A_i ... množina slov, kde na i-tém místě začíná skupina BBB.

Počet slov, kde je BBB je

$$|A_1 \cup \dots \cup A_{10}| = |A_1| + \dots + |A_{10}| - |A_1 \cap A_2| \dots$$

BBABBABBABBA

1, 1

2, $12 = \binom{12}{1}$

3, $\binom{12}{2}$

4, $\binom{12}{3} - 10$

5, $\binom{12}{4} - \left(P(8,1,1) - 9\right) = \binom{12}{4} - \left(\frac{10!}{8!} - 9\right) = 414$

$\underbrace{BBB}_{\text{celek}}, \underbrace{A, A, \dots, A}_{8}, B$

AABBBBAA...

6) $\binom{12}{5} - \left(\underbrace{P(7,1,2)}_{3\times} - \underbrace{P(7,1,1)}_{2\times} \right) = 504$

BBBB

BBBBBB

7) $6 \times B$ spočítáme přímo slova obsahující 6 písmen B a žádnou skupinu BBB

i) $6\times$ samostatně stojící B

BABABABABAB $\binom{7}{1}$

ii) $1\times BB$, $4\times B$

BABBABABAB $\binom{7}{2}\cdot 5$

iii) $2\times BB$, $2\times B$

BABBABBAB $\binom{7}{3}\binom{4}{2}$

iv) $3\times BB$ BBABBABB $\binom{7}{4}$

8, Jedno písmeno B

0 $7\times B$ B B B B B B B

$1\times BB$ $5\times B$ BB A B A B A B A B A B 6

$2\times BB$ $3\times B$ BB A BB A B A B A B $\binom{6}{1}\binom{5}{2}$

$3\times BB$ $1\times B$ BB A BB A BB A B $\binom{6}{2}\cdot 4$

Jiný způsob: spočítejme počet slov délky 6 neobsahujících skupinu BBB.

A_i .... množina slov délky 6, kde na i-tém místě začíná skupina BBB

Počet slov délky 6 obsahujících skupinu BBB je

$$|A_1\cup A_2\cup A_3\cup A_4| = |A_1|+|A_2|+|A_3|+|A_4| - |A_1\cap A_2| - |A_2\cap A_3| - |A_3\cap A_4| - |A_1\cap A_3| - |A_1\cap A_4| - |A_2\cap A_4|$$

$$+|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + |A_1 \cap A_3 \cap A_4| - |A_1 \cap A_2 \cap A_3 \cap A_4|$$

$$+|A_2 \cap A_3 \cap A_4|$$

$$= 4\cdot 2^3 - (3\cdot 2^2 + \cancel{2} + \cancel{2} + \cancel{1}) + (\cancel{2} + \cancel{2} + \cancel{1} + \cancel{1}) - \cancel{1}$$

$$= 32 - (12) = 20$$

BBBXXX
XBBBXX

$A_1 \cap A_2$: BBBBXX
$A_1 \cap A_3$: BBBBBX
$A_2 \cap A_4$ XB....B
$A_1 \cap A_4$ BBBBBB

Slov neobsahujících BBB je tedy

$$2^6 - 20 = 64 - 20 = 44$$

$\overbrace{XXXX}^{4}AB$ BBA$\overbrace{XXX}^{3}$

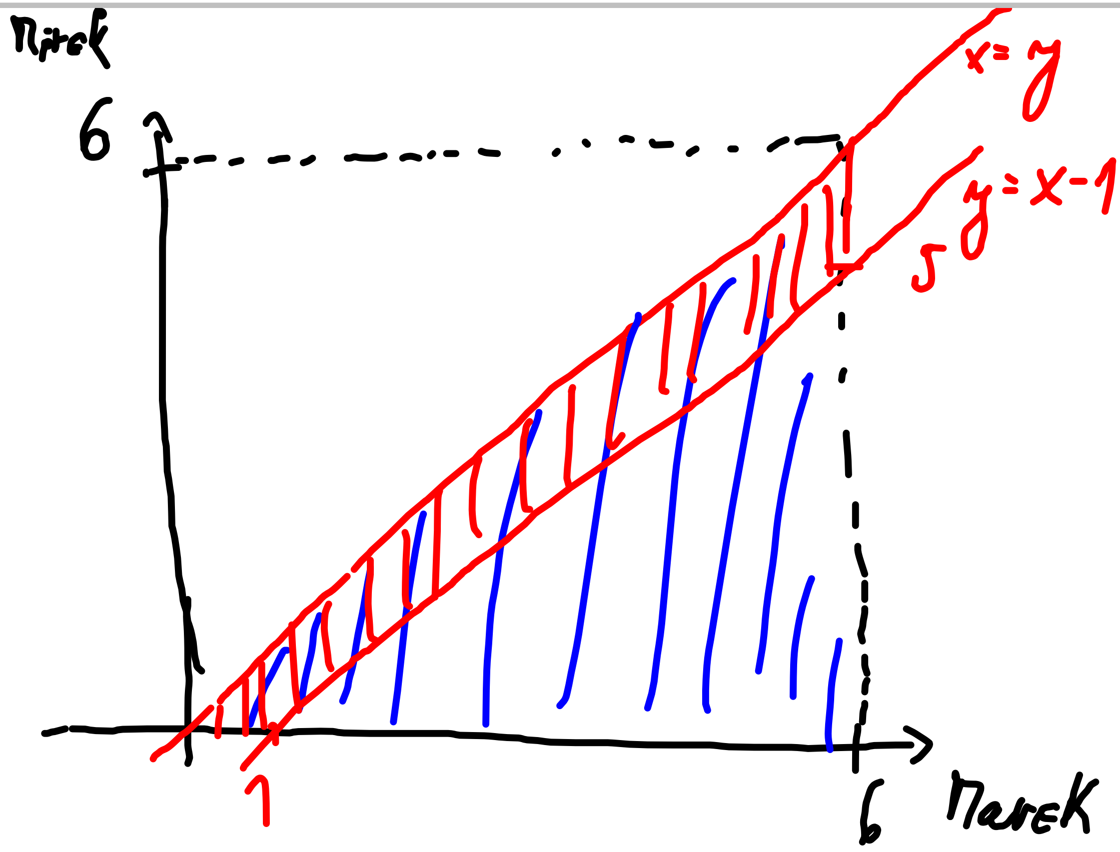
$\underbrace{\quad}_{3}$ ABB   BA$\underbrace{\quad}_{4}$

$\underbrace{\quad}_{3}$ ABB   BBA$\underbrace{\quad}_{3}$

$$a_{12} = a_6 \cdot a_6 - 2a_4 \cdot a_3 - a_3^2 =$$

$$= 44^2 - 2\cdot 13\cdot 7 - 7^2 =$$

$$= 1705$$



$y \leq x$

$x - 1 \leq y$

$$\frac{5{,}5}{36} = \frac{11}{72}$$

$$p: \begin{array}{l} x = 2 + 3t \\ y = t \end{array} \quad \Big|\cdot(-3)$$

$$x - 3y = 2$$

$$\underline{x - 3y - 2 = 0}$$

$$r: \begin{array}{l} x = -1 + s \\ y = 2 + 3s \end{array}$$

$$P = p \cap r: \quad (-1+s) - 3(2+3s) - 2 = 0$$

$$-9 - 8s = 0 \Rightarrow s = -\frac{9}{8}$$

Dosadím do parametr. vyjádření r:

$$x = -1 + \left(-\frac{9}{8}\right) = -\frac{17}{8}$$

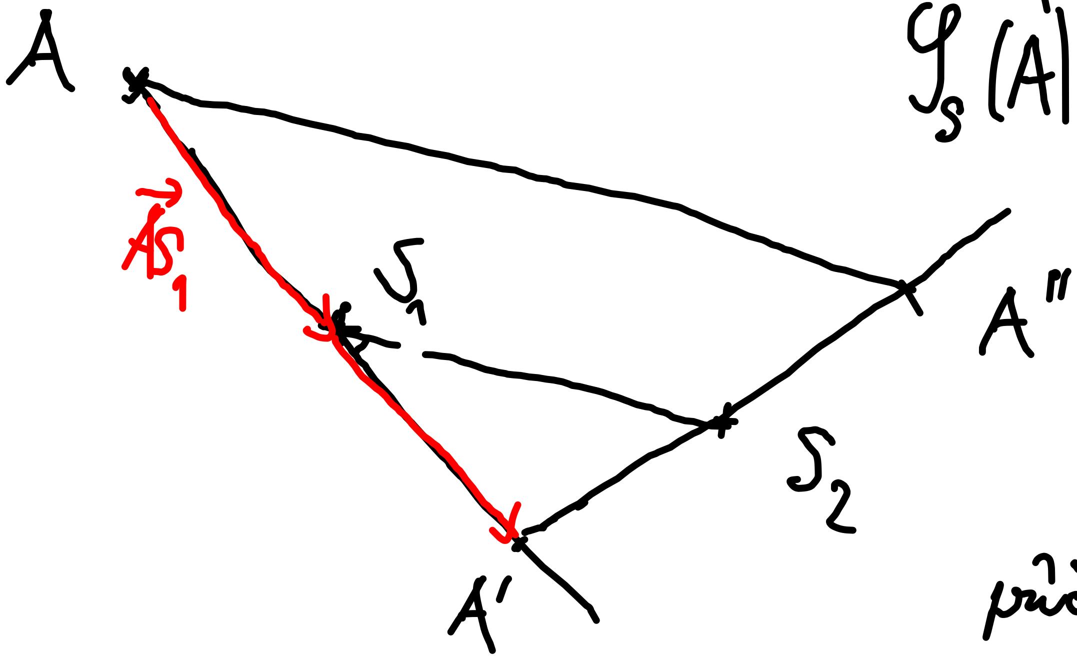
$$y = 2 + 3\cdot\left(-\frac{9}{8}\right) = -\frac{11}{8}$$

$$P = \left[-\frac{17}{8}, -\frac{11}{8}\right]$$

Ověření: $-\frac{17}{8} + 3\cdot\frac{11}{8} - 2 = 0$ ✓

$A = [1,1] \quad S = [3,4]$

$\varphi_S(A) = 2\cdot[3,4]-[1,1] =$
$[5,7]$



S_1S_2 je střední příčkou v $\triangle AA'A''$ rovnoběžnou s AA''

$|AA''| = 2|S_1S_2|$

$\overrightarrow{AA''} = 2\cdot\overrightarrow{S_1S_2}$

$A + 2(S-A) = 2S - A$

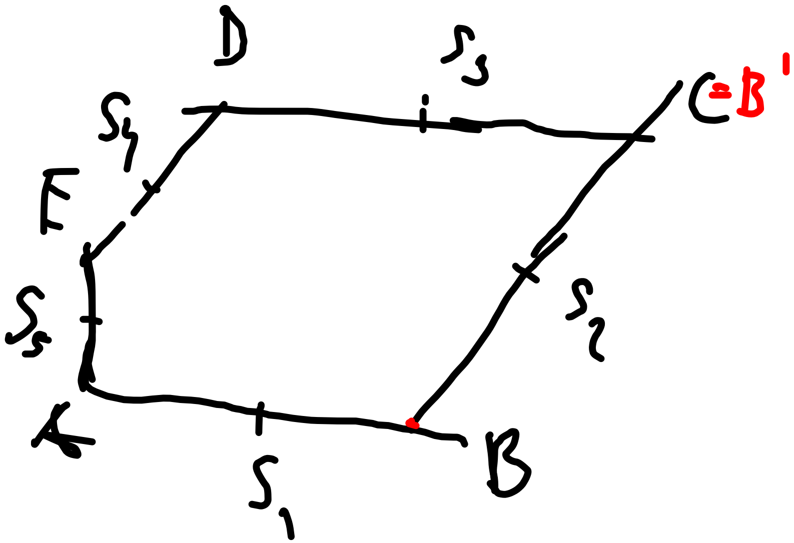
$\varphi_S(A) = 2S - A$

$$\varphi_{S_2}(\varphi_{S_1}(A)) = \varphi_{S_2}(2S_1 - A) = 2S_2 - (2S_1 - A) =$$
$$= \boxed{A + 2(S_2 - S_1)}$$

$$\varphi_{S_3}\left(\varphi_{S_2}\left(\varphi_{S_1}(A)\right)\right) = \varphi_{S_3}\left(A + 2(S_2 - S_1)\right) = 2S_3 - \left(A + 2(S_2 - S_1)\right) =$$

$$= 2(S_3 - S_2 + S_1) - A = \varphi_{(S_3 - S_2 + S_1)}(A)$$

$$\varphi_{S_4} \circ \underbrace{\varphi_{S_3} \circ \varphi_{S_2} \circ \varphi_{S_1}}_{\varphi_{S_3 - S_2 + S_1}} = \varphi_{S_4} \circ \varphi_{S_3 - S_2 + S_1} = \tau_{2(S_4 + S_2 - S_3 - S_1)}$$



$\varphi_{S_1}(A) = B$

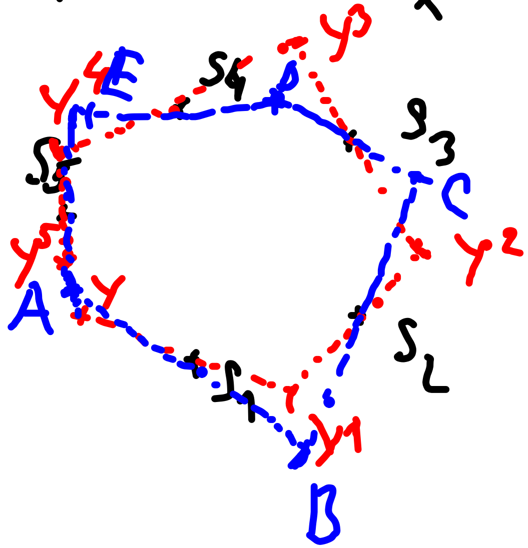
$\varphi_{S_2}(B) = C$

$\varphi_{S_3}(C) = D$

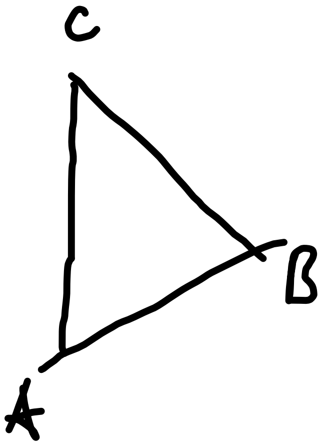
$\varphi_{S_4}(D) = E$

$\varphi_{S_5}(E) = A$

$$\underbrace{\varphi_{S_5} \circ \varphi_{S_4} \circ \varphi_{S_3} \circ \varphi_{S_2} \circ \varphi_{S_1}}_{\varphi_X}(A) = A \Rightarrow \varphi_X = \varphi_A$$

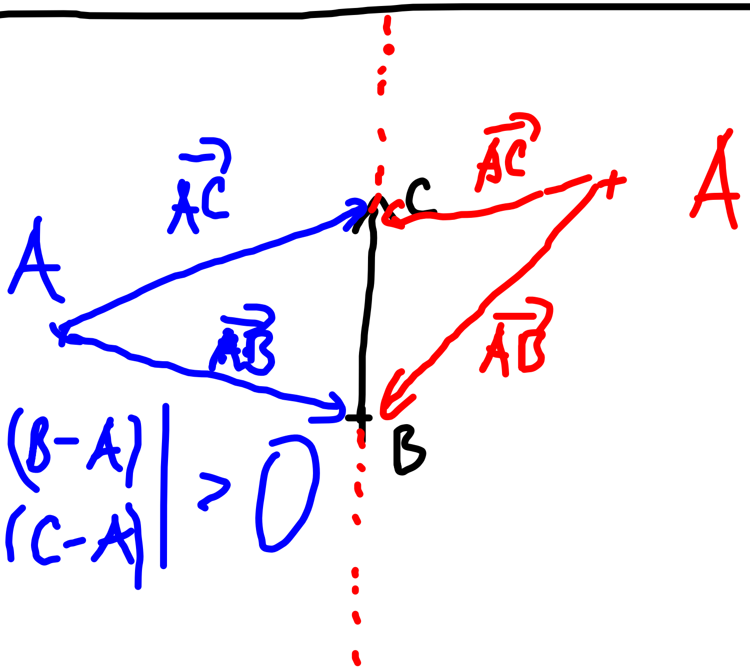


$A = \frac{1}{2}(Y + Y^5)$



$$S_{ABC} = \frac{1}{2}\left|\begin{matrix}(B-A)\\(C-A)\end{matrix}\right|$$

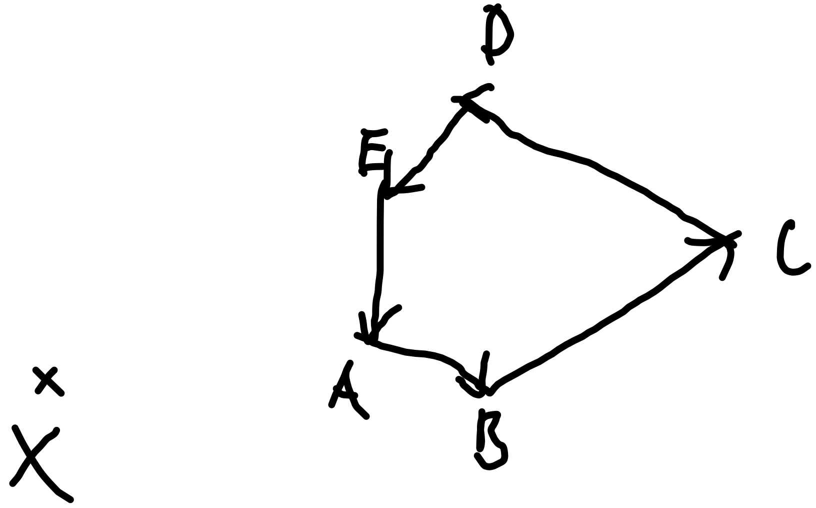
$$S = \frac{1}{2}\left|\begin{matrix}[8,8]-[2,2]\\ [3,5]-[2,2]\end{matrix}\right| = \frac{1}{2}\left|\begin{matrix}6 & 6\\ 1 & 3\end{matrix}\right| = \frac{1}{2}(18-6) = 6$$



$$\left|\begin{matrix}(B-A)\\(C-A)\end{matrix}\right| > 0$$

$$\left|\begin{matrix}(B-A)\\(C-A)\end{matrix}\right| < 0$$

Konvexní mnohoúhelník



X leží vpravo od $\vec{EA}$ ⇒ z X je vidět strana EA

X leží vlevo od BC ⇒ z X není vidět BC

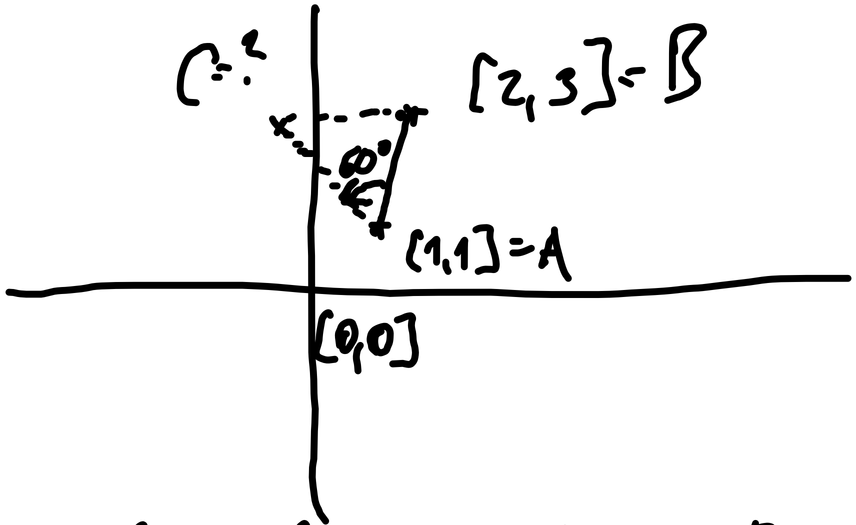
$X = [-4, 1]$ $\quad A = [3, 4]$, $B = [5, 7]$, $C = [4, 10]$

$$\begin{vmatrix} A-X \\ B-X \end{vmatrix} = \begin{vmatrix} 7 & 3 \\ 9 & 6 \end{vmatrix} = 42 - 27 > 0$$

$$\begin{vmatrix} B-X \\ C-X \end{vmatrix} = \begin{vmatrix} 9 & 6 \\ 8 & 9 \end{vmatrix} > 0$$

$$\begin{vmatrix} C-X \\ A-X \end{vmatrix} = \begin{vmatrix} 8 & 9 \\ 7 & 3 \end{vmatrix} < 0$$

⇒ AC je vidět, BC a AB nejsou vidět

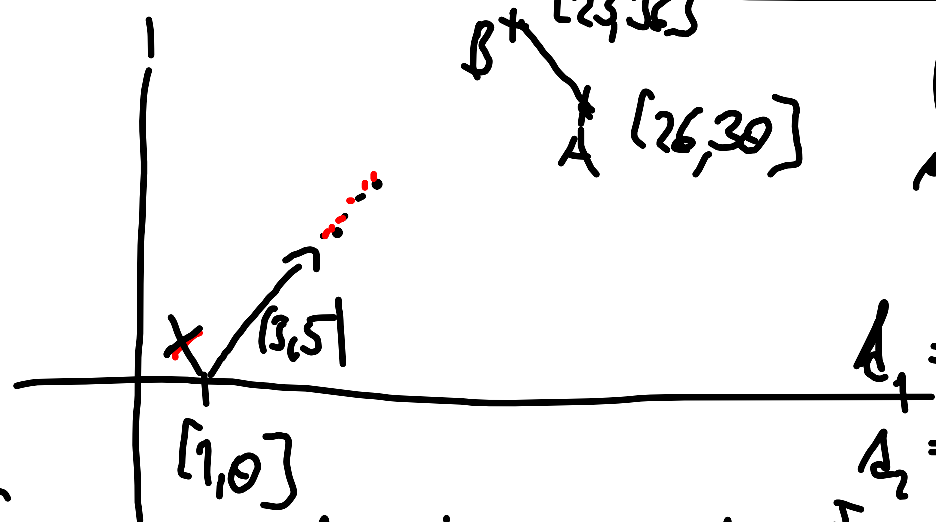


C je obrazem bodu B v otočení o 60° v kladném smyslu kolem bodu A

$$R_{\varphi,0} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

$$\varphi_1 = 60^\circ \qquad \begin{pmatrix} \cos 60^\circ & -\sin 60^\circ \\ \sin 60^\circ & \cos 60^\circ \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$



Porovnáme směrnic vektorů XA, XB a (3,5)

$$k_1 = \frac{30}{25} = \frac{6}{5}$$

$$k_2 = \frac{(36-0)}{(23-1)} = \frac{36}{22} = \frac{18}{11}$$

$$k_3 = \frac{5}{3}$$

$\frac{6}{5} < \frac{18}{11} < \frac{5}{3} \Rightarrow$ míč jde mimo