PV211: Introduction to Information Retrieval

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IIR 14: Vector Space Classification
Handout version

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Overview

1. Intro vector space classification
2. Rocchio
3. kNN
4. Linear classifiers
5. > two classes

Take-away today

- Vector space classification: Basic idea of doing text classification for documents that are represented as vectors
- Rocchio classifier: Rocchio relevance feedback idea applied to text classification
- k nearest neighbor classification
- Linear classifiers
- More than two classes

Roadmap for today

- Naive Bayes is simple and a good baseline.
- Use it if you want to get a text classifier up and running in a hurry.
- But other classification methods are more accurate.
- Perhaps the simplest well performing alternative: kNN
- kNN is a vector space classifier.
- Plan for rest of today
  1. intro vector space classification
  2. very simple vector space classification: Rocchio
  3. kNN
  4. general properties of classifiers

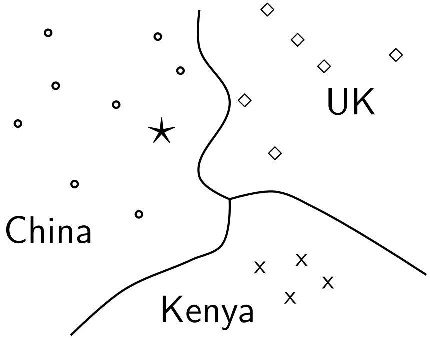
Recall vector space representation

- Each document is a vector, one component for each term.
- Terms are axes.
- High dimensionality: 100,000s of dimensions
- Normalize vectors (documents) to unit length
- How can we do classification in this space?

Vector space classification

- As before, the training set is a set of documents, each labeled with its class.
- In vector space classification, this set corresponds to a labeled set of points or vectors in the vector space.
- Premise 1: Documents in the same class form a contiguous region.
- Premise 2: Documents from different classes don't overlap.
- We define lines, surfaces, hyper-surfaces to divide regions.

Classes in the vector space



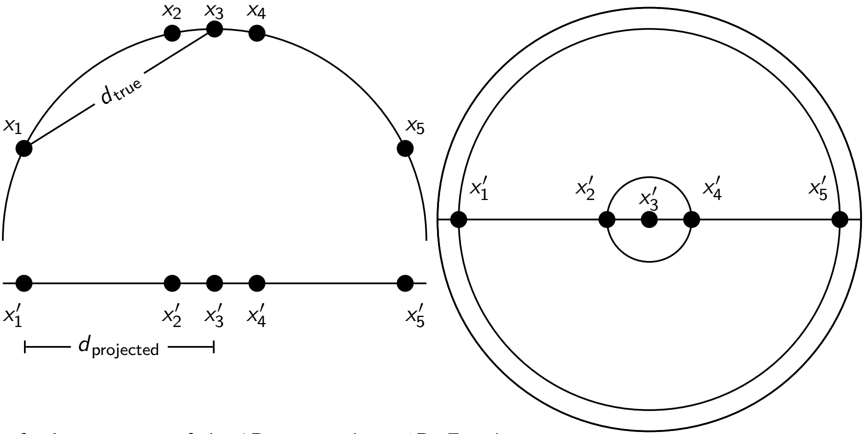
Should the document $\star$ be assigned to *China*, *UK* or *Kenya*?

Find separators between the classes

Based on these separators: $\star$ should be assigned to *China*

How do we find separators that do a good job at classifying new documents like $\star$? – Main topic of today

Aside: 2D/3D graphs can be misleading



Left: A projection of the 2D semicircle to 1D. For the points x_1, x_2, x_3, x_4, x_5 at x coordinates $-0.9, -0.2, 0, 0.2, 0.9$ the distance $|x_2x_3| \approx 0.201$ only differs by 0.5% from $|x'_2x'_3| = 0.2$; but $|x_1x_3|/|x'_1x'_3| = d_{\text{true}}/d_{\text{projected}} \approx 1.06/0.9 \approx 1.18$ is an example of a large distortion (18%) when projecting a large area. *Right:* The corresponding projection of the 3D hemisphere to 2D.

Relevance feedback

- In relevance feedback, the user marks documents as relevant/non-relevant.
- Relevant/non-relevant can be viewed as classes or categories.
- For each document, the user decides which of these two classes is correct.
- The IR system then uses these class assignments to build a better query ("model") of the information need ...
- ... and returns better documents.
- Relevance feedback is a form of text classification.
- The notion of text classification (TC) is very general and has many applications within and beyond information retrieval.

Using Rocchio for vector space classification

- The principal difference between relevance feedback and text classification:
  - The training set is given as part of the input in text classification.
  - It is interactively created in relevance feedback.

Rocchio classification: Basic idea

- Compute a centroid for each class
  - The centroid is the average of all documents in the class.
- Assign each test document to the class of its closest centroid.

Recall definition of centroid

$$\vec{\mu}(c) = \frac{1}{|D_c|} \sum_{d \in D_c} \vec{v}(d)$$

where D_c is the set of all documents that belong to class c and $\vec{v}(d)$ is the vector space representation of d.

Rocchio algorithm

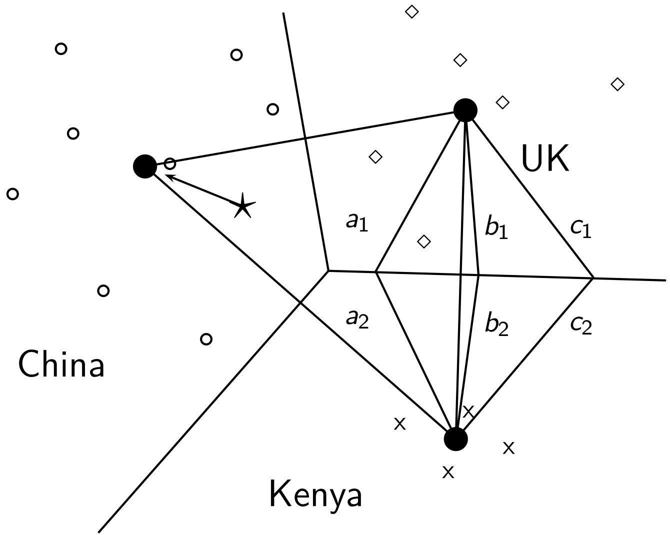
$\textsc{TrainRocchio}(\mathbb{C}, \mathbb{D})$

for each $c_j \in \mathbb{C}$
do $D_j \leftarrow \{d : \langle d, c_j \rangle \in \mathbb{D}\}$
$\quad \vec{\mu}_j \leftarrow \frac{1}{|D_j|} \sum_{d \in D_j} \vec{v}(d)$
return $\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}$

$\textsc{ApplyRocchio}(\{\vec{\mu}_1, \ldots, \vec{\mu}_J\}, d)$

return $\arg\min_j |\vec{\mu}_j - \vec{v}(d)|$

Rocchio illustrated: $a_1 = a_2, b_1 = b_2, c_1 = c_2$



Rocchio properties

- Rocchio forms a simple representation for each class: the centroid
  - We can interpret the centroid as the prototype of the class.
- Classification is based on similarity to / distance from centroid/prototype.
- Does not guarantee that classifications are consistent with the training data!

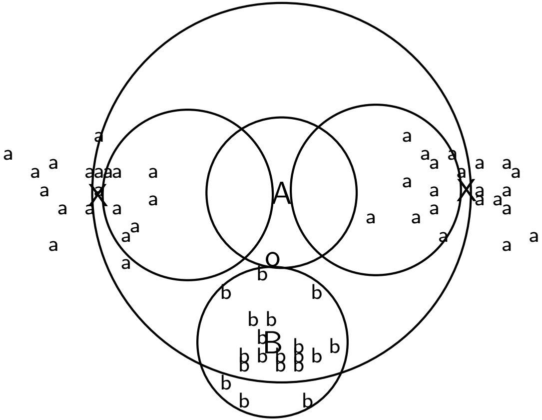
Time complexity of Rocchio

mode	time complexity
training	$\Theta(\lvert\mathbb{D}\rvert L_{\mathrm{ave}} + \lvert\mathbb{C}\rvert\lvert V\rvert) \approx \Theta(\lvert\mathbb{D}\rvert L_{\mathrm{ave}})$
testing	$\Theta(L_{\mathrm{a}} + \lvert\mathbb{C}\rvert M_{\mathrm{a}}) \approx \Theta(\lvert\mathbb{C}\rvert M_{\mathrm{a}})$

Rocchio vs. Naive Bayes

- In many cases, Rocchio performs worse than Naive Bayes.
- One reason: Rocchio does not handle nonconvex, multimodal classes correctly.

Rocchio cannot handle nonconvex, multimodal classes



Exercise: Why is Rocchio not expected to do well for the classification task a vs. b here?

- A is centroid of the a's, B is centroid of the b's.
- The point o is closer to A than to B.
- But o is a better fit for the b class.
- A is a multimodal class with two prototypes.
- But in Rocchio we only have one prototype.

kNN classification

- kNN classification is another vector space classification method.
- It also is very simple and easy to implement.
- kNN is more accurate (in most cases) than Naive Bayes and Rocchio.
- If you need to get a pretty accurate classifier up and running in a short time ...
- ... and you don't care about efficiency that much ...
- ... use kNN.

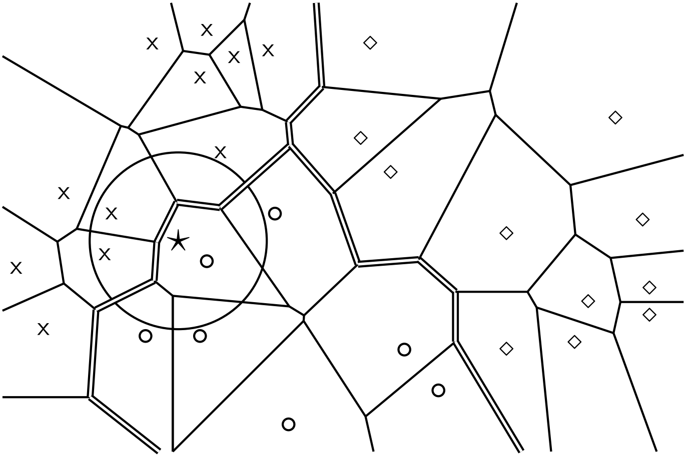
kNN classification

- kNN $= k$ nearest neighbors
- kNN classification rule for $k = 1$ (1NN): Assign each test document to the class of its nearest neighbor in the training set.
- 1NN is not very robust – one document can be mislabeled or atypical.
- kNN classification rule for $k > 1$ (kNN): Assign each test document to the majority class of its k nearest neighbors in the training set.
- Rationale of kNN: contiguity hypothesis
    - We expect a test document d to have the same label as the training documents located in the local region surrounding d.

Probabilistic kNN

- Probabilistic version of kNN: $P(c|d) =$ fraction of k neighbors of d that are in c
- kNN classification rule for probabilistic kNN: Assign d to class c with highest $P(c|d)$

kNN is based on Voronoi tessellation



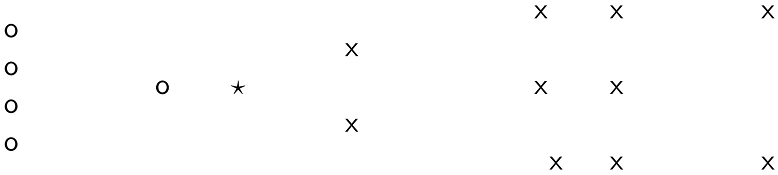
1NN, 3NN classification decision for star?

kNN algorithm

Train-kNN($\mathbb{C}, \mathbb{D}$)
$\mathbb{D}' \leftarrow$ Preprocess($\mathbb{D}$)
$k \leftarrow$ Select-k($\mathbb{C}, \mathbb{D}'$)
return $\mathbb{D}', k$

Apply-kNN($\mathbb{D}', k, d$)
$S_k \leftarrow$ ComputeNearestNeighbors($\mathbb{D}', k, d$)
for each $c_j \in \mathbb{C}(\mathbb{D}')$
do $p_j \leftarrow |S_k \cap c_j|/k$
return $\arg\max_j p_j$

Exercise



How is star classified by:
(i) 1-NN (ii) 3-NN (iii) 9-NN (iv) 15-NN (v) Rocchio?

Time complexity of kNN

kNN with preprocessing of training set

training $\Theta(|\mathbb{D}|L_{\text{ave}})$

testing $\Theta(L_{\text{a}} + |\mathbb{D}|M_{\text{ave}}M_{\text{a}}) = \Theta(|\mathbb{D}|M_{\text{ave}}M_{\text{a}})$

- kNN test time proportional to the size of the training set!
- The larger the training set, the longer it takes to classify a test document.
- kNN is inefficient for very large training sets.
- Question: Can we divide up the training set into regions, so that we only have to search in one region to do kNN classification for a given test document? (which perhaps would give us better than linear time complexity)

Curse of dimensionality

- Our intuitions about space are based on the 3D world we live in.
- Intuition 1: some things are close by, some things are distant.
- Intuition 2: we can carve up space into areas such that: within an area things are close, distances between areas are large.
- These two intuitions don't necessarily hold for high dimensions.
- In particular: for a set of k uniformly distributed points, let dmin be the smallest distance between any two points and dmax be the largest distance between any two points.
- Then

$$\lim_{d \to \infty} \frac{\text{dmax} - \text{dmin}}{\text{dmin}} = 0$$

Curse of dimensionality: Simulation

- Simulate
$$\lim_{d\to\infty} \frac{\text{dmax} - \text{dmin}}{\text{dmin}} = 0$$
- Pick a dimensionality d
- Generate 10 random points in the d-dimensional hypercube (uniform distribution)
- Compute all 45 distances
- Compute $\frac{\text{dmax}-\text{dmin}}{\text{dmin}}$
- We see that intuition 1 (some things are close, others are distant) is not true for high dimensions.

Intuition 2: Space can be carved up

- Intuition 2: we can carve up space into areas such that: within an area things are close, distances between areas are large.
- If this is true, then we have a simple and efficient algorithm for kNN.
- To find the k closest neighbors of data point $< x_1, x_2, \ldots, x_d >$ do the following.
- Using binary search find all data points whose first dimension is in $[x_1 - \epsilon, x_1 + \epsilon]$. This is $O(\log n)$ where n is the number of data points.
- Do this for each dimension, then intersect the d subsets.

Intuition 2: Space can be carved up

- Size of data set $n = 100$
- Again, assume uniform distribution in hypercube
- Set $\epsilon = 0.05$: we will look in an interval of length 0.1 for neighbors on each dimension.
- What is the probability that the nearest neighbor of a new data point $\vec{x}$ is in this neighborhood in $d = 1$ dimension?
- for $d = 1$: $1 - (1 - 0.1)^{100} \approx 0.99997$
- In $d = 2$ dimensions?
- for $d = 2$: $1 - (1 - 0.1^2)^{100} \approx 0.63$
- In $d = 3$ dimensions?
- for $d = 3$: $1 - (1 - 0.1^3)^{100} \approx 0.095$
- In $d = 4$ dimensions?
- for $d = 4$: $1 - (1 - 0.1^4)^{100} \approx 0.0095$
- In $d = 5$ dimensions?
- for $d = 5$: $1 - (1 - 0.1^5)^{100} \approx 0.0009995$

Intuition 2: Space can be carved up

- In $d = 5$ dimensions?
- for $d = 5$: $1 - (1 - 0.1^5)^{100} \approx 0.0009995$
- In other words: with enough dimensions, there is only one "local" region that will contain the nearest neighbor with high certainty: the entire search space.
- We cannot carve up high-dimensional space into neat neighborhoods . . .
- . . . unless the "true" dimensionality is much lower than d.

kNN: Discussion

- No training necessary
  - But linear preprocessing of documents is as expensive as training Naive Bayes.
  - We always preprocess the training set, so in reality training time of kNN is linear.
- kNN is very accurate if training set is large.
- Optimality result: asymptotically zero error if Bayes rate is zero.
- But kNN can be very inaccurate if training set is small.

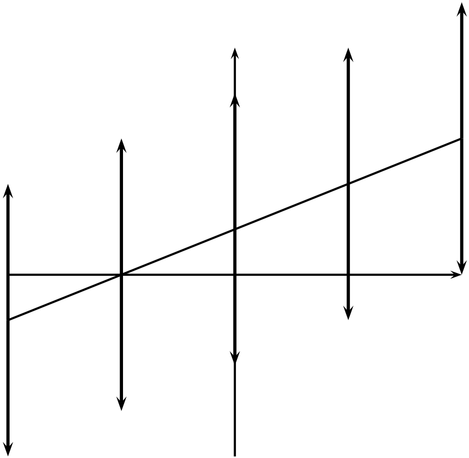
Linear classifiers

- Definition:
  - A linear classifier computes a linear combination or weighted sum $\sum_i w_i x_i$ of the feature values.
  - Classification decision: $\sum_i w_i x_i > \theta$?
  - ... where θ (the threshold) is a parameter.
- (First, we only consider binary classifiers.)
- Geometrically, this corresponds to a line (2D), a plane (3D) or a hyperplane (higher dimensionalities), the separator.
- We find this separator based on training set.
- Methods for finding separator: Perceptron, Rocchio, Naive Bayes – as we will explain on the next slides
- Assumption: The classes are linearly separable.

A linear classifier in 1D

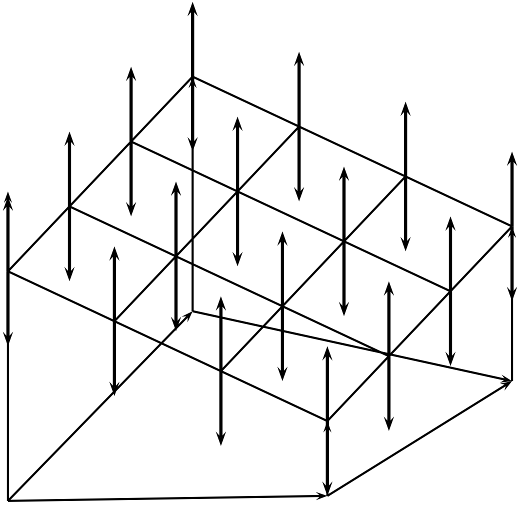
- A linear classifier in 1D is a point described by the equation $w_1 d_1 = \theta$
- The point at θ / w_1
- Points (d_1) with $w_1 d_1 \geq \theta$ are in the class c.
- Points (d_1) with $w_1 d_1 < \theta$ are in the complement class $\overline{c}$.

A linear classifier in 2D



- A linear classifier in 2D is a line described by the equation $w_1 d_1 + w_2 d_2 = \theta$
- Example for a 2D linear classifier
- Points $(d_1 \ d_2)$ with $w_1 d_1 + w_2 d_2 \geq \theta$ are in the class c.
- Points $(d_1 \ d_2)$ with $w_1 d_1 + w_2 d_2 < \theta$ are in the complement class $\overline{c}$.

A linear classifier in 3D



- A linear classifier in 3D is a plane described by the equation $w_1 d_1 + w_2 d_2 + w_3 d_3 = \theta$
- Example for a 3D linear classifier
- Points $(d_1 \; d_2 \; d_3)$ with $w_1 d_1 + w_2 d_2 + w_3 d_3 \geq \theta$ are in the class c.
- Points $(d_1 \; d_2 \; d_3)$ with $w_1 d_1 + w_2 d_2 + w_3 d_3 < \theta$ are in the complement class $\overline{c}$.

Rocchio as a linear classifier

- Rocchio is a linear classifier defined by:

$$\sum_{i=1}^{M} w_i d_i = \vec{w}\vec{d} = \theta$$

where $\vec{w}$ is the normal vector $\vec{\mu}(c_1) - \vec{\mu}(c_2)$ and $\theta = 0.5 * (|\vec{\mu}(c_1)|^2 - |\vec{\mu}(c_2)|^2)$.

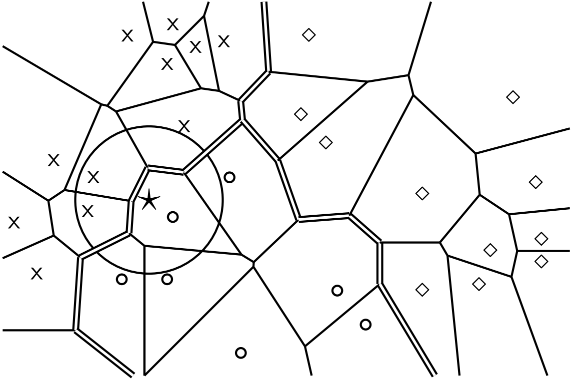
Naive Bayes as a linear classifier

Multinomial Naive Bayes is a linear classifier (in log space) defined by:

$$\sum_{i=1}^{M} w_i d_i = \theta$$

where $w_i = \log[\hat{P}(t_i|c)/\hat{P}(t_i|\bar{c})]$, d_i = number of occurrences of t_i in d, and $\theta = -\log[\hat{P}(c)/\hat{P}(\bar{c})]$. Here, the index i, $1 \leq i \leq M$, refers to terms of the vocabulary (not to positions in d as k did in our original definition of Naive Bayes)

kNN is not a linear classifier



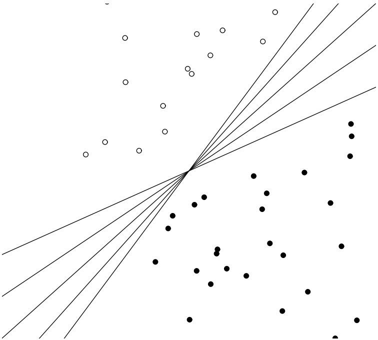
- Classification decision based on majority of k nearest neighbors.
- The decision boundaries between classes are piecewise linear . . .
- . . . but they are in general not linear classifiers that can be described as $\sum_{i=1}^{M} w_i d_i = \theta$.

Example of a linear two-class classifier

t_i	w_i	d_{1i}	d_{2i}	t_i	w_i	d_{1i}	d_{2i}
prime	0.70	0	1	dlrs	-0.71	1	1
rate	0.67	1	0	world	-0.35	1	0
interest	0.63	0	0	sees	-0.33	0	0
rates	0.60	0	0	year	-0.25	0	0
discount	0.46	1	0	group	-0.24	0	0
bundesbank	0.43	0	0	dlr	-0.24	0	0

- This is for the class *interest* in Reuters-21578.
- For simplicity: assume a simple 0/1 vector representation
- d_1: "rate discount dlrs world"
- d_2: "prime dlrs"
- $\theta = 0$
- Exercise: Which class is d_1 assigned to? Which class is d_2 assigned to?
- We assign document $\vec{d}_1$ "rate discount dlrs world" to *interest* since $\vec{w}^T\vec{d}_1 = 0.67 \cdot 1 + 0.46 \cdot 1 + (-0.71) \cdot 1 + (-0.35) \cdot 1 = 0.07 > 0 = \theta$.
- We assign $\vec{d}_2$ "prime dlrs" to the complement class (not in *interest*) since $\vec{w}^T\vec{d}_2 = -0.01 \leq \theta$.

Which hyperplane?



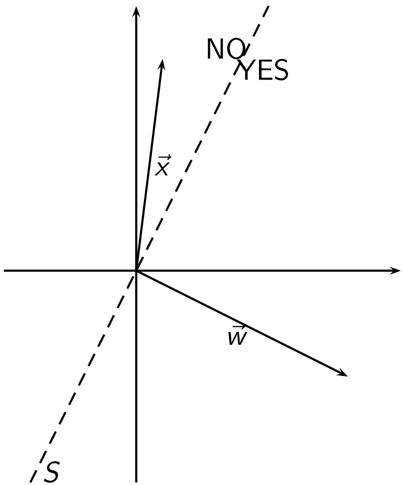
Learning algorithms for vector space classification

- In terms of actual computation, there are two types of learning algorithms.
- (i) Simple learning algorithms that estimate the parameters of the classifier directly from the training data, often in one linear pass.
  - Naive Bayes, Rocchio, kNN are all examples of this.
- (ii) Iterative algorithms
  - Support vector machines
  - Perceptron (example available as PDF on website: `http://cislmu.org`)
- The best performing learning algorithms usually require iterative learning.

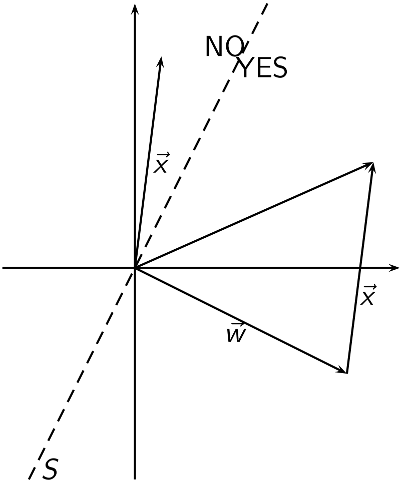
Perceptron update rule

- Randomly initialize linear separator $\vec{w}$
- Do until convergence:
  - Pick data point $\vec{x}$
  - If sign($\vec{w}^T\vec{x}$) is correct class (1 or -1): do nothing
  - Otherwise: $\vec{w} = \vec{w} - \text{sign}(\vec{w}^T\vec{x})\vec{x}$

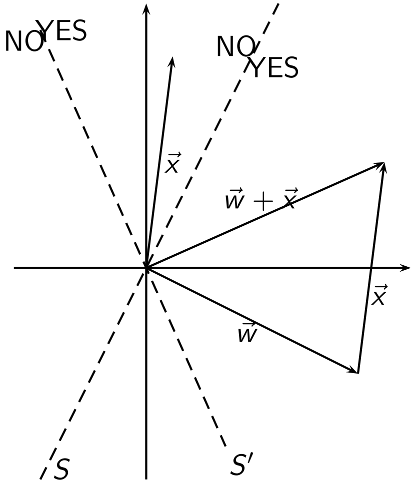
Perceptron



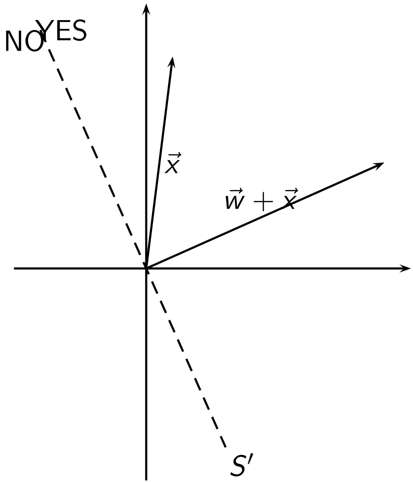
Perceptron



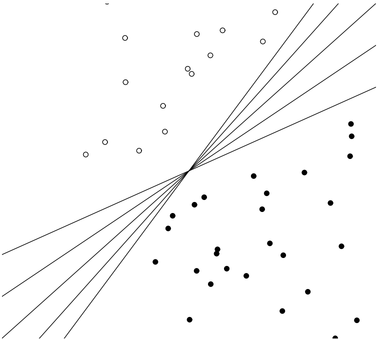
Perceptron



Perceptron



Which hyperplane?



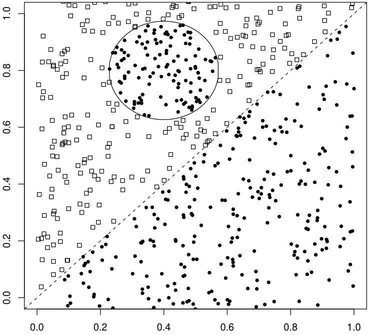
Which hyperplane?

- For linearly separable training sets: there are infinitely many separating hyperplanes.
- They all separate the training set perfectly . . .
- . . . but they behave differently on test data.
- Error rates on new data are low for some, high for others.
- How do we find a low-error separator?
- Perceptron: generally bad; Naive Bayes, Rocchio: ok; linear SVM: good

Linear classifiers: Discussion

- Many common text classifiers are linear classifiers: Naive Bayes, Rocchio, logistic regression, linear support vector machines, etc.
- Each method has a different way of selecting the separating hyperplane
  - Huge differences in performance on test documents
- Can we get better performance with more powerful nonlinear classifiers?
- Not in general: A given amount of training data may suffice for estimating a linear boundary, but not for estimating a more complex nonlinear boundary.

A nonlinear problem

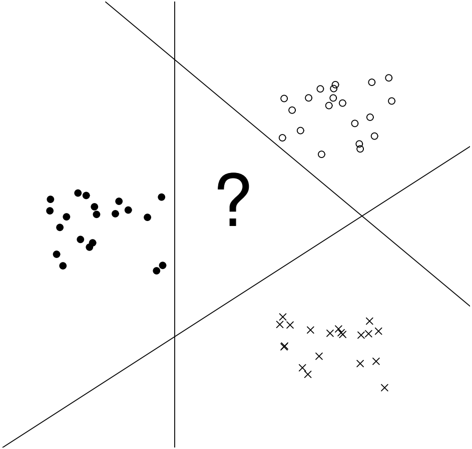


- Linear classifier like Rocchio does badly on this task.
- kNN will do well (assuming enough training data)

Which classifier do I use for a given TC problem?

- Is there a learning method that is optimal for all text classification problems?
- No, because there is a trade-off between bias and variance.
- Factors to take into account:
  - How much training data is available?
  - How simple/complex is the problem? (linear vs. nonlinear decision boundary)
  - How noisy is the problem?
  - How stable is the problem over time?
    - For an unstable problem, it's better to use a simple and robust classifier.

How to combine hyperplanes for > 2 classes?



One-of problems

- One-of or multiclass classification
  - Classes are mutually exclusive.
  - Each document belongs to exactly one class.
  - Example: language of a document (assumption: no document contains multiple languages)

One-of classification with linear classifiers

- Combine two-class linear classifiers as follows for one-of classification:
  - Run each classifier separately
  - Rank classifiers (e.g., according to score)
  - Pick the class with the highest score

Any-of problems

- Any-of or multilabel classification
  - A document can be a member of 0, 1, or many classes.
  - A decision on one class leaves decisions open on all other classes.
  - A type of "independence" (but not statistical independence)
  - Example: topic classification
  - Usually: make decisions on the region, on the subject area, on the industry and so on "independently"

Any-of classification with linear classifiers

- Combine two-class linear classifiers as follows for any-of classification:
  - Simply run each two-class classifier separately on the test document and assign document accordingly

Take-away today

- Vector space classification: Basic idea of doing text classification for documents that are represented as vectors
- Rocchio classifier: Rocchio relevance feedback idea applied to text classification
- k nearest neighbor classification
- Linear classifiers
- More than two classes

Resources

- Chapter 13 of IIR (feature selection)
- Chapter 14 of IIR
- Resources at `http://cislmu.org`
  - Perceptron example
  - General overview of text classification: Sebastiani (2002)
  - Text classification chapter on decision trees and perceptrons: Manning & Schütze (1999)
  - One of the best machine learning textbooks: Hastie, Tibshirani & Friedman (2003)