i) $B := A^T A$ $\qquad 1 \leq i,j \leq n$

$b_{ij} = (i\text{-tý řádek } A^T) \cdot (j\text{-tý sloupec } A)$

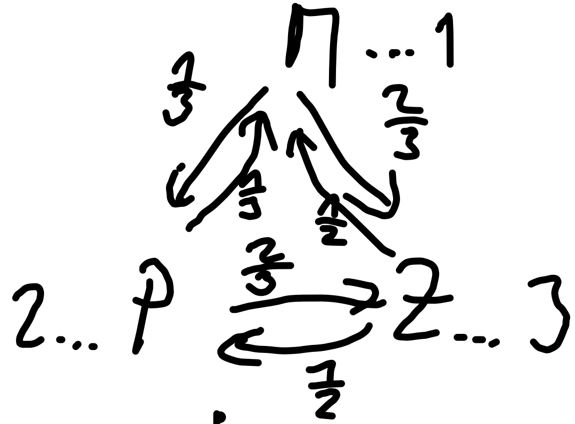
$b_{ji} = (j\text{-tý řádek } A^T) \cdot (i\text{-tý sloupec } A)$

$\qquad\qquad \| \qquad\qquad\qquad\qquad \|$

$(j\text{-tý sloupec } A) \cdot (i\text{-tý řádek } A^T) = b_{ij}$

ii) $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\qquad \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 \Rightarrow \lambda_{1,2} = 1$

$$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{2}{3} & \frac{2}{3} & 0 \end{pmatrix}$$



$s = 6\lambda$

$$\begin{vmatrix} -\lambda & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{3} & -\lambda & \frac{1}{2} \\ \frac{2}{3} & \frac{2}{3} & -\lambda \end{vmatrix} = 0 \iff \begin{vmatrix} -s & 2 & 3 \\ 2 & -s & 3 \\ 4 & 4 & -s \end{vmatrix} = 0$$

$$-s^3 + 48 + 12s + 12s + 4s = -s^3 + 28s + 48 =$$

$$= -(s-6)(s^2 + 6s + 8)$$

	-1	0	28	48
6	-1	-6	-8	0

$s_1 = 6$

$$s_{2,3} = \frac{-6 \pm \sqrt{4}}{2} = \begin{cases} -2 \\ -4 \end{cases}$$

$\lambda_1 = 1, \quad \lambda_2 = -\frac{1}{3}, \quad \lambda_3 = -\frac{2}{3}$

Vlastní vektor příslušný $\lambda = 1$: $(s=6)$

$$\begin{pmatrix} -6 & 2 & 3 \\ 2 & -6 & 3 \\ 4 & 4 & -6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 0 & -16 & 12 \\ 2 & -6 & 3 \\ 0 & 16 & -12 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-4x_2 + 3x_3 = 0 \Rightarrow x_3 = +\frac{4}{3}x_2$$

$$-6x_1 + 2x_2 + 3x_3 = 0$$

$$2x_1 - 6x_2 + 3x_3 = 0$$

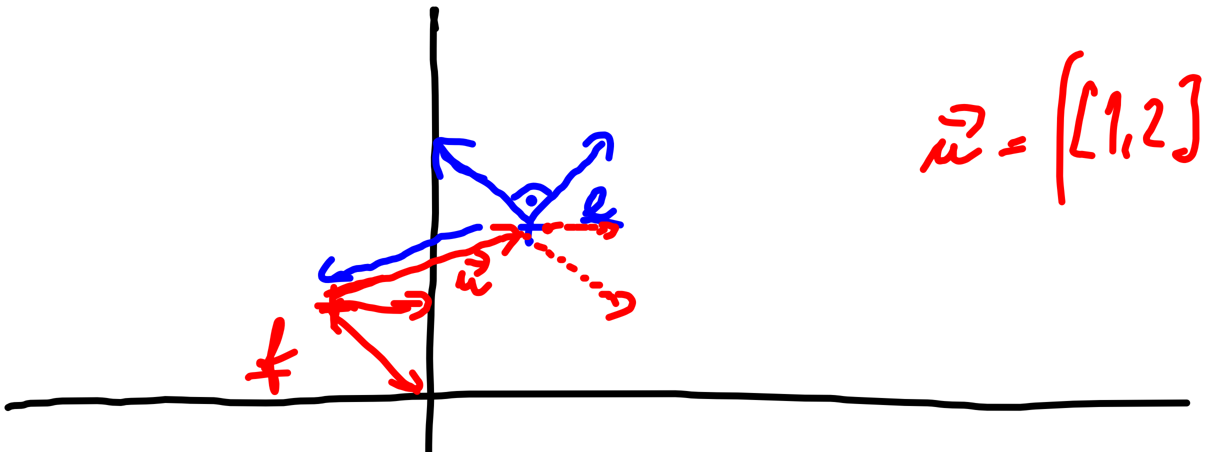
$$4x_1 + 4x_2 - 6x_3 = 0 \Leftrightarrow 2x_1 + 2x_2 - 3x_3 = 0$$

$$2x_1 - 6x_2 + 4x_2 = 0 \Leftrightarrow$$

$$x_1 = x_2$$

$$\left(1, 1, \frac{4}{3}\right)$$

Hledaná pst je $\dfrac{\frac{4}{3}}{1+1+\frac{4}{3}} = \dfrac{\frac{4}{3}}{\frac{10}{3}} = \dfrac{2}{5}$



$\vec{u} = \left([1,2] - [-1,1]\right)_f = (3,-1)$

1, nechť $B=(x,y)$ jsou souřadnice nějakého bodu v bázi e v $\mathbb{R}^2$. Pak souřadnice B v bázi $([1,2],(1,0),(1,-1))$ dostanu analogicky jako ve vekt. prostoru:

vyjádřím vektory z e pomocí vektorů z f:

$(-1,1) = a(1,0) + b(1,-1) \Rightarrow b=-1,\ a=0$

$(1,1) = a(1,0) + b(1,-1) \Rightarrow b=-1 \Rightarrow a=2 \qquad (0,-1)_f$

$$T = \begin{pmatrix} 0 & 2 \\ -1 & -1 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 0 & 2 \\ -1 & -1 \end{pmatrix}}_{T}\begin{pmatrix} x \\ y \end{pmatrix} + \underbrace{\begin{pmatrix} 3 \\ -1 \end{pmatrix}}_{\vec{u}}$$

2, posun počátku do bodu $[-1,1]$:

$[1,2] - [-1,1] = (2,1)$ ... vyjádření v bázi f:

$(2,1) = a(1,0) + b(1,-1) \Rightarrow b = -1,\ a = 3$

Tra-ce od f k e: potřebuji T^{-1}:

$$\left(\begin{matrix} 0 & 2 \\ -1 & -1 \end{matrix}\ \middle|\ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix}\ \middle|\ \begin{matrix} 0 & -1 \\ \frac{1}{2} & 0 \end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}\ \middle|\ \begin{matrix} -\frac{1}{2} & -1 \\ +\frac{1}{2} & 0 \end{matrix}\right) = T^{-1}$$

$(-2,-1) = a\cdot(-1,1) + b(1,1) \Leftrightarrow$

$$\left.\begin{aligned} -2 &= -a + b \\ -1 &= a + b \end{aligned}\right\} \Rightarrow -3 = 2b \Rightarrow b = -\frac{3}{2}$$

$\Rightarrow a = \frac{1}{2}$

$$T^{-1}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix}$$

Celkově je f v bázi f dáno jako $\qquad B = (x,y)_f$

$$= \begin{pmatrix} -\frac{1}{2} & -1 \\ +\frac{1}{2} & 0 \end{pmatrix} \overbrace{\left[\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \underbrace{\left[\begin{pmatrix} 0 & 2 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} \right]}_{(x,y)_e} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right]}^{[f(x,y)]_e} + \begin{pmatrix} \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 \end{pmatrix} \left[\begin{pmatrix} -3 & -1 \\ -1 & +3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right] + \begin{pmatrix} \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{5}{2} & -\frac{5}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -9 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{3}{2} \end{pmatrix} =$$

$$\begin{pmatrix} \frac{5}{2} & -\frac{5}{2} \\ -\frac{3}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -\frac{17}{2} \\ -\frac{1}{2} \end{pmatrix}$$

$$(2,1,1)\times(6,0,2) = \left(\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}, -\begin{vmatrix} 2 & 1 \\ 6 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 6 & 0 \end{vmatrix} \right) =$$

$$= (2, 2, -6)$$

vektor kolmý k oběma mimoběžkám je $(1,1,-3)$

Hledáme $P \in p$, $Q \in q$ tak, aby $(P-Q) = k \cdot (1,1,-3)$

$$P = [-1,2,2] + (2,1,1)t$$

$$Q = [0,2,-2] + (6,0,2)s$$

$P-Q:$ $\quad -1+2t-6s = k \Rightarrow -1+t-6s = 0$

$2+t-2 = k \Rightarrow t = k$

$2+t+2-2s = -3k \Rightarrow 2+4t+2-2s = 0 \Leftrightarrow$

$t = 1+6s \Rightarrow t = -\frac{13}{11}$ $\qquad 2+2t-s=0$

$2+2+12s-s = 0 \Rightarrow s = -\frac{4}{11}$

$$P = [-1, 2, 2] + (2, 1, 1) \cdot \left(-\frac{13}{11}\right) = \left[-\frac{37}{11}, \frac{9}{11}, \frac{9}{11}\right]$$

$$Q = [0, 2, -2] + (6, 0, 2) \cdot \left(-\frac{4}{11}\right) = \left[-\frac{24}{11}, 2, -\frac{30}{11}\right]$$

$$Q - P = \left[\frac{13}{11}, \frac{13}{11}, -\frac{39}{11}\right] = \frac{13}{11} \cdot (1, 1, -3)$$

Hledaná příčka PQ leží v rovině ρ dané přímkou p a bodem $X = [1,3,5]$. Bod Q tedy leží na průsečíku $\rho \cap q$.

$$\rho = [2,2,2] + (1,0,1)t + \overbrace{(-1,1,3)}^{[1,3,5]-[2,2,2]} s$$

$$q = [3,3,9] + (2,-1,0)u$$

$$2 + t - s = 3 + 2u \Rightarrow 2 + t - 1 + u = 3 + 2u \Rightarrow u = t - 2$$
$$t = u + 2$$
$$2 + s = 3 - u \Rightarrow s = 1 - u$$
$$2 + t + 3s = 9$$

$$2 + u + 2 + 3 - 3u = 9$$

$$-2 = 2u \Rightarrow u = -1 \Rightarrow t = 1$$

$$\Rightarrow Q = [3,3,9] - (2,-1,0) = [1,4,9]$$

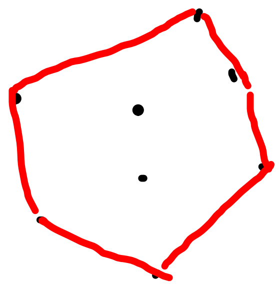
Bod P je průsečíkem přímky $\overrightarrow{QX}$ a p:

$$p = [2,2,2] + (1,0,1)t$$

$$\overrightarrow{QX}: [1,3,5] + (0,1,4)s$$

$$2 + t = 1 \Rightarrow t = -1 \Rightarrow P = [1,2,1]$$

$$PQ = [1,2,1][1,4,9]$$



Konvexní kombinace bodů $[1,0,-1]$, $[-1,4,5]$, $[3,4,5]$, $[7,-3,1]$:

$$s_1[1,0,-1]+s_2[-1,4,5]+s_3[3,4,5]+s_4[7,-3,1],$$

$$s_1, s_2, s_3, s_4 \geq 0, \quad s_1+s_2+s_3+s_4=1$$

Zkusíme vyjádřit bod $[1,2,2]$ jako afinní kombinaci daných bodů:

$$[1,2,2]=s_1[1,0,-1]+s_2[-1,4,5]+s_3[3,4,5]+s_4[7,-3,1]$$

$$1 = a_1 - a_2 + 3a_3 + 7a_4$$
$$2 = 4a_2 + 4a_3 - 3a_4$$
$$2 = -a_1 + 5a_2 + 5a_3 + a_4$$
$$1 = a_1 + a_2 + a_3 + a_4$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 3 & 7 & 1 \\ 0 & 4 & 4 & -3 & 2 \\ -1 & 5 & 5 & 1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 2 & 6 & 0 \\ 0 & 4 & 4 & -3 & 2 \\ 0 & 6 & 6 & 2 & 3 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 2 & 6 & 0 \\ 0 & 0 & 8 & 9 & 2 \\ 0 & 0 & 12 & 20 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & 2 & 6 & 0 \\ 0 & 0 & 8 & 9 & 2 \\ 0 & 0 & 0 & -\frac{7}{2} & 0 \end{array}\right) \Rightarrow$$

$$\Rightarrow a_4 = 0 \Rightarrow 8a_3 = 2 \Rightarrow a_3 = \frac{1}{4}$$
$$-2a_2 + \frac{1}{2} = 0 \Rightarrow a_2 = \frac{1}{4}$$
$$\Rightarrow a_1 = \frac{1}{2}$$

Daný bod $[1,2,2]$ lze vyjádřit jako afinní kombinaci daných bodů, tato kombinace je dokonce konvexní, tedy bod $[1,2,2]$ leží v konvexním obalu bodů $[1,0,-1]$, $[-1,4,5]$, $[3,4,5]$ a $[7,-3,1]$