$$a_{ij} = -a_{ji}, \quad j = i \Rightarrow a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$$

$n = 3$

$$\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} = a \cdot \underline{\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}} + b \cdot \underline{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}} + c \underline{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}}$$

$a, b, c \in \mathbb{R}$

Báze prostoru antisym. matic $n \times n$ nad $\mathbb{R}$:

$$\{A_{ij}\}_{1 \le i < j \le n}, \text{ kde } A_{ij} = \begin{array}{c} \\ i \\ \\ j \\ \\ \end{array} \begin{pmatrix} 0 & \cdots & 0 & 0 \\ 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & 0 \\ \cdots & -1 & \cdots & 0 \\ 0 & \cdots & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ -1 & & & & \\ 0 & & & & \\ \vdots & & & & \\ 0 & \cdots & & & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & & \vdots \\ -1 & 0 & 0 & & \\ 0 & \cdots & & & 0 \end{pmatrix}, \ldots$$

$$x + iy = a(2+i) + b(1-2i)$$

lib. kompl. číslo

hledáme $a, b \in \mathbb{R}$

$x \in \mathbb{R},\ y \in \mathbb{R}$

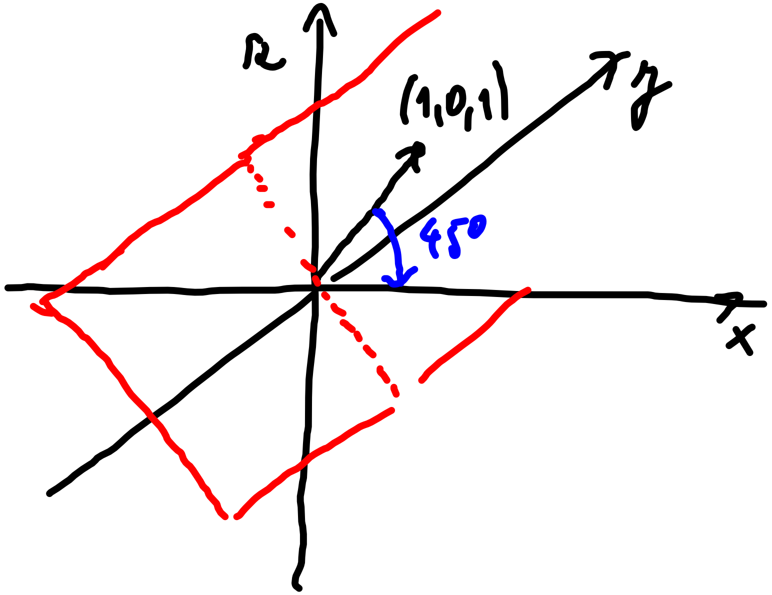
$$\left.\begin{aligned} \mathrm{Re}:\ & x = 2a + b \\ \mathrm{Im}:\ & y = a - 2b \end{aligned}\right\} \Rightarrow \begin{aligned} 2x + y &= 5a \Rightarrow a = \frac{2x+y}{5} \\ x - 2y &= 5b \Rightarrow b = \frac{x-2y}{5} \end{aligned}$$

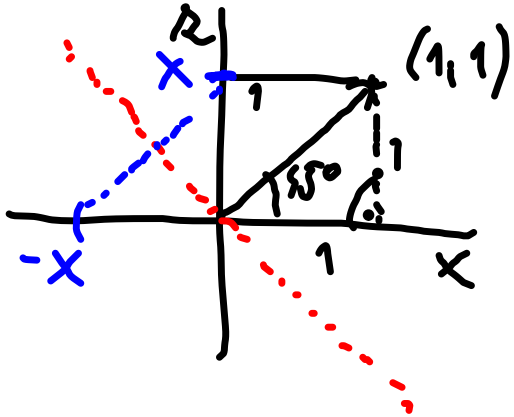
konkrétně pro $x = 4,\ b = 1 \Rightarrow a = \frac{9}{5},\ b = \frac{2}{5}$

$$\left(\frac{9}{5}, \frac{2}{5}\right)$$

jiná báze $1, i$

$$\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 0 & 5 \\ 1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 \\ 0 & 5 \end{pmatrix}$$





$$\begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 1 & a & \boxed{1} & a \\ a & 1 & \boxed{0} & 1 \\ a & a & \boxed{0} & a \\ a & a & \boxed{a} & 1 \end{vmatrix} = 1 \cdot (-1)^{1+3} \cdot \begin{vmatrix} a & 1 & 1 \\ a & a & a \\ a & a & 1 \end{vmatrix} + \cancel{0 \cdot (-1)^{2+3} \begin{vmatrix} 1 & a & a \\ a & a & a \\ a & a & 1 \end{vmatrix}}$$

rozvineme determinant podle 3. sloupce
(sloupec s největším počtem nul)

$$+ \cancel{0 \cdot (-1)^{3+3} \begin{vmatrix} 1 & a & a \\ a & 1 & 1 \\ a & a & 1 \end{vmatrix}} + a \cdot (-1)^{4+3} \begin{vmatrix} 1 & a & a \\ a & 1 & 1 \\ a & a & a \end{vmatrix} =$$

$$= a^2 + a^2 + a^2 - a^2 - a^3 - a - a\left(\underbrace{a + a^3 + a^2 - a^2 - a - a^3}_{0}\right)$$

$$= -a^3 + 2a^2 - a$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 2 + 2 + 0 - 6 - 0 - 6 = -8$$

$$A^* = \begin{pmatrix} 2 & -2 & -4 \\ -2 & -2 & 8 \\ -2 & 2 & -4 \end{pmatrix}$$

$$(A^*)^T = \begin{pmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} \end{pmatrix} = \begin{pmatrix} 2 & -2 & -2 \\ -2 & -2 & 2 \\ -4 & 8 & -4 \end{pmatrix}$$

$$A^{-1} = \frac{(A^*)}{|A|} = \begin{pmatrix} -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} & -1 \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{cccc|cccc} 1 & a & 1 & a & 1 & 0 & 0 & 0 \\ a & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ a & a & 0 & a & 0 & 0 & 1 & 0 \\ a & a & a & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & a & 1 & a & 1 & 0 & 0 & 0 \\ 0 & 1-a^2 & -a & 1-a^2 & -a & 1 & 0 & 0 \\ 0 & a-a^2 & -a & a-a^2 & -a & 0 & 1 & 0 \\ 0 & a-a^2 & 0 & 1-a^2 & -a & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|} 1 & a & 1 & a \\ 0 & 1-a^2 & -a & 1-a^2 \\ 0 & 0 & \frac{a^2}{(1+a)}-a & \\ 0 & & & \end{array}\right)$$

inverze existuje

$\Leftrightarrow$ $|A| \neq 0$

$2a^2 - a^3 - a =$

$-a(a-1)^2 \neq 0$

$a \neq 0, a \neq 1$

$$\frac{a-a^2}{1-a^2} = \frac{a}{(1+a)}$$

Výpočet složitý, použijeme jiný způsob

$$
(A^*)^T = \begin{pmatrix}
\begin{vmatrix} 1 & 0 & 1 \\ a & 0 & a \\ a & a & 1 \end{vmatrix} & -\begin{vmatrix} a & 0 & 1 \\ a & 0 & a \\ a & a & 1 \end{vmatrix} & \begin{vmatrix} a & 1 & 1 \\ a & a & a \\ a & a & 1 \end{vmatrix} & -\begin{vmatrix} a & 1 & 0 \\ a & a & 0 \\ a & a & a \end{vmatrix} \\
-\begin{vmatrix} a & 1 & a \\ a & 0 & a \\ a & a & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 & a \\ a & 0 & a \\ a & a & 1 \end{vmatrix} & -\begin{vmatrix} 1 & a & a \\ a & a & a \\ a & a & 1 \end{vmatrix} & \begin{vmatrix} 1 & a & 1 \\ a & a & 0 \\ a & a & a \end{vmatrix} \\
\begin{vmatrix} a & 1 & a \\ 1 & 0 & 1 \\ a & a & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 & a \\ a & 0 & 1 \\ a & a & 1 \end{vmatrix} & \begin{vmatrix} 1 & a & a \\ a & 1 & 1 \\ a & a & 1 \end{vmatrix} & -\begin{vmatrix} 1 & a & 1 \\ a & 1 & 0 \\ a & a & a \end{vmatrix} \\
-\begin{vmatrix} a & 1 & a \\ 1 & 0 & 1 \\ a & 0 & a \end{vmatrix} & \begin{vmatrix} 1 & 1 & a \\ a & 0 & 1 \\ a & 0 & a \end{vmatrix} & -\begin{vmatrix} 1 & a & a \\ a & 1 & 1 \\ a & a & a \end{vmatrix} & \begin{vmatrix} 1 & a & 1 \\ a & 1 & 0 \\ a & a & 0 \end{vmatrix}
\end{pmatrix}
$$

$$
A^{-1} = \frac{(A^*)}{|A|} = \frac{1}{-a(a-1)^2} \begin{pmatrix} 0 & a^2-a & a^2+a-1-a^2 & \cdots \\ & & & \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{(a-1)} & \frac{1}{a(a-1)} \\ & & \end{pmatrix}
$$

$$A\vec{x} = \vec{b} \Rightarrow \vec{x} = (A^{-1})\cdot\vec{b}$$

$$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & -2 & -1 \\ 1 & 3 & 2 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$$

$$|A| = -8+1+9+2+6+6 = 16$$

$$x = \frac{\begin{vmatrix} 1 & -1 & 1 \\ 3 & -2 & -1 \\ 5 & 3 & 2 \end{vmatrix}}{|A|} = \frac{-4+5+9+10+3+6}{16} = \frac{29}{16}$$

$$y = \frac{\begin{vmatrix} 2 & 1 & 1 \\ 3 & 3 & -1 \\ 1 & 5 & 2 \end{vmatrix}}{|A|} = \frac{12-1+15-3-6+10}{16} = \frac{27}{16}$$

$$z = \frac{\begin{vmatrix} 2 & -1 & 1 \\ 3 & -2 & 3 \\ 1 & 3 & 5 \end{vmatrix}}{16} = \frac{-20+9-3+2-18+15}{16} = -\frac{15}{16}$$