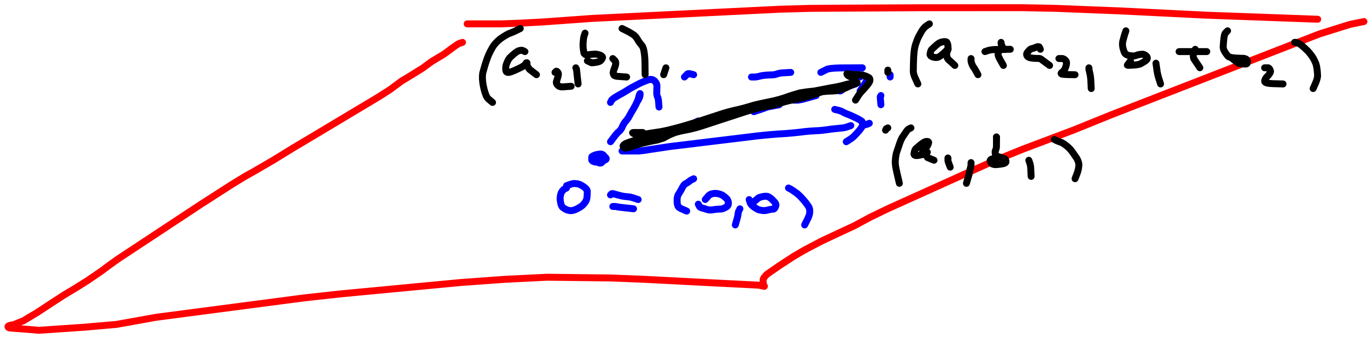
$\underbrace{(1,2,3,17)} \in \mathbb{R}^4$ $\qquad$ $\begin{pmatrix} 1 \\ 2 \\ 3 \\ 17 \end{pmatrix} \in \mathbb{R}^4$

$\mathbb{R}^n$: $\qquad \{1,2,3,4\} \longrightarrow \mathbb{R}$

$\mathbb{R} \longleftrightarrow \mathbb{K}$ $\qquad \underbrace{\mathbb{Z}_2, \mathbb{Z}, \mathbb{R}, \mathbb{Q}, \mathbb{C}}$

$\sim \mathbb{Z}_4$; $\quad 2 \cdot 2 = 0 \Rightarrow 2^{-1}$ neexistuje



$$a\big((b_1,\dots,b_n) + (c_1,\dots,c_n)\big) =$$

$$(b_1,\dots,b_n) = v, \qquad (c_1,\dots,c_n) = w$$

$$= a\,(b_1+c_1,\dots,b_n+c_n)$$ sčítání vektorů

$$= (a(b_1+c_1),\dots,a(b_n+c_n))$$ násobení vekt.

$$= (ab_1+ac_1,\dots,ab_n+ac_n)$$ distrib. [illegible]

$$= (ab_1,\dots,ab_n) + (ac_1,\dots,ac_n)$$ sčítání vekt.

$$= a\,(b_1,\dots,b_n) + a\,(c_1,\dots,c_n)$$ násobení vekt.

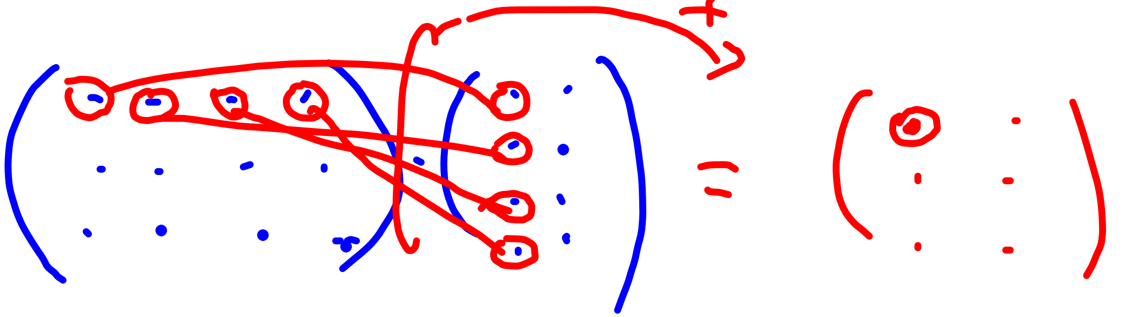
$$a(b_1,\dots,b_n) = av, \qquad a(c_1,\dots,c_n) = aw$$

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad a \cdot E = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A \cdot x = y$$

známé, hledáme, známé

$$y_i = a_{i1} x_1 + a_{i2} x_2 + \dots + a_{in} x_n$$



$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2\cdot 2-1\cdot 1=3 & 2\cdot 1+1\cdot 0=2 & 2\cdot 1+1\cdot 1=3 \\ & & \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

$Mat_n(\mathbb{K})$ .. matice typu n/n nad $\mathbb{K}$

$$+ : Mat_n(\mathbb{K}) \times Mat_n(\mathbb{K}) \to Mat_n(\mathbb{K})$$

$$\cdot : Mat_n(\mathbb{K}) \times Mat_n(\mathbb{K}) \to Mat_n(\mathbb{K})$$

$$\boxed{\begin{array}{l} A + B = B + A, \quad \exists -A \ \forall A \\ A + (B + C) = (A + B) + C, \quad \exists\, 0 \end{array}}$$

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C \qquad (O1)$$

$$\forall A \quad A \cdot E = E \cdot A = A \qquad (O3)$$

$$\left.\begin{array}{l} A \cdot (B + C) = A \cdot B + A \cdot C \\ (B + C) \cdot A = B \cdot A + C \cdot A \end{array}\right\} \qquad (O4)$$

$$A \cdot B \neq B \cdot A \quad (\text{obecně})$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|ccc} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \hline \multicolumn{2}{c|}{0} & \multicolumn{3}{c}{0} \end{array}\right) \cdot \left(\begin{array}{cc|ccc} 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \hline \multicolumn{2}{c|}{0} & \multicolumn{3}{c}{0} \end{array}\right)$$

$$= \left(\begin{array}{cc|c} 0 & 1 & \\ 0 & 0 & 0 \\ \hline \multicolumn{2}{c|}{0} & 0 \end{array}\right) \Rightarrow$$

obecně

$A \cdot B = 0$

$A \neq 0, \; B \neq 0$

$$\underbrace{A\cdot(B\cdot C)} = (A\cdot B)\cdot C$$

$A = (a_{ij})$ $\quad B = (b_{jk})$ $\quad C = (c_{kl})$

$$A\cdot B = \Big(\underbrace{\sum_j a_{ij} b_{jk}}_{(A\cdot B)_{ik}}\Big) \qquad B\cdot C = \sum_k b_{jk} c_{kl}$$

$$A\cdot(B\cdot C) = \sum_j a_{ij}\Big(\sum_k (b_{jk} c_{kl})\Big) = \sum_j \sum_k (a_{ij} b_{jk}) c_{kl}$$

$$= \sum_{k,j} (a_{ij} b_{jk}) c_{kl} = (A\cdot B)\cdot C$$

$$A \cdot X = E$$

$$A: \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right) \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 \\ 0 & 0 & 2 & -2 \end{array}\right)$$

$$x + y + z = 1 \qquad \Rightarrow \quad x = 2$$

$$x - 2y + 2z = 0 \qquad / -(1)$$

$$y - z = 1$$

$$3y - z = 1$$

$$y - z = 1 \qquad (-3) \qquad \Rightarrow \quad y = 0$$

$$2z = -2 \Rightarrow z = -1$$

$$A=\begin{pmatrix}0&2&1\\2&3&0\\1&2&1\end{pmatrix}\overset{P_1=\boxed{\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}}}{\sim}\begin{pmatrix}1&2&1\\2&3&0\\0&2&1\end{pmatrix}\overset{P_2=\begin{pmatrix}1&0&0\\-2&1&0\\0&0&1\end{pmatrix}}{\sim}\begin{pmatrix}1&2&1\\0&-1&-2\\0&2&1\end{pmatrix}\sim$$

$$\overset{P_3}{\sim}\begin{pmatrix}1&2&1\\0&-1&-2\\0&0&-3\end{pmatrix}\overset{P_4}{\sim}\begin{pmatrix}1&2&0\\0&-1&0\\0&0&1\end{pmatrix}\overset{P_5}{\sim}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

$$\underbrace{(P_5P_4P_3P_2P_1)}_{A^{-1}}A=E$$

$$\left(\begin{matrix}0&2&1\\2&3&0\\1&2&1\end{matrix}\;\middle|\;\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right)$$

$$\left(\begin{matrix}1&2&1\\2&3&0\\0&2&1\end{matrix}\;\middle|\;\begin{matrix}0&0&1\\0&1&0\\1&0&0\end{matrix}\right)$$

$$\begin{pmatrix}1&2&1\\0&-1&-2\\0&2&1\end{pmatrix}\Bigg|\cdot\begin{pmatrix}0&0&1\\0&1&-2\\1&0&0\end{pmatrix}=P_2P_1$$

$$\left(\begin{matrix}1&2&1\\0&-1&2\\0&0&-3\end{matrix}\;\middle|\;\begin{matrix}0&0&1\\0&1&2\\1&2&-4\end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 2 & 1 \\ 0 & -1 & 2 \end{matrix}\right.$$