>2 2 | 2 4<br>3 2 | 4 5<br>3 3 |
| s | 2 2 | 1 2<br>3 1 | 2 3<br>2 2 | 3 3<br>3 3 | 3 4<br>4 3 |
| l | 3 3 | 3 2<br>4 2 | 2 3<br>3 2 | 3 4<br>3 3 | 4 4<br>4 4 |
| o | 4 4 | 4 3<br>5 3 | 3 3<br>4 3 | 2 4<br>4 2 | 4 5<br>3 3 |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 1    | delete    | o     | *      |
| 0    | (copy)    | s     | s      |
| 1    | replace   | l     | n      |
| 0    | (copy)    | o     | o      |
| 1    | insert    | *     | w      |

|   |     | c          | a          | t          | c          | a          | t          |
|---|-----|------------|------------|------------|------------|------------|------------|
|   | 0   | 1 1        | 2 2        | 3 3        | 4 4        | 5 5        | 6 6        |
| c | 1 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 | 5 6<br>4 4 | 6 7<br>5 5 |
| a | 2 2 | 2 1<br>3 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 | 5 6<br>4 4 |
| t | 3 3 | 3 2<br>4 2 | 2 1<br>3 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 |

|   |     |            |            |            |            |            |            |
|---|-----|------------|------------|------------|------------|------------|------------|
|   |     | c          | a          | t          | c          | a          | t          |
|   | 0   | 1 1        | 2 2        | 3 3        | 4 4        | 5 5        | 6 6        |
| c | 1 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 | 5 6<br>4 4 | 6 7<br>5 5 |
| a | 2 2 | 2 1<br>3 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 | 5 6<br>4 4 |
| t | 3 3 | 3 2<br>4 2 | 2 1<br>3 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 1    | insert    | *     | c      |
| 1    | insert    | *     | a      |
| 1    | insert    | *     | t      |
| 0    | (copy)    | c     | c      |
| 0    | (copy)    | a     | a      |
| 0    | (copy)    | t     | t      |



|   |        |            |            |            |            |            |            |
|---|--------|------------|------------|------------|------------|------------|------------|
|   |        | c          | a          | t          | c          | a          | t          |
|   | 0      | 1 1        | 2 2        | 3 3        | 4 4        | 5 5        | 6 6        |
| c | 1<br>1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 | 5 6<br>4 4 | 6 7<br>5 5 |
| a | 2<br>2 | 2 1<br>3 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 | 5 6<br>4 4 |
| t | 3<br>3 | 3 2<br>4 2 | 2 1<br>3 1 | 0 2<br>2 0 | 2 3<br>1 1 | 3 4<br>2 2 | 3 5<br>3 3 |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 0    | (copy)    | c     | c      |
| 1    | insert    | *     | a      |
| 1    | insert    | *     | t      |
| 1    | insert    | *     | c      |
| 0    | (copy)    | a     | a      |
| 0    | (copy)    | t     | t      |

|   |        |                |                |              |              |              |              |
|---|--------|----------------|----------------|--------------|--------------|--------------|--------------|
|   |        | c              | a              | t            | c            | a            | t            |
|   | 0      | 1 1            | 2 2            | 3 3          | 4 4          | 5 5          | 6 6          |
| c | 1<br>1 | 0   2<br>2   0 | 2 3<br>1 1     | 3 4<br>2 2   | 3 5<br>3 3   | 5 6<br>4 4   | 6 7<br>5 5   |
| a | 2<br>2 | 2 1<br>3 1     | 0   2<br>2   0 | 2 3<br>1   1 | 3 4<br>2   2 | 3 5<br>3   3 | 5 6<br>4 4   |
| t | 3<br>3 | 3 2<br>4 2     | 2 1<br>3 1     | 0 2<br>2 0   | 2 3<br>1 1   | 3 4<br>2 2   | 3 5<br>3   3 |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 0    | (copy)    | c     | c      |
| 0    | (copy)    | a     | a      |
| 1    | insert    | *     | t      |
| 1    | insert    | *     | c      |
| 1    | insert    | *     | a      |
| 0    | (copy)    | t     | t      |

|   |   |     |     |     |     |     |     |     |
|---|---|-----|-----|-----|-----|-----|-----|-----|
|   |   |     | c   | a   | t   | c   | a   | t   |
|   |   | 0   | 1 1 | 2 2 | 3 3 | 4 4 | 5 5 | 6 6 |
| c | 1 | 0 2 | 2 3 | 3 4 | 3 5 | 5 6 | 6 7 |     |
|   | 1 | 2 0 | 1 1 | 2 2 | 3 3 | 4 4 | 5 5 |     |
| a | 2 | 2 1 | 0 2 | 2 3 | 3 4 | 3 5 | 5 6 |     |
|   | 2 | 3 1 | 2 0 | 1 1 | 2 2 | 3 3 | 4 4 |     |
| t | 3 | 3 2 | 2 1 | 0 2 | 2 3 | 3 4 | 3 5 |     |
|   | 3 | 4 2 | 3 1 | 2 0 | 1 1 | 2 2 | 3 3 |     |

| cost | operation | input | output |
|------|-----------|-------|--------|
| 0    | (copy)    | c     | c      |
| 0    | (copy)    | a     | a      |
| 0    | (copy)    | t     | t      |
| 1    | insert    | *     | c      |
| 1    | insert    | *     | a      |
| 1    | insert    | *     | t      |

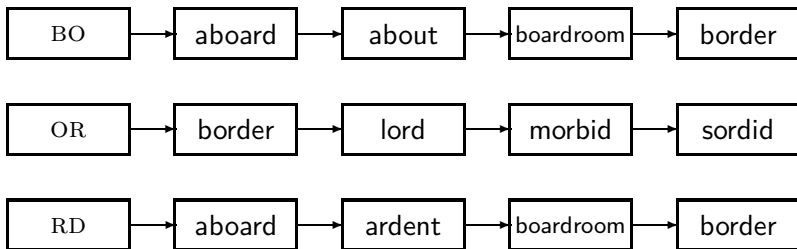
# Spelling correction

- Now that we can compute edit distance: how to use it for isolated word spelling correction – this is the last slide in this section.
- $k$ -gram indexes for isolated word spelling correction.
- Context-sensitive spelling correction
- General issues

# $k$ -gram indexes for spelling correction

- Enumerate all  $k$ -grams in the query term
- Example: bigram index, misspelled word *bordroom*
- Bigrams: *bo, or, rd, dr, ro, oo, om*
- Use the  $k$ -gram index to retrieve “correct” words that match query term  $k$ -grams
- Threshold by number of matching  $k$ -grams
- E.g., only vocabulary terms that differ by at most 3  $k$ -grams

# $k$ -gram indexes for spelling correction: *boardroom*



# Context-sensitive spelling correction

- Our example was: *an asteroid that fell **form** the sky*
- How can we correct *form* here?
- One idea: **hit-based** spelling correction
  - Retrieve “correct” terms close to each query term
  - for *flew form munich*: *flea* for *flew*, *from* for *form*, *munch* for *munich*
  - Now try all possible resulting phrases as queries with one word “fixed” at a time
  - Try query “*flea form munich*”
  - Try query “*flew from munich*”
  - Try query “*flew form munch*”
  - The correct query “*flew from munich*” has the most hits.
- Suppose we have 7 alternatives for *flew*, 20 for *form* and 3 for *munich*, how many “corrected” phrases will we enumerate?

# Context-sensitive spelling correction

- The “hit-based” algorithm we just outlined is not very efficient.
- More efficient alternative: look at “collection” of queries, not documents.



# General issues in spelling correction

- User interface
  - automatic vs. suggested correction
  - *Did you mean* only works for one suggestion.
  - What about multiple possible corrections?
  - Tradeoff: simple vs. powerful UI
- Cost
  - Spelling correction is potentially expensive.
  - Avoid running on every query?
  - Maybe just on queries that match few documents.
  - Guess: Spelling correction of major search engines is efficient enough to be run on every query.

# Exercise: Understand Peter Norvig's spelling corrector

```
import re, collections
def words(text): return re.findall('[a-z]+', text.lower())
def train(features):
    model = collections.defaultdict(lambda: 1)
    for f in features:
        model[f] += 1
    return model
NWORDS = train(words(file('big.txt').read()))
alphabet = 'abcdefghijklmnopqrstuvwxyz'
def edits1(word):
    splits      = [(word[:i], word[i:]) for i in range(len(word) + 1)]
    deletes     = [a + b[1:] for a, b in splits if b]
    transposes  = [a + b[1] + b[0] + b[2:] for a, b in splits if len(b) > 1]
    replaces    = [a + c + b[1:] for a, b in splits for c in alphabet if b]
    inserts     = [a + c + b      for a, b in splits for c in alphabet]
    return set(deletes + transposes + replaces + inserts)
def known_edits2(word):
    return set(e2 for e1 in edits1(word) for e2 in edits1(e1) if e2 in NWORDS)
def known(words): return set(w for w in words if w in NWORDS)
def correct(word):
    candidates = known([word]) or known(edits1(word)) or known_edits2(word) or [word]
    return max(candidates, key=NWORDS.get)
```

```
import re, collections

def words(text): return re.findall('[a-z]+', text.lower())

def train(features):
    model = collections.defaultdict(lambda: 1)
    for f in features:
        model[f] += 1
    return model

NWORDS = train(words(file('big.txt').read()))

alphabet = 'abcdefghijklmnopqrstuvwxyz'

def edits1(word):
    n = len(word)
    return set([word[0:i]+word[i+1:] for i in range(n)] + # deletion
               [word[0:i]+word[i+1]+word[i]+word[i+2:] for i in range(n-1)] + # transposition
               [word[0:i]+c+word[i+1:] for i in range(n) for c in alphabet] + # alteration
               [word[0:i]+c+word[i:] for i in range(n+1) for c in alphabet]) # insertion

def known_edits2(word):
    return set(e2 for e1 in edits1(word) for e2 in edits1(e1) if e2 in NWORDS)

def known(words): return set(w for w in words if w in NWORDS)

def correct(word):
    candidates = known([word]) or known(edits1(word)) or known_edits2(word) or [word]
    return max(candidates, key=lambda w: NWORDS[w])
```

<https://norvig.com/spell-correct.html>

# Soundex

- Soundex is the basis for finding **phonetic** (as opposed to orthographic) alternatives.
- Example: *chebyshev* / *tchebyscheff*
- Algorithm:
  - Turn every token to be indexed into a 4-character reduced form
  - Do the same with query terms
  - Build and search an index on the reduced forms

# Soundex algorithm

- 1 Retain the first letter of the term.
- 2 Change all occurrences of the following letters to '0' (zero): A, E, I, O, U, H, W, Y
- 3 Change letters to digits as follows:
  - B, F, P, V to 1
  - C, G, J, K, Q, S, X, Z to 2
  - D, T to 3
  - L to 4
  - M, N to 5
  - R to 6
- 4 Repeatedly remove one out of each pair of consecutive identical digits
- 5 Remove all zeros from the resulting string; pad the resulting string with trailing zeros and return the first four positions, which will consist of a letter followed by three digits

## Example: Soundex of *HERMAN*

- Retain H
- *ERMAN* → *ORMON*
- *ORMON* → *06505*
- *06505* → *06505*
- *06505* → *655*
- Return *H655*
- Note: *HERMANN* will generate the same code

# How useful is Soundex?

- *Not very – for information retrieval, obsolete*
- Ok for “high recall” tasks in other applications (e.g., Interpol).
- Zobel and Dart (1996) suggest better alternatives for phonetic matching in IR.

# Exercise

- Compute Soundex code of your last name



# Take-away

- **Tolerant retrieval:** What to do if there is no exact match between query term and document term
- Wildcard queries
- Spelling correction

# Resources

- Chapter 3 of IIR
- Resources at <https://www.fi.muni.cz/~sojka/PV211/> and <http://cis1mu.org>, materials in MU IS and FI MU library
  - trie vs hash vs ternary tree
  - Soundex demo
  - Edit distance demo
  - Peter Norvig's spelling corrector
  - Google: wild card search, spelling correction gone wrong, a misspelling that is more frequent than the correct spelling
  - NLP IR boolean search system example: Sketch Engine – <https://ske.fi.muni.cz>