$$\begin{vmatrix} 8-\lambda & 0 & 6 \\ -3 & 2-\lambda & -3 \\ -9 & 0 & -7-\lambda \end{vmatrix} = (8-\lambda)(2-\lambda)(-7-\lambda) + 54(2-\lambda) =$$

$$= (16-10\lambda+\lambda^2)(-7-\lambda) + 108 - 54\lambda =$$

$$= -112 + \cancel{70\lambda} - 7\lambda^2 - 16\lambda + 10\lambda^2 - \lambda^3 + 108 - \cancel{54\lambda} =$$

$$= -\lambda^3 + 3\lambda^2 - 4 = (\lambda+1)(-\lambda^2+4\lambda-4) =$$

$$= -(\lambda+1)(\lambda^2-4\lambda+4) =$$

$$= -(\lambda+1)(\lambda-2)^2$$

$$\begin{array}{r|rrrr} & -1 & 3 & 0 & -4 \\ \hline -1 & -1 & 4 & -4 & 0 \end{array}$$

$|A-\lambda E|$

$\lambda_1 = -1$

$\lambda_{2,3} = 2$

$\lambda = 2$:

$$\begin{pmatrix} 6 & 0 & 6 \\ -3 & 0 & -3 \\ -9 & 0 & -9 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \quad x_1 + x_3 = 0 \Rightarrow x_1 = -x_3$$

$$\langle (0,1,0), (-1,0,1) \rangle \setminus \vec{o}$$

$\lambda = -1$:

$$\begin{pmatrix} 9 & 0 & 6 \\ -3 & 3 & -3 \\ -9 & 0 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \quad \langle (-2,1,3) \rangle \setminus \vec{o}$$

$$a(1,0,1,0)+b(2,3,-1,1)+c(1,3,0,1)=d(1,3,-2,1)+e(1,0,-1,0)$$

$$(2)\quad \begin{aligned} a+2b+c-d-e&=0\\ 3b+3c-3d&=0\\ a-b+2d+e&=0 \Rightarrow a-b+2b+2c+e=0 \Rightarrow e=-a-b-2c\\ b+c-d&=0 \Rightarrow d=b+c \end{aligned}$$

$$0=0$$

$$(1)\quad a+2b+c-b-c+a+b+2c=0$$

$$2a+2b+2c=0 \Leftrightarrow a+b+c=0$$

$$a=-b-c$$

$$\langle(-1,1,0),(-2,0,1)\rangle$$

$$V_1\cap V_2 = -b(1,0,1,0)+b(2,3,-1,1)+c(1,3,0,1)-c(1,0,1,0)$$

$$\cong V_2 \qquad = \langle(1,3,0,1),(0,3,-1,1)\rangle$$

Hledáme ortonorm. bázi $V_2 = \langle (1,3,-2,1), (1,0,-1,0) \rangle$:

1, 1. vektor volím $(1,0,-1,0)$

2, 2. vektor $\vec{v}_2$ volím ve tvaru $(1,3,-2,1) + a(1,0,-1,0)$

podmínka: $(1,3,-2,1)\cdot(1,0,-1,0) + a(1,0,-1,0)\cdot(1,0,-1,0) = 0$

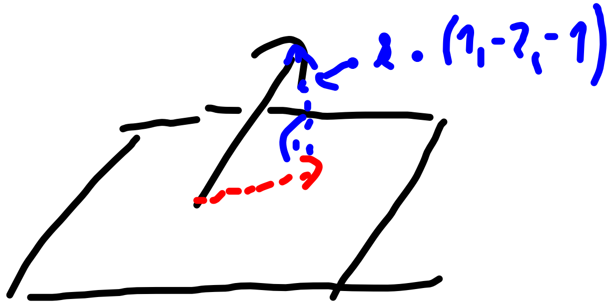
$$\Rightarrow a = -\frac{(1,3,-2,1)\cdot(1,0,-1,0)}{\|(1,0,-1,0)\|^2} = \frac{3}{2}$$

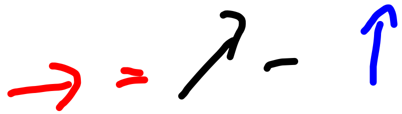
$$\Rightarrow \vec{v}_2 = (1,3,-2,1) - \frac{3}{2}(1,0,-1,0) = \left(-\frac{1}{2}, 3, -\frac{1}{2}, 1\right)$$

$$(\vec{v}_1 \cdot \vec{v}_2) = \left(-\frac{1}{2}, 3, -\frac{1}{2}, 1\right)\cdot(1,0,-1,0) = -\frac{1}{2} + \frac{1}{2} = 0$$

$\Rightarrow$ ortonormální báze je

$$\sqrt{\frac{2}{21}}\left(-\frac{1}{2}, 3, -\frac{1}{2}, 1\right),\ \frac{1}{\sqrt{2}}(1,0,-1,0)$$



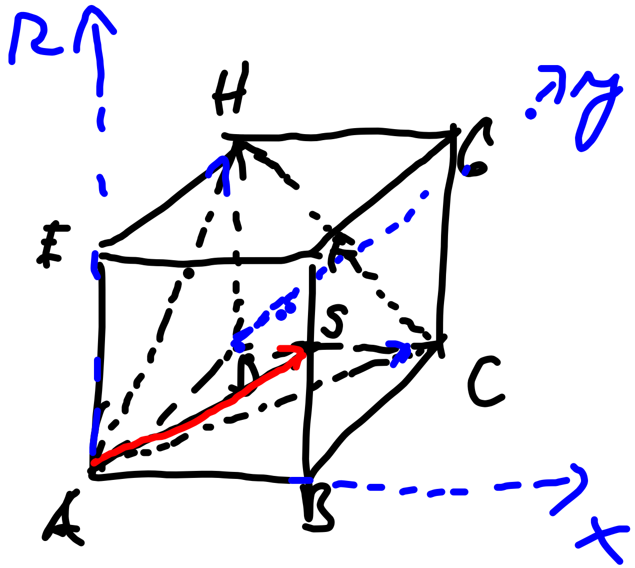


$$(x_1,x_2,x_3) - \underbrace{\tfrac{1}{6}(x_1,x_2,x_3)(1,-2,-1)}_{\text{číslo}}(1,-2,-1) =$$

$$= (x_1,x_2,x_3) - \tfrac{1}{6}(x_1-2x_2-x_3)(1,-2,-1) =$$

$$= (x_1,x_2,x_3) + \left(-\tfrac{1}{6}x_1+\tfrac{1}{3}x_2+\tfrac{1}{6}x_3,\ \tfrac{1}{3}x_1-\tfrac{2}{3}x_2-\tfrac{1}{3}x_3,\ \tfrac{1}{6}x_1-\tfrac{1}{3}x_2-\tfrac{1}{6}x_3\right)$$

$$= \left(\tfrac{5}{6}x_1+\tfrac{1}{3}x_2+\tfrac{1}{6}x_3,\ \tfrac{1}{3}x_1+\tfrac{1}{3}x_2-\tfrac{1}{3}x_3,\ \tfrac{1}{6}x_1-\tfrac{1}{3}x_2+\tfrac{5}{6}x_3\right)$$



Rovina ACH $= \langle \vec{AH}, \vec{AC} \rangle$

Zjistíme kolmý průmět $\vec{AS}$ do dané roviny

$$\vec{AS} = (1, 0, \tfrac{1}{2})$$
$$\vec{AH} = (0, 1, 1)$$
$$\vec{AC} = (1, 1, 0)$$

Najdeme ortogonální bázi roviny / vekt. prostoru ACH:

1. $v_1 = (0,1,1)$
2. $v_2 = (1,1,0) + a\,(0,1,1) =$

$$v_1 \cdot v_2 = 0 \Rightarrow a = -\frac{(1,1,0)\cdot(0,1,1)}{\|(0,1,1)\|^2} = -\frac{1}{2}$$

$$\Rightarrow v_2 = (1,1,0) - \tfrac{1}{2}(0,1,1) = (1, \tfrac{1}{2}, -\tfrac{1}{2}) \checkmark$$

projekce $\vec{AS}$: $\vec{p} = \frac{1}{2}\left((1,0,\tfrac{1}{2})\cdot(0,1,1)\right)(0,1,1) + \frac{2}{3}\left((1,0,\tfrac{1}{2})\cdot(1,\tfrac{1}{2},-\tfrac{1}{2})\right)(1,\tfrac{1}{2},-\tfrac{1}{2})$

$= \frac{1}{4}(0,1,1) + \frac{1}{2}\left(1,\tfrac{1}{2},-\tfrac{1}{2}\right) = \left(\tfrac{1}{2},\tfrac{1}{2},0\right)$

Ověření $\vec{AS}-\vec{p}$ musí být kolmé na ACH

$\left(\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2}\right)$ ✓

$$\underbrace{(x_1, x_2, \dots, x_n)}_{\vec{x}} \cdot (\underbrace{y_1, y_2, \dots, y_n}_{\vec{y}}) = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

$$(x_1, x_2, \dots, x_n)\, E \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

T ... matice přechodu od $\underbrace{\text{nové báze}}_{f}$ k ortonorm.

$$(x_1, x_2, \dots, x_n) \underbrace{T^T E\, T}_{\text{matice sk. součinu v bázi } f} \vec{y}$$

matice sk. součinu v bázi f.

Nechť $f = ((1,0,1), (1,-1,1), (1,1,0))$. V této bázi má uvažovaný skalární součin jednotkovou matici. Nechť T je matice přechodu od st. báze k bázi f, pak matice uvažovaného sk. součinu ve st. bázi je $T^T T$.

matice přechodu od $\underline{f}$ k $\underline{e}$

$$
\left(\begin{array}{ccc|ccc}
1 & 1 & 1 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 1
\end{array}\right)
$$

$$
\left(\begin{array}{ccc|ccc}
1 & 1 & 1 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 & 1 & 0 \\
0 & 0 & -1 & -1 & 0 & 1
\end{array}\right)
$$

$$
\left(\begin{array}{ccc|ccc}
1 & 1 & 0 & 0 & 0 & 1 \\
0 & -1 & 0 & -1 & 1 & 1 \\
0 & 0 & -1 & -1 & 0 & 1
\end{array}\right)
$$

$$
\left(\begin{array}{ccc|ccc}
1 & 0 & 0 & -1 & 1 & 2 \\
0 & -1 & 0 & -1 & 1 & 1 \\
0 & 0 & -1 & -1 & 0 & 1
\end{array}\right)
$$

$$
T = \begin{pmatrix} -1 & 1 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix}
$$

$$
T^T \cdot T =
$$

$$
\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & -1 & -1 \end{pmatrix}
\begin{pmatrix} -1 & 1 & 2 \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix}
= \begin{pmatrix} 3 & -2 & -4 \\ -2 & 2 & 3 \\ -4 & 3 & 6 \end{pmatrix}
$$

$$
(1,0,1)\begin{pmatrix} 3 & -2 & -4 \\ -2 & 2 & 3 \\ -4 & 3 & 6 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = (1,0,1)\cdot\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} = 1
$$

1, řešíme zhomogenizovanou rovnici

$$X_{n+2} = X_{n+1} - X_n$$

char. polynom $x^2 - x + 1$, a kořeny $r_{1,2} = \frac{1 \pm \sqrt{-3}}{2} =$

$$= \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$= \cos\left(\frac{\pi}{3}\right) \pm i \sin\left(\frac{\pi}{3}\right)$$

reálná báze prostoru všech řešení:

$$\left\{\cos\left(\frac{n\pi}{3}\right)\right\}_{n=0}^{\infty}, \left\{\sin\left(\frac{n\pi}{3}\right)\right\}_{n=0}^{\infty}.$$

$$a \cdot \left\{\cos\left(\frac{n\pi}{3}\right)\right\}_{n=0}^{\infty} + b \cdot \left\{\sin\left(\frac{n\pi}{3}\right)\right\}_{n=0}^{\infty}, \ a, b \in \mathbb{C}$$

2, ~~řeš~~ najdeme partikulární řešení nehomogenní rce:

$$an + b, \ a, b \in \mathbb{C}$$

$$a(n+2) + b = a(n+1) + b - an - b + 2n$$

$n^1: a = a - a + 2 \Rightarrow a = 2$, $n^0: 2a + b = a + b - b \Rightarrow b = -a$

$b = -2$

partikulárním řešením je polynom $2n-2$

všechna řešení rekurence bez poč. podmínek jsou

$$2n-2+a\cdot\left\{\cos\frac{n\pi}{3}\right\}_{n=0}^{\infty}+b\cdot\left\{\sin\frac{n\pi}{3}\right\}_{n=0}^{\infty},\quad a,b\in\mathbb{C}$$

$$x_0=x_1=x_2=-1$$

$$n=0:\quad -1=-2+a\cdot\cos 0+b\cdot\sin 0=-2+a\Rightarrow a=1$$

$$n=1:\quad 1=0+a\cdot\cos\left(\frac{\pi}{3}\right)+b\cdot\sin\frac{\pi}{3}=\frac{1}{2}+b\cdot\frac{\sqrt{3}}{2}\Rightarrow$$

$$b=\frac{1}{\sqrt{3}}$$

Jediná posloupnost splňující zadání je

$$x_n=2n-2+\cos\frac{n\pi}{3}+\frac{1}{\sqrt{3}}\sin\frac{n\pi}{3}$$

$(x_1, \dots, x_n)$

$f_1, \dots, f_n$ .. porodnosti

$\tau_1, \dots, \tau_n$ ... koef. přežití

$$\left(f_1 x_1 + f_2 x_2 + \dots + f_n x_n,\ \tau_1 x_1,\ \tau_2 x_2,\ \dots,\ \tau_{n-1} x_{n-1} \right)$$

← jediná dominantní hodnota

$$\vec{v}_1 + \vec{v}_2 + \dots + \vec{v}_n = \dots = \lambda_1^n \vec{v}_1 + \dots + \lambda_n^n \vec{v}_n$$

$\lambda_1 > 1$, $\lambda_n < 1$

$$\approx \lambda_1^n \vec{v}_1$$

$$\begin{vmatrix} -\lambda & 2 & 5 \\ 1 & -\lambda & 0 \\ 0 & \frac{4}{5} & -\lambda \end{vmatrix} = -\lambda^3 + 4 + 2\lambda = $$
$$= (\lambda - 2)(-\lambda^2 - 2\lambda - 2)$$

	-1	0	2	4
2	-1	-2	-2	0

$\Rightarrow$ $\lambda = 2$ je hlavní dominantní vl. hodnota

odpovídající vl. vektor

$$\begin{pmatrix} -2 & 2 & 5 \\ 1 & -2 & 0 \\ 0 & \frac{4}{5} & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \vec{0}$$

$$\begin{pmatrix} -2 & 2 & 5 \\ 1 & -2 & 0 \\ 0 & 4 & -10 \end{pmatrix} \sim \begin{pmatrix} -2 & 2 & 5 \\ 1 & -2 & 0 \\ -4 & 8 & 0 \end{pmatrix}$$

$$-2x_1 + 2x_2 + 5x_3 = 0 \Rightarrow 5x_3 - 2x_2 = 0 \Rightarrow x_3 = \frac{2}{5}x_2$$

$$x_1 - 2x_2 = 0 \Rightarrow x_1 = 2x_2$$

vlastní vektory jsou 1-dim. prostorem

$$\left(2, 1, \frac{2}{5}\right)$$