Exercise 1 Consider the following formulae.

(a) $(A \leftrightarrow B) \rightarrow (\neg A \wedge C)$

(b) $(A \rightarrow B) \rightarrow C$

(c) $A \leftrightarrow B$

(d) $(A \rightarrow B) \leftrightarrow (A \rightarrow C)$

(e) $(A \wedge B) \vee (A \wedge C)$

(f) $[A \rightarrow (B \vee \neg A)] \rightarrow (B \rightarrow A)$

(g) $[(A \vee B) \rightarrow (C \rightarrow A)] \leftrightarrow (A \vee B \vee C)$

For each of them

(1) use a truth table to determine if the formula is valid and/or satisfiable;

(2) convert the formula into CNF using the truth table;

(3) convert the formula into CNF using equivalence transformations instead;

(4) write the formula as a set of clauses.

Exercise 2 Which of the following formulae imply each other?

(a) $A \wedge B$

(b) $A \vee B$

(c) $A \rightarrow B$

(d) $A \leftrightarrow B$

(e) $\neg A \wedge \neg B$

(f) $\neg A$

(g) $\neg(A \rightarrow B)$

Exercise 3 Encode the following problems as a satisfiability problem for propositional logic:

(a) the independent set problem: given a graph $\mathfrak{G}$ and a number k, does the graph contain k vertices which are pairwise not connected by an edge to each other?

(b) the domino tiling problem: given a finite set D of square *dominoes*, two relations $H, V \subseteq D \times D$ specifying which pairs of dominoes can be put horizontally/vertically next to each other, and a number n, does there exist a tiling of the $n \times n$ grid, i.e., a function $\tau : n \times n \rightarrow D$ such that

$$\big(\tau(i,j), \tau(i+1,j)\big) \in H \quad \text{and} \quad \big(\tau(i,j), \tau(i,j+1)\big) \in V\,, \quad \text{for all } i,j?$$

Exercise 4 We can encode n-bit numbers via an n-tuple of propositional variables $A_{n-1}, \dots, A_0$.

(a) Write a formula $\varphi(A_1, A_0,\ B_1, B_0,\ C_2, C_1, C_0)$ for the addition of 2-bit numbers $(A + B = C)$.

(b) Write a formula $\varphi(A_{n-1}, \dots, A_0,\ B_{n-1}, \dots, B_0,\ C_n, \dots, C_0)$ for the addition of n-bit numbers.

Exercise 5 Use the DPLL algorithm to determine whether the following formulae are satisfiable.

(a) $\neg[(A \to B) \leftrightarrow (A \to C)]$

(b) $(A \lor B \lor C) \land (B \lor D) \land (A \to D) \land (B \to A)$

(c) $(A \leftrightarrow B) \to (\neg A \land C)$

(d) $[A \to (B \lor \neg A)] \to (B \to A)$

(e) $[(A \lor B) \to (C \to A)] \leftrightarrow (A \lor B \lor C)$

Exercise 6 Use the resolution method to determine which of the following formulae are valid.

(a) $(A \land \neg B \land C) \lor (\neg A \land \neg B \land \neg C) \lor (B \land C) \lor (A \land \neg C) \lor (\neg A \land B \land \neg C) \lor (\neg A \land \neg B \land C)$

(b) $(A \land B) \lor (B \land C \land D) \lor (\neg A \land B) \lor (\neg C \land \neg D)$

(c) $(\neg A \land B \land \neg C) \lor (\neg B \land \neg C \land D) \lor (\neg C \land \neg D) \lor (A \land B) \lor (\neg A \land B \land C) \lor (\neg A \land C)$
$\lor (A \land \neg B \land C) \lor (A \land \neg B \land D)$

Exercise 7 Construct the game for the following set of Horn-formulae and determine the winning regions.

$$B \land C \land D \to A \qquad C \land F \to B \qquad A \land F \to C \qquad D \to B \qquad A \land B \to F$$
$$E \to A \qquad D \land E \to B \qquad C \qquad D$$

Exercise 8 (optional) Given a finite automaton $\mathcal{A}$ and an input word w, write down a formula $\varphi_{\mathcal{A},w}$ that is satisfiable if, and only if, the automaton $\mathcal{A}$ accepts w.