

Evaluation in Information Retrieval (Chapter 8)

Exercise 8/1

The following ordered list of 20 letters R and N represents relevant (R) and non-relevant (N) retrieved documents as an answer for a query on a collection of 10 000 documents.

The leftmost document is expected to be the most relevant. The list contains 6 relevant documents. Assume that the collection contains 8 documents relevant to the query.

IR(Q) -> $\overline{R}RNNNNNNRNRNNNNRNNNR$
 1 2 9 11 15 20

- Precision? Recall?
- F-measure?
- MAP?
- Best possible MAP?
- Worst possible MAP?

$$P = \frac{6}{20} = \frac{3}{10} = 0,3 \quad (30\%)$$

$$R = \frac{6}{8} = \frac{3}{4} = 0,75 \quad (75\%)$$

$$F = \frac{2 \cdot R \cdot P}{R + P} = \frac{2 \cdot 0,75 \cdot 0,3}{0,3 + 0,75} = \dots = \frac{3}{7} \approx 43\%$$

$$MAP = \frac{1}{7} \cdot \left[\frac{1}{8} \cdot \left(\frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \frac{4}{4} + \frac{5}{5} + \frac{6}{6} + 0 + 0 \right) \right] = \dots = 0,4163$$

$$RRNNNNNNRNNNNRNNNR \overset{10000}{RRN..N} \overset{2x}{R} \overset{10000-22}{R} \overset{7}{R} \overset{N}{N}$$

$$MAP = \frac{1}{7} \cdot \left[\frac{1}{8} \cdot \left(1 + 1 + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8} \right) \right] = \dots = 0,5074$$

$$RRNNNNNNRNNNNRNNNR \overset{9999}{N..NRR} \overset{10000-22}{R} \overset{7}{R} \overset{N}{N}$$

$$MAP = \frac{1}{7} \cdot \left[\frac{1}{8} \cdot \left(1 + 1 + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{9999} + \frac{8}{10000} \right) \right] = \dots = 0,4165$$

Definition 1 (Recall)
 Recall describes how many of the relevant documents are retrieved.

$$recall = R = \frac{\#relevant\ retrieved}{\#relevant}$$

Definition 2 (Precision)
 Precision describes how many of the retrieved documents are relevant.

$$precision = P = \frac{\#relevant\ retrieved}{\#retrieved}$$

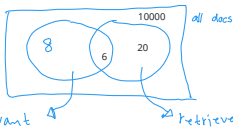
Definition 3 (F-measure)
 A balanced F-measure (F₁-measure) defines a recall-precision relationship represented by their weighted harmonic mean:

$$F = \frac{2 \cdot R \cdot P}{R + P}$$

Definition 4 (Mean Average Precision)
 MAP expresses the precision in each point a new relevant document is included in the result. It is counted as

$$MAP(Q) = \frac{1}{|Q|} \cdot \left(\sum_{i=1}^{rel_Q} \frac{1}{prec_i} \right)$$

where rel_Q is the number of relevant documents for query Q and prec_i is the precision at the i-th relevant document.



Exercise 8/4

Below is a table showing how two judges judged the relevance (0 = non-relevant, 1 = relevant) of the set of 12 documents with respect to a query. Assume that you developed an IR system, that for this query returns the documents {4, 5, 6, 7, 8}.

IR(Q) -> {4, 5, 6, 7, 8}

- Calculate the κ statistic.
- Precision, if judges agree -> {1, 2, 3, 4}
- Recall, if at least one judge thinks so

$$P = \frac{1}{5}$$

$$R = \frac{5}{10} = \frac{1}{2}$$

Definition 5 (κ statistic)
 Let N be the total number of documents, J is a set of judges and P(A) = $\frac{\#agree}{N}$ the number of documents on which the judges agree. Let also define R_j and NR_j be the number of relevant and non-relevant documents, respectively, according to the judge j ∈ J and

$$P(R_j) = \frac{\sum_{i \in J} R_i}{|J| \cdot N} \quad \text{and} \quad P(NR_j) = \frac{\sum_{i \in J} NR_i}{|J| \cdot N}$$

as the number of relevant and non-relevant documents, respectively. Let finally define

$$P(E) = P(R)^2 + P(NR)^2$$

as the approximate number of disagreements between the judges. Then the κ statistic is defined as the measure of agreement between the judges

$$\kappa = \frac{P(A) - P(E)}{1 - P(E)}$$

Doc ID	Judge 1	Judge 2
1	0	0
2	0	0
3	1	1
4	1	1
5	1	0
6	1	0
7	1	0
8	1	0
9	0	1
10	0	1
11	0	1
12	0	1

Table 1: Judges judging the relevance of documents.

agreement

$$P(A) = \frac{4}{12} = \frac{1}{3}$$

$$P(R) = \frac{6+6}{2 \cdot 12} = \frac{1}{2}$$

$$P(NR) = \frac{6+6}{2 \cdot 12} = \frac{1}{2}$$

$$P(E) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 2 \cdot \left(\frac{1}{2}\right)^2 = 2 \cdot \frac{1}{4} = \frac{1}{2}$$

$$\kappa = \frac{1/3 - 1/2}{1 - 1/2} = \frac{-1/6}{1/2} = -\frac{1}{3}$$

$$= -\frac{1}{3}$$

they more disagree than agree