

## 2007 - Exercises V.

1. Consider the following assignment of numerical equivalents to a 40-letter alphabet. The letters A, B, ..., Z are given their numerical equivalents 0 to 25, respectively, a blank space = 26, . = 27, ? = 28, \$ = 29 and the numerals 0, 1, ..., 9 are assigned 30, 31, ..., 39, respectively. Using this alphabet, a message is encoded into a string of numbers by taking each pair of symbols in the message, converting the pair to their numerical equivalents, say  $a$  and  $b$ , and replacing the pair by the number  $40a + b$ . If this method is used to convert a message to a numerical sequence and then the sequence is encrypted by the RSA cryptosystem, explain how the message could be decrypted without knowing the factorization of the public modulus.
2. Suppose that  $n = 1363$  in the RSA cryptosystem and it has been revealed that  $\varphi(n) = 1288$ . Use this information to factor  $n$ .
3. Suppose that  $X' = (1987, 439, 1394, 724, 339, 2303, 1256, 810, 650)$  and  $m = 2503$  in the Knapsack cryptosystem. Decrypt the cryptotext  $c = 3155$ .
4. Suppose that Alice, Bob and Charles have the same encryption exponent  $e = 3$  for the RSA cryptosystem. Let their moduli be  $n_A$ ,  $n_B$  and  $n_C$ . Suppose that  $\gcd(n_i, n_j) = 1$  for each  $i, j \in \{A, B, C\}$ . Suppose that Dennis sends the same message  $m$  to all of them. This means he sends the cryptotexts  $c_A = m^3 \bmod n_A$ ,  $c_B = m^3 \bmod n_B$  and  $c_C = m^3 \bmod n_C$ . Show how it is possible to compute  $m$  from the public information without factoring the moduli.
5. Let  $(n = pq, e)$  be a public key and let  $d$  be the corresponding secret key for the RSA cryptosystem. Let  $m$  be a plaintext of the form  $m = kp + 1$  for some  $k \in \mathbb{N}$ . Show how one can efficiently compute  $d$  knowing only the cryptotext  $c = m^e \bmod n$ .
6. Two users, Ari-Pekka and Saku, use the RSA cryptosystem with the same modulus  $n$  and encryption exponents  $e_A$  and  $e_S$  with  $\gcd(e_A, e_S) = 1$ . Hannu sends the message  $m$  to both Ari-Pekka and Saku. Olli intercepts  $c_A = m^{e_A} \bmod n$  and  $c_S = m^{e_S} \bmod n$ . Olli then computes  $x_1 = e_A^{-1} \bmod e_S$  and  $x_2 = (x_1 e_A - 1)/e_S$ .
  - (a) How can Olli compute  $m$  from intercepted ciphertexts  $c_A$ ,  $c_S$  using  $x_1$  and  $x_2$ ?
  - (b) Use the proposed method to compute  $m$  if  $n = 18721$ ,  $e_A = 43$ ,  $e_S = 7717$  and you intercept ciphertexts  $c_A = 12677$  and  $c_S = 14702$ .
7.
  - (a) Consider a Diffie-Hellman scheme with  $q = 3$  and  $p = 353$ . Alice chooses  $x = 97$  and Bob chooses  $y = 233$ . Compute  $X$  and  $Y$  and the key  $k$ .
  - (b) Consider a Diffie-Hellman scheme with  $q = 2$ ,  $p = 11$ ,  $X = 9$  and  $Y = 3$ . Compute  $x$  and  $y$ .
  - (c) Design the extension of the Diffie-Hellman key exchange that allows three parties Alice, Bob and Charlie to generate a common secret key.