

**Exercise 1** Suppose we are given a predicate  $\text{flight}(\text{From}, \text{To}, \text{Time}, \text{Price})$  containing information about direct flights including the starting airport, the destination, the flight time, and the price of a ticket. Write a Prolog program computing a predicate  $\text{travel}(\text{From}, \text{To}, \text{Stops}, \text{Time}, \text{Price})$  indicating all possibilities to travel from one city to another using one or several flights.

**Exercise 2** Write a Prolog predicate  $\text{fib}(N, X)$  computing the Fibonacci sequence. Evaluate  $\text{fib}(3, X)$  and  $\text{fib}(N, 5)$ .

**Exercise 3** Write Prolog definitions of the following predicates.

$\text{length}(\text{List}, N)$	$N$ is the length of $\text{List}$ .
$\text{append}(X, Y, Z)$	$Z$ is the concatenation of the lists $X$ and $Y$ .
$\text{reverse}(X, Y)$	$Y$ is the reverse of the list $X$ .
$\text{map}(X, Y)$	maps a list $X = [X_1, \dots, X_n]$ to $Y = [f(X_1), \dots, f(X_n)]$ .
$\text{fold\_left}(X, Y, Z)$	maps $Y = [Y_1, \dots, Y_n]$ to $Z = f(\dots f(f(X, Y_1), Y_2) \dots, Y_n)$ .
$\text{fold\_right}(X, Y, Z)$	maps $Y = [Y_1, \dots, Y_n]$ to $Z = f(Y_1, f(Y_2, \dots, f(Y_n, X) \dots))$ .

The Prolog notation for lists is as follows:

$[]$   $[X, Y, Z]$   $[X|Y]$   $[X, Y|Z]$ .

**Exercise 4**

(a) Write a naive sort function

$\text{naive\_sort}(X, Y) :- \text{permute}(X, Y), \text{sorted}(Y)$ .

by implementing the relations

$\text{sorted}(X)$	checks that the list $X$ is sorted.
$\text{insert}(X, Y, Z)$	if the list $Z$ is obtained from $Y$ by inserting $X$ at an arbitrary position.
$\text{permute}(X, Y)$	if the list $Y$ is a permutation of $X$ .

(b) (Optional) Implement merge sort using the relations

$\text{merge}(X, Y, Z)$	merges two sorted lists $X$ and $Y$ into $Z$ .
$\text{split}(X, Y, Z)$	splits the list $X$ into two lists $Y$ and $Z$ .

**Exercise 5** We consider undirected graphs of the form  $\langle V, E \rangle$ . Express the following relation in relational algebra.

- $x$  and  $y$  are not connected by an edge.
- The edge  $\langle x, y \rangle$  is part of a triangle.
- $x$  has at least two neighbours.
- Every neighbour of  $x$  is also a neighbour of  $y$ .

**Exercise 6** Evaluate the following Datalog program on the tree  $\langle V, E, P \rangle$  to the right.

$$\begin{aligned}
 U &\leftarrow S(x, y) \wedge W(x) \wedge W(y) \\
 W(x) &\leftarrow P(x) \\
 W(x) &\leftarrow E(x, y) \wedge W(y) \\
 S(x, y) &\leftarrow E(z, x) \wedge E(z, y) \wedge x \neq y \\
 R(x, y) &\leftarrow P(x) \wedge x = y \\
 R(x, y) &\leftarrow E(x, z) \wedge R(z, y) \\
 R(x, y) &\leftarrow R(x, z) \wedge E(z, y)
 \end{aligned}$$
