2007 - Exercises II.

1. Decide which of the following codes is linear. Find the standard form of generator matrix for codes that are linear.
   (a) quaternary code $C_1 = \{000, 123, 202, 321\}$
   (b) 6-ary code $C_2 = \{00, 03, 43, 23, 20, 40\}$
   (c) ternary code $C_3 = \{000, 221, 112\}$

2. Compare the set of perfect codes with the set of maximum distance separable codes (*ie.* use $\subseteq, =, \ldots$).

3. Let C be a binary code with generator matrix
$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$
   (a) Write a generator matrix for the code obtained from C by puncturing the 1st and 4th coordinate.
   (b) Write a generator matrix for the extended code $\hat{C}$.
   (If C is a linear code over $\mathbb{F}_q^n$ then $\hat{C} = \{x_1x_2 \ldots x_n x_{n+1} \in \mathbb{F}_q^{n+1} | x_1 \ldots x_n \in C$ and $x_1 + \ldots x_n + x_{n+1} = 0\}$ is the extended code.)
   (c) Describe the code that is obtained from a linear code first by extending this code and then puncturing on the new coordinate. What happens if we change the order of these operations?

4. Let C be a binary linear $[4, 2]$-code such that $C = C^{\perp}$. Show that C contains at least two words of weight 2.

5. Let C be a perfect binary linear $[7, 4]$-code. Find the value of d. Suppose that C is transmitted over a binary symmetric channel with error probability $p = 0.01$ using the syndrome decoding strategy. Calculate the word error probability.

6. Consider the linear code C over the field $\mathbb{Z}_7$ spanned by the codewords 43352, 24545 and 31433.
   (a) Find a generator matrix in the standard form for C.
   (b) How many different messages can C encode?
   (c) Show that a received word containing at most one error can be corrected.