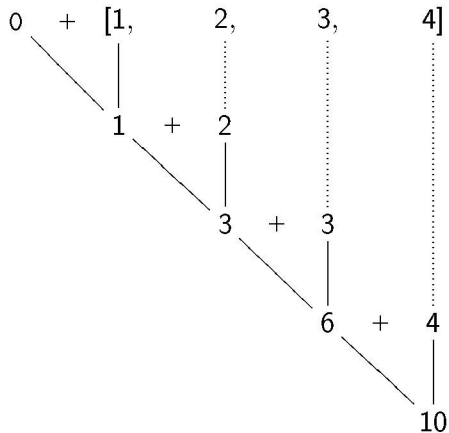


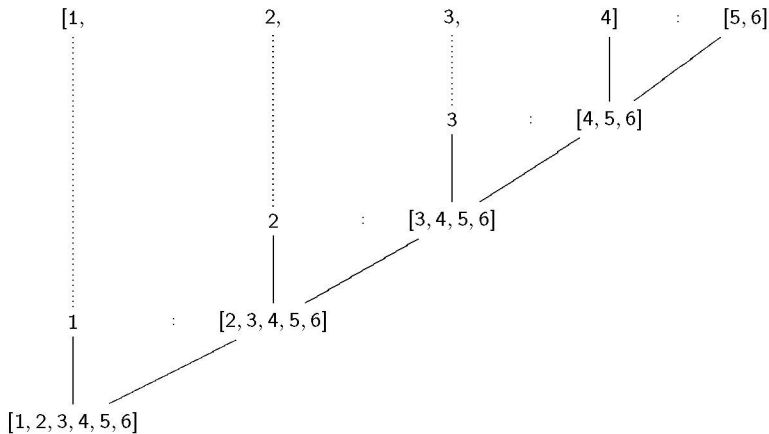
# foldl, foldr

```
foldl (+) 0 [1, 2, 3, 4]
```



# foldl, foldr

```
foldr (:) [5,6] [1,2,3,4]
```



## Lambda abstrakce, částečná aplikace

```
f x y = x + y
f      = (\x y -> x + y)
```

```
a ⊕ b = ...
```

```
(⊕) = (\a b -> ...)
```

```
(a ⊕) = (\b -> a ⊕ b) = (⊕) a
```

```
(⊕ b) = (\a -> a ⊕ b) = flip (⊕) b
```

```
flip :: (a -> b -> c) -> b -> a -> c
```

```
flip f x y = f y x
```

$(.) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$

$(f . g) x = f (g x)$

$(.) f g x = f (g x)$

$h x = \underline{f} (\underline{g} x)$

$h x = (\underline{f} . \underline{g}) x$

$h = f . g$

$\eta$ -redukce  $x$

# Pointfree – příklady

$$h\ x = \underline{f}\ (\underline{g}\ x)$$

$$h\ x = (f\ .\ g)\ x$$

$$h = f\ .\ g$$

$$f\ u\ v\ w\ x = u\ v\ (w\ x)$$

$$f\ u\ v\ w\ x = \underline{u\ v}\ (\underline{w}\ x)$$

$$f\ u\ v\ w\ x = (\underline{u\ v}\ .\ w)\ x$$

$$(\lambda a \rightarrow b \oplus a) = (b \oplus)$$

$$f\ u\ v\ w\ x = (u\ v\ .) w\ x$$

$$f\ u\ v = (u\ v\ .)$$

po  $\eta$ -redukci  $w\ a\ x$

$$f\ u\ v = \underline{(\underline{.})}\ (\underline{u}\ v)$$

$$f\ u\ v = ((\underline{.})\ .\ u)\ v$$

$$f\ u\ v = ((\underline{.})\ .)\ u\ v$$

$\eta$ -redukujeme  $u\ a\ v$

$$f = \underline{((\underline{.})\ .)}$$

$$(f \oplus) = ((\oplus) f)$$

$$f = ((\underline{.})\ (\underline{.}))$$

$$f = (\underline{.})\ (\underline{.})$$

## Pointfree – příklady

$$h\ x = \underline{f}\ (\underline{g}\ x)$$

$$h\ x = (f\ .\ g)\ x$$

$$h = f\ .\ g$$

$$h\ x\ y = q\ y\ .\ q\ x$$

$$h\ x\ y = \quad \quad \quad q\ y\ .\ q\ x$$

$$h\ x\ y = \quad (\cdot)\ (q\ y)\ (q\ x)$$

$$h\ x\ y = \underline{\text{flip}}\ (\cdot)\ (\underline{q\ x})\ (\underline{q}\ y)$$

$$h\ x\ y = (\underline{\text{flip}}\ (\cdot)\ \underline{q\ x})\ .\ \underline{q}\ y$$

$$h\ x = \text{flip}\ (\cdot)\ (q\ x)\ .\ q$$

$$f\ b\ a = \text{flip}\ f\ a\ b$$

po  $\eta$ -redukci y

$$h\ x = (\cdot\ q)\ (\underline{\text{flip}}\ (\cdot)\ (\underline{q}\ x))$$

$$h\ x = (\cdot\ q)\ ((\underline{\text{flip}}\ (\cdot))\ .\ \underline{q})\ x$$

$$h\ x = \underline{(\cdot\ q)}\ (\underline{((\text{flip}\ (\cdot))\ .\ q)}\ x)$$

$$h\ x = ((\cdot\ q)\ .\ (\text{flip}\ (\cdot)\ .\ q))\ x$$

$$f.\!(g.h) = f.g.h$$

$$h = (\cdot\ q)\ .\ \text{flip}\ (\cdot)\ .\ q$$

po  $\eta$ -redukci x

## Pointfree – příklady

$$h\ x = \underline{f}\ (\underline{g}\ x)$$

$$h\ x = (\underline{f}\ .\ \underline{g})\ x$$

$$h = f . g$$

$$f\ x\ y = q\ y\ (q\ x)$$

$$f\ x\ y = \quad q\ y\ \quad (q\ x) \quad \boxed{f\ b\ a = \text{flip}\ f\ a\ b}$$

$$f\ x\ y = \text{flip}\ q\ (q\ x)\ y$$

$$f\ x = \text{flip}\ q\ (q\ x) \quad \boxed{\text{po } \eta\text{-redukci } y}$$

$$f\ x = \underline{\text{flip}\ q}\ (\underline{q}\ x)$$

$$f\ x = (\underline{\text{flip}\ q}\ .\ \underline{q})\ x$$

$$f = \text{flip}\ q . q \quad \boxed{\text{po } \eta\text{-redukci } x}$$