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Statistical complexity of software systems represented as multi-layer networks

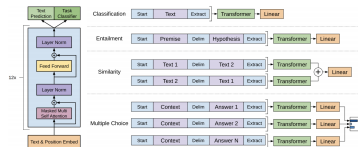
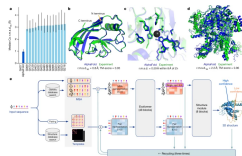
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April 4, 2024

Can we measure a complexity of a Software System?

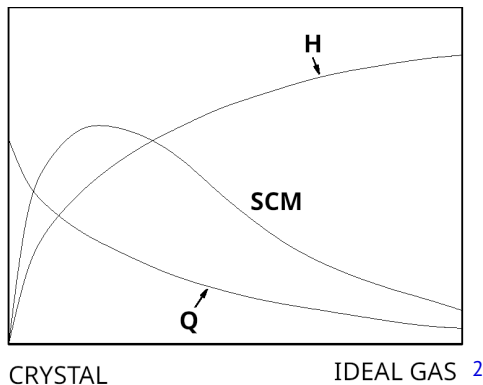
- To compare software systems
- To analyze change of properties on complexity
- To predict emergent behaviors
- To design complex software systems



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¹ <https://www.nature.com/articles/s41586-021-03819-2>, <https://www.youtube.com/watch?v=WXuK6gekU1Y>, <https://paperswithcode.com/method/gpt>

Intuitive notion of magnitude of "complexity"



² Sketch of the intuitive notion of the magnitudes of "information" (H) and "disequilibrium" (Q) for the physical systems and the behavior intuitively required for the magnitude of "complexity." The quantity $SCM = H \cdot Q$ is proposed to measure such a magnitude. [LopezRui1995]

Statistical Complexity Measure

- Statistical Complexity Measure (SCM)

$$SCM = H \cdot Q \quad (1)$$

- Shannon entropy

$$H = - \sum_i p_i \log(p_i) \quad (2)$$

- The disequilibrium, using Jensen-Shannon divergence

$$Q = JSD(P||R) \quad (3)$$

Jensen-Shannon Divergence

- Jensen-Shannon divergence

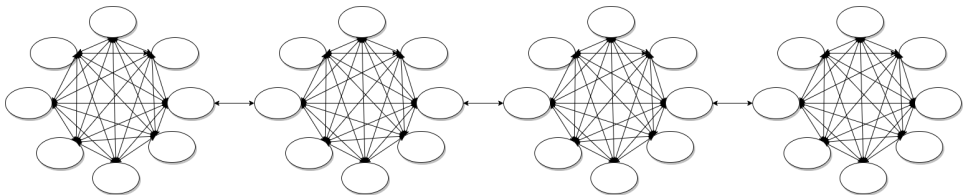
$$JSD(P||R) = \frac{1}{2}D(P||M) + \frac{1}{2}D(R||M) \quad (4)$$

- Kullback-Leibler divergence

$$D(P||M) = \sum_i p_i \log \left(\frac{p_i}{m_i} \right) \quad (5)$$

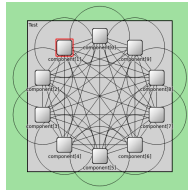
$$D(R||M) = \sum_i r_i \log \left(\frac{r_i}{m_i} \right) \quad (6)$$

Layered system

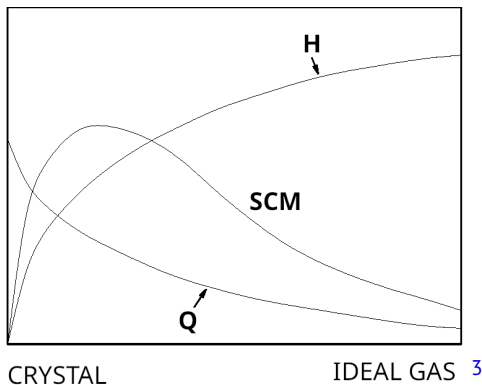


Model design

- Number of components: 128, 256, 512, 1024
- Number of layers: 1, 2, 4, 8, 16, 32, 64
- Simulation time: 10, 100, 200, 500 seconds
- Uniformly distributed communication between components on layer
- Ordered communication between layers
- Implemented using OMNet++ Systems



Dimensionality of a multi-layer network

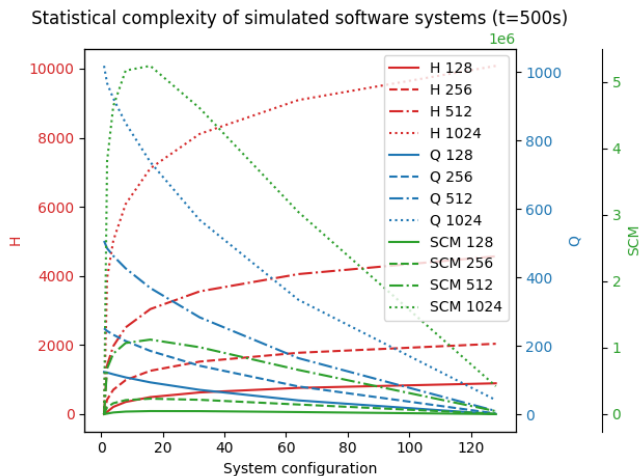


³ Sketch of the intuitive notion of the magnitudes of “information” (H) and “disequilibrium” (Q) for the physical systems and the behavior intuitively required for the magnitude of “complexity.” The quantity $SCM = H \cdot Q$ is proposed to measure such a magnitude. [LopezRui1995]

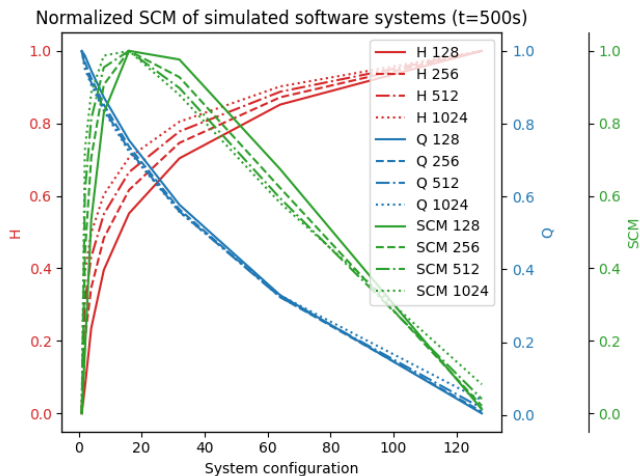
System configuration dimensionality

Configuration	Dimension	Abbreviation
Ordered	1 (2^0)	Or
64 Layers	2 (2^1)	L64
32 Layers	4 (2^2)	L32
16 Layers	8 (2^3)	L16
8 Layers	16 (2^4)	L8
4 Layers	32 (2^5)	L4
2 Layers	64 (2^6)	L2
Chaotic	128 (2^7)	Ch

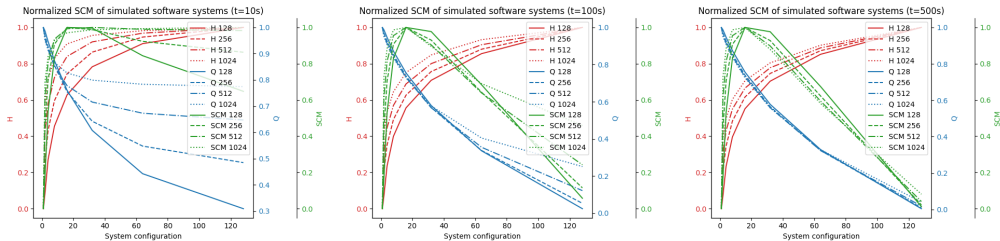
Statistical complexity of simulated systems



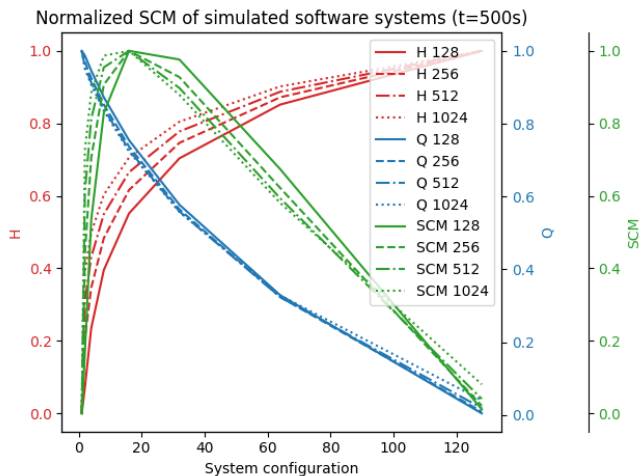
Normalized statistical complexity of simulated systems



Normalized statistical complexity of simulated systems



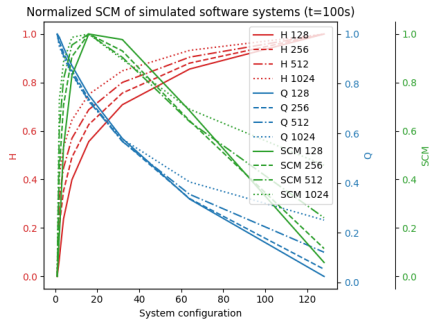
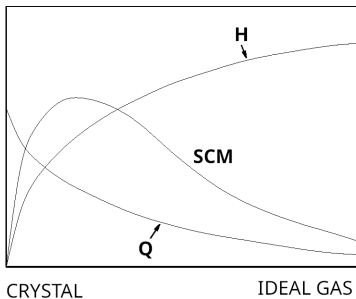
System with maximum SCM



Normalized simulation results for $t=500s$

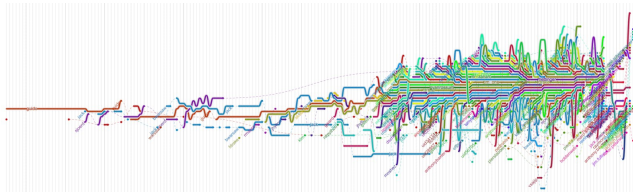
	128			256			512			1024		
	\bar{H}	\bar{Q}	\overline{SCM}	\bar{H}	\bar{Q}	\overline{SCM}	\bar{H}	\bar{Q}	\overline{SCM}	\bar{H}	\bar{Q}	\overline{SCM}
Or	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000	0.000	1.000	0.000
L64	0.072	0.987	0.170	0.206	0.969	0.439	0.309	0.957	0.610	0.391	0.950	0.727
L32	0.235	0.946	0.534	0.346	0.925	0.705	0.431	0.913	0.812	0.498	0.907	0.884
L16	0.395	0.873	0.829	0.483	0.853	0.907	0.550	0.842	0.954	0.603	0.836	0.987
L8	0.552	0.755	1.000	0.616	0.738	1.000	0.665	0.728	1.000	0.706	0.724	1.000
L4	0.703	0.578	0.976	0.746	0.565	0.928	0.778	0.559	0.898	0.805	0.558	0.879
L2	0.852	0.329	0.674	0.872	0.324	0.622	0.887	0.324	0.593	0.902	0.329	0.580
Ch	1.000	0.005	0.012	1.000	0.010	0.021	1.000	0.019	0.040	1.000	0.041	0.080

Comparison



Future work

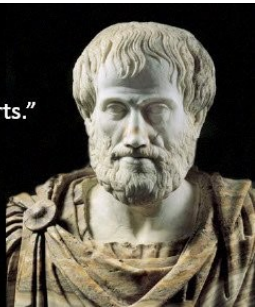
- Measure SCM for real software systems, for example ANN
- Explore how properties such as memory effect SCM
- Define how system state vectors can be constructed for software systems
- Research how system dimensionality may be calculated Systems



⁴<https://hackernoon.com/complex-adaptive-systems-and-the-future-of-app-development-2bb0288f05e0>

"The whole is greater
than the sum of its parts."

-Aristotle



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^a <https://www.linkedin.com/pulse/more-than-sum-its-parts-amarjit-s-basra/>

Thurner2018:

*"A Complex System is co-evolving
multilayer network."*

For Software Systems represented
as a multi-layer network use

SCM as measure of complexity

Ordered system has zero SCM

Chaotic system has low SCM

There is a system with maximum SCM

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