$$\mu_4^6 = 4^6 - \underline{\binom{4}{3}3^6} + \underline{\binom{4}{2}2^6} - \binom{4}{1}\cdot 1$$

$$\mu_1^6 = 1$$

$$\mu_2^6 = 2^6 - \binom{2}{1}\cdot\mu_1^6 = 2^6 - 2$$

$$\mu_3^6 = 3^6 - \binom{3}{2}\mu_2^6 - \binom{3}{1}\mu_1^6 = 3^6 - \binom{3}{2}(2^6-2) - 3$$

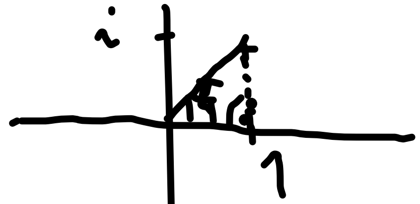
$$\mu_4^6 = 4^6 - \underline{\binom{4}{3}\mu_3^6} - \underline{\binom{4}{2}\mu_2^6} - \underline{\binom{4}{1}\mu_1^6} =$$

$$= 4^6 - \binom{4}{3}\left(3^6 - \binom{3}{2}(2^6-2) - 3\right) - \binom{4}{2}(2^6-2) - 4$$

$$= 4^6 - \binom{4}{3}3^6 + 2^6\left(\binom{4}{3}\binom{3}{2} - \binom{4}{2}\right) - \binom{4}{3}\binom{3}{2}\cdot 2 + \binom{4}{3}\cdot 3 + \binom{4}{2}\cdot 2 - 4$$

$$= 4^6 - \binom{4}{3}3^6 + 2^6\cdot\binom{4}{2} - 24 + 12 + 12 - 4$$

$$\begin{vmatrix} 1 & 0 & 0 & a \\ 0 & a & 1 & 0 \\ 0 & 1 & a & 0 \\ a & 0 & 0 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} a & 1 & 0 \\ 1 & a & 0 \\ 0 & 0 & 1 \end{vmatrix} - a \begin{vmatrix} 0 & a & 1 \\ 0 & 1 & a \\ a & 0 & 0 \end{vmatrix} =$$



$$= a^2 - 1 - a(a^3 - a) = -a^4 + 2a^2 - 1$$

$$-a^4 + 2a^2 - 1 = 1$$

$$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$a^4 - 2a^2 + 2 = 0 \qquad b = a^2$$

$$b^2 - 2b + 2 \qquad b_{1,2} = \frac{2 \pm \sqrt{-4}}{2} = \underline{1 \pm i}$$

Zapíšeme $b_{1,2}$ v goniometrickém tvaru:

$$b_{1,2} = \sqrt{2}\left(\cos\frac{\pi}{4} \pm i \sin\frac{\pi}{4}\right)$$

$$a_1 = \sqrt[4]{2}\left(\cos\frac{\pi}{8} + i \sin\frac{\pi}{8}\right)$$

$$a_2 = \sqrt[4]{2}\left(\cos\frac{9\pi}{8} + i \sin\frac{9\pi}{8}\right)$$

Moivreova veta:

$$[r(\cos\varphi + i\sin\varphi)]^n = r^n(\cos n\varphi + i \sin n\varphi)$$

$$a_3 = \sqrt[4]{2}\left(\cos\left(-\frac{\pi}{8}\right) + i\sin\left(-\frac{\pi}{8}\right)\right) = \sqrt[4]{2}\left(\cos\frac{\pi}{8} - i\sin\frac{\pi}{8}\right)$$

$$a_4 = \sqrt[4]{2}\left(\cos\left(\frac{7\pi}{8}\right) + i\sin\left(\frac{7\pi}{8}\right)\right)$$

$$(\cos\varphi + i\sin\varphi)^2 = (\cos 2\varphi + i\sin 2\varphi)$$

$$= \cos^2\varphi + 2i\cos\varphi\sin\varphi - \sin^2\varphi$$

$$\mathrm{Re}: \cos^2\varphi - \sin^2\varphi = \cos 2\varphi$$

$$\mathrm{Im}: 2\cos\varphi\sin\varphi = \sin 2\varphi$$

$$\left(\begin{matrix} 1 & 2 & 2 \\ 1 & 1 & 2 \\ -1 & 1 & 1 \end{matrix}\;\middle|\;\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 2 & 2 \\ 0 & -1 & 0 \\ 0 & 3 & 3 \end{matrix}\;\middle|\;\begin{matrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 2 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{matrix}\;\middle|\;\begin{matrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 3 & 1 \end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\;\middle|\;\begin{matrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -\frac{2}{3} & 1 & \frac{1}{3} \end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\;\middle|\;\begin{matrix} \frac{1}{3} & 0 & -\frac{2}{3} \\ 1 & -1 & 0 \\ -\frac{2}{3} & 1 & \frac{1}{3} \end{matrix}\right)$$

Báze v daném vekt. prostoru mají stejný počet prvků.

Náznak důkazu:

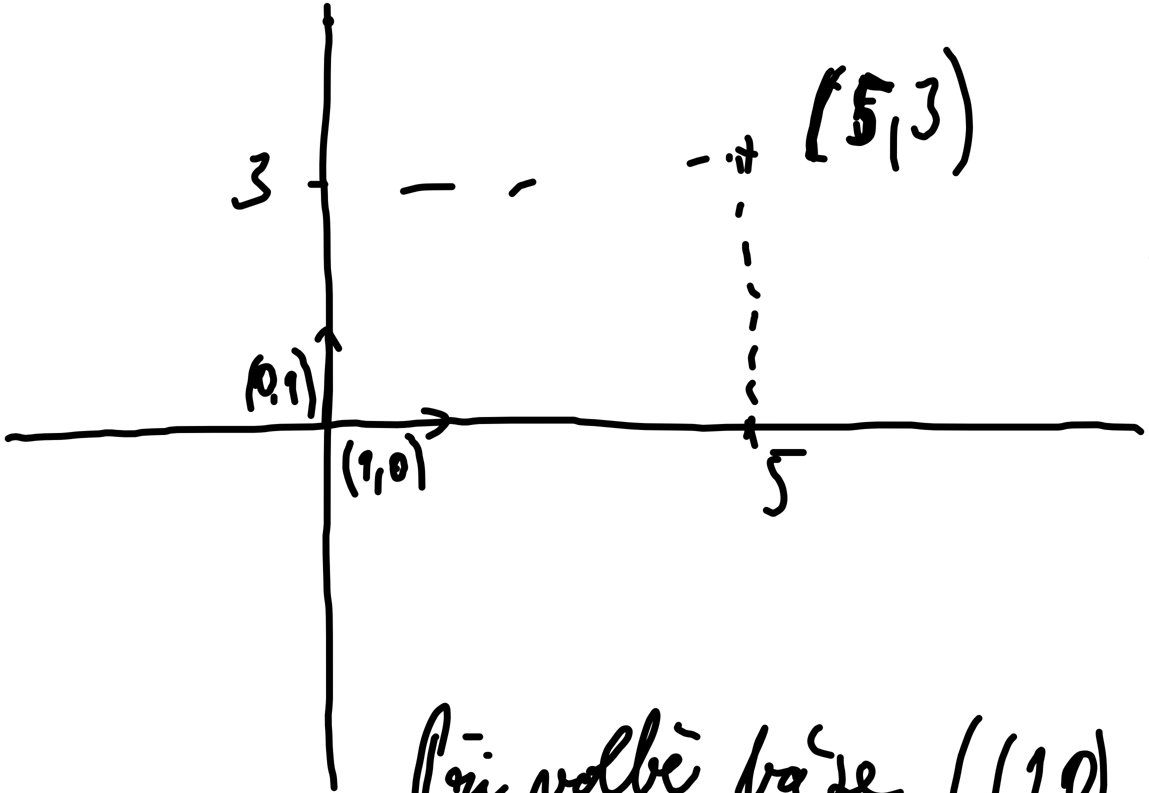
nechť $(e_1, \dots, e_n)$ a $(f_1, \dots, f_s)$ jsou 2 různé báze a $n < s$.

Pak $e_1 = \sum_{j=1}^{s} a^1_j f_j = a^1_1 f_1 + a^1_2 f_2 + \dots + a^1_s f_s$

$$e_i = \sum_{j=1}^{s} a^i_j f_j$$

$$f_1 = \sum_{i=1}^{n} b_i e_i = \sum_{i=1}^{n} b_i \sum_{j=1}^{s} a^i_j f_j$$

$$\sum_{i=1}^{n} \sum_{j=1}^{s} b_i a^i_j f_j - f_1 = 0 \qquad \lightning$$



$(5,3) = 5(1,0) + 3(0,1)$

souřadnice vektoru $(5,3)$ v bázi $\{(1,0),(0,1)\}$

Při volbě báze $((1,0),(1,1))$ tak platí

$(5,3) = 2\cdot(1,0) + 3\cdot(1,1)$

souřadnice vektoru $(5,3)$ v bázi $\{(1,0),(0,1)\}$

Char. polynom dané dif. rce je

$x^2 - 2x + 2$

len má kořeny $(1+i)$ a $(1-i)$, tedy množinou

všech řešení dané dif. rce je vektorový prostor s

bazí $\left(\underbrace{\{(1+i)^n\}_{n=1}^{\infty}}_{e_1}, \underbrace{\{(1-i)^n\}_{n=1}^{\infty}}_{e_2} \right)$.

Je-li (e_1, e_2) báze kompl. vekt. prostoru V, pak i

$\left(\underbrace{\frac{1}{2}(e_1 + e_2)}_{f_1}, \underbrace{\frac{1}{2i}(e_1 - e_2)}_{f_2} \right)$ je báze daného

prostoru.

$e_1 = f_1 + i f_2$

$e_2 = f_1 - i f_2$

V našem případě dostaváme

$$f_1 = \frac{1}{2}\left[\{(1+i)^n\}_{n=1}^{\infty} + \{(1-i)^n\}_{n=1}^{\infty} \right] =$$

$$f_1 = \frac{1}{2}\left(\left\{ \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^n \right\}_{n=1}^{\infty} + \left\{ \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^n \right\}_{n=1}^{\infty} \right)$$

$$+ i \sin\left(-\frac{\pi}{4}\right)$$

$$= \frac{1}{2}\left(\left\{ (\sqrt{2})^n \left(\cos\frac{n\pi}{4} + \cancel{i\sin\frac{n\pi}{4}}\right) \right\}_{n=1}^{\infty} + \left\{ (\sqrt{2})^n \left(\cos\frac{n\pi}{4} - \cancel{i\sin\frac{n\pi}{4}}\right) \right\}_{n=1}^{\infty} \right)$$

$$= \frac{1}{2}\left(\left\{ 2(\sqrt{2})^n \left(\cos\frac{n\pi}{4}\right) \right\}_{n=1}^{\infty} \right) = \left\{ (\sqrt{2})^n \cos\frac{n\pi}{4} \right\}_{n=1}^{\infty}$$

$$f_2 = \frac{1}{2i}\left(e_1 - e_2\right) = \left\{ (\sqrt{2})^n \sin\frac{n\pi}{4} \right\}_{n=1}^{\infty}$$

$$x_2 = 2x_1 - 2x_0 \Rightarrow x_0 = \frac{1}{2}(2x_1 - x_2) = 0$$

ii) vyjádříme jedinou posloupnost vyhovující dané lin. dif. rci jako lineární kombinaci vektorů e_1, e_2.

$$a \cdot e_1 + b \cdot e_2, \quad a, b \in \mathbb{C}$$

$$x_0: \quad 0 = a \cdot 1 + b \cdot 1 \Rightarrow a + b = 0$$

$$x_1 = 1 = a\cdot(1+i)^1 + b\cdot(1-i)^1 = \underbrace{a+b}_{0} + i(a-b) \Rightarrow$$

$$\Rightarrow (a-b) = -i$$
$$a+b = 0$$
$$2a = -i \Rightarrow a = -\frac{i}{2} \Rightarrow b = \frac{i}{2} \Rightarrow$$

jediným

Řešením dané rek. rce s poč. podmínkami je posloupnost

$$\left\{-\frac{i}{2}(1+i)^n + \frac{i}{2}(1-i)^n\right\}_{n=1}^{\infty}$$

Tuto posloupnost můžeme vyjádřit i pomocí báze

f_1, f_2: $\quad x_0 = 0 = c\cdot(\sqrt{2})^0\cos 0 + d\cdot(\sqrt{2})^0\sin 0 = c$

$$x_1 = 1 = c\cdot\sqrt{2}\cos\frac{\pi}{4} + d\cdot(\sqrt{2})\sin\frac{\pi}{4} =$$
$$= c + d \Rightarrow d = 1$$

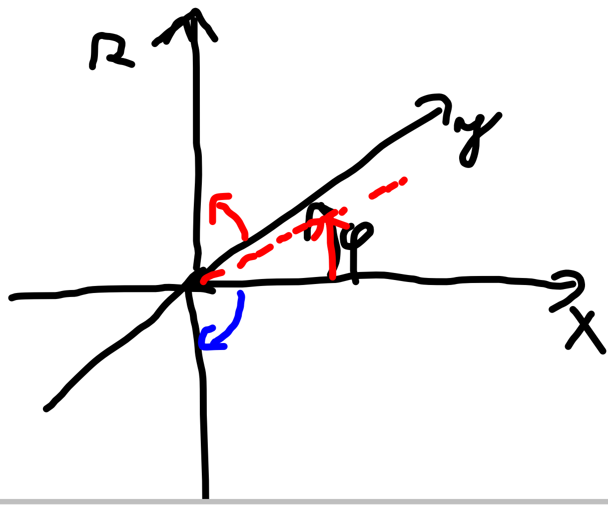
$\Rightarrow f_1$ je řešením dané rce.

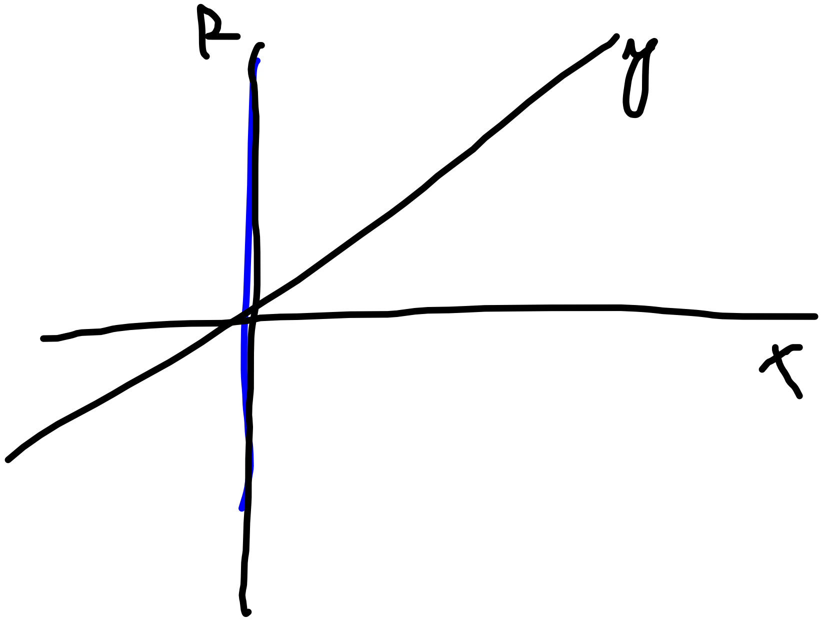
$$x_2 = x_3$$

$$2x_1 + 3x_3 + x_3 - x_4 = 1$$

$$x_4 = 2x_1 + 4x_3 - 1$$

Pro libovolnou volbu $x_1, x_3 \in \mathbb{R}$ $(\in \mathbb{C})$ dostáváme řešení. Řešení je tedy dvojparametrický systém (afinní prostor) $\{(x_1, x_3, x_3, 2x_1 + 4x_3 - 1) \mid x_1, x_3 \in \mathbb{R}\}$





Zrcadlení podle roviny zy : $[x, y, z] \mapsto [-x, y, z]$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$