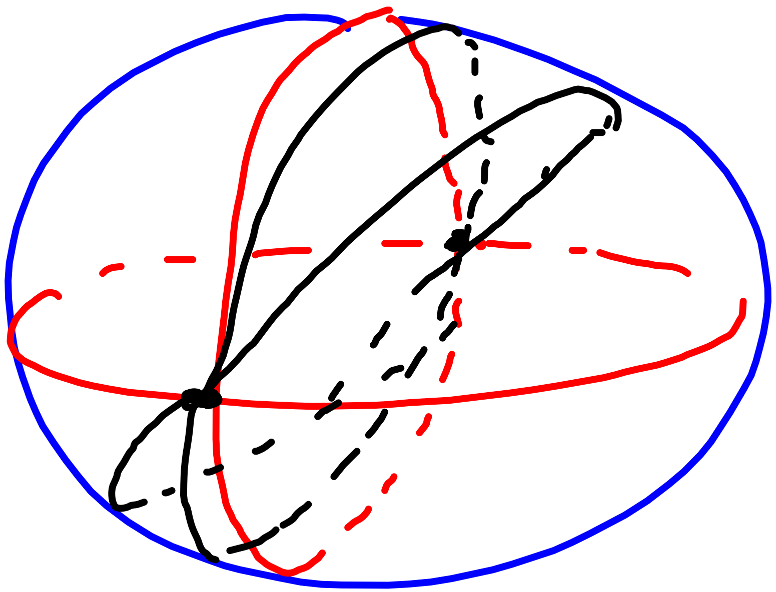
úroky:

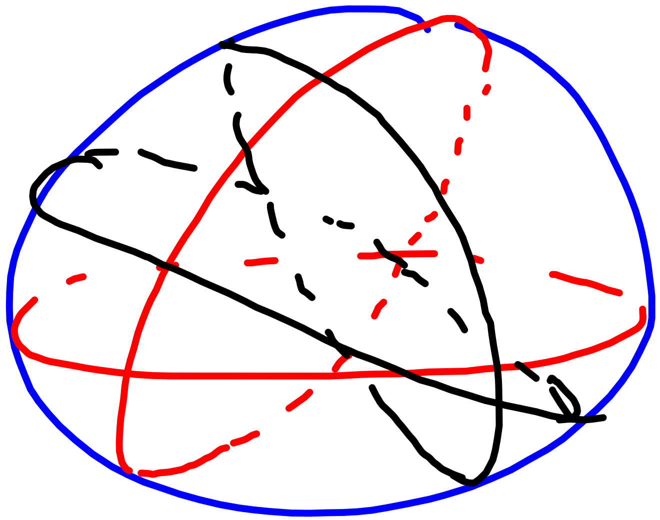
a) měsíční? $(1{,}03)^{1/12}$ = koeficient

3% úrok za rok

3/12 % úrok za měsíc

b) roční? – „průměrný vklad"





$\cancel{a_0 = 1}$

$a_1 = 2$

$a_2 = 4$

min:

$$\boxed{a_n = a_{n-1} + 2}$$

$$a_n = 2n$$

max:

$$a_3 = 8$$

$$\boxed{a_n = a_{n-1} + 2(n-1)}$$

$$a_n = a_{n-2} + 2(n-1) + 2(n-2) = a_1 + 2\sum_{i=1}^{n-1} i$$

$$\boxed{= 2\cdot\frac{1}{2}\, n\cdot(n-1) + 2 = n^2 - n + 2}$$

$$\sum_{i=1}^{n} i^4 = a n^5 + b n^4 + \dots + f$$

$$n=1: \quad 1 = a + b + c + d + e + f$$

$$17 = a\,32 + b\,16 + \dots$$

$$\sum_{i=1}^{n} i^k = \frac{1}{k+1}(n+1)^{k+1} - \frac{1}{2}(n+1)^k + \dots$$

$$(a+b)^k = a^k + k a^{k-1} b + \binom{k}{2} a^{k-2} b^2 + \dots$$

$$(n+1)^5 = n^5 + 5n^4 + 10n^3 + 10n^2 + 5n + 1$$

$$n^5 = (n-1)^5 + 5(n-1)^4 + 10(n-1)^3 + \dots$$

$$\vdots$$

$$2^5 = 1 + 5 + 10 + \dots$$

$$\sum_{i=2}^{n+1} i^5 = \sum_{i=1}^{n} i^5 + 5 \boxed{\sum_{i=1}^{n} i^4} + \boxed{10 \sum_{i=1}^{n} i^3 + \dots}$$

$$(n+1)^5 - 1 = 5 \boxed{} + \dots$$

1

1 2 1

1 3 3 1

1 4 6 4 1

→ 1 5 10 10 5 1

Název: IX 24-12:40 (4 z 10)

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + a^{n-2}b^2\binom{n}{2} + \dots$$

$a=1, \quad b=\pm 1:$

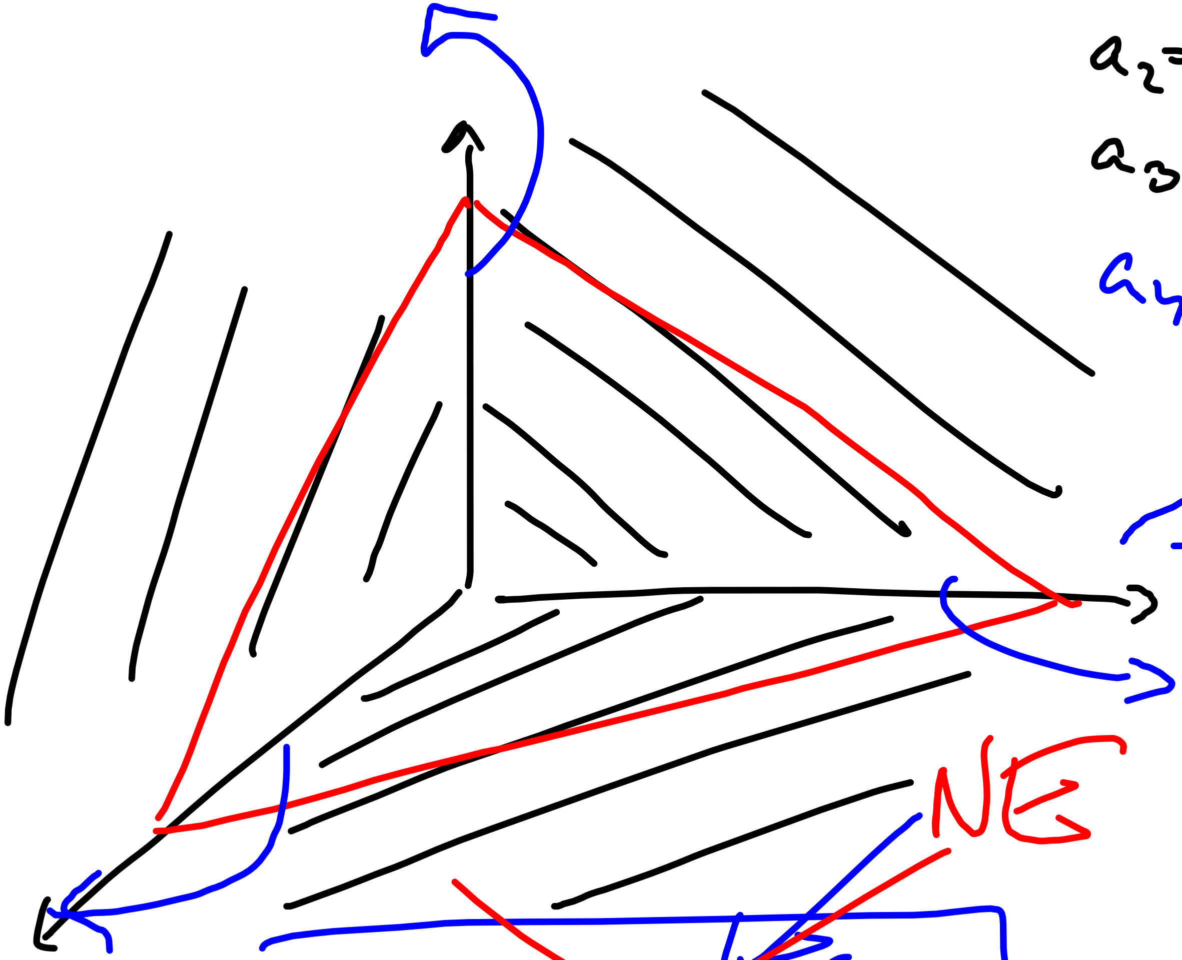
$$2^n = (1+1)^n = \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n} = \sum_{i=0}^{n}\binom{n}{i}$$

$$0 = (1-1)^n = \binom{n}{0} + (-1)\binom{n}{1} + (-1)^2\binom{n}{2} + \dots$$

$$= \sum_{i=0}^{n}(-1)^i\binom{n}{i}.$$

$$\binom{n+1}{i+1} = \binom{n}{i+1} + \binom{n}{i}$$

rozdělení $\mathbb{R}^3$ rovinami

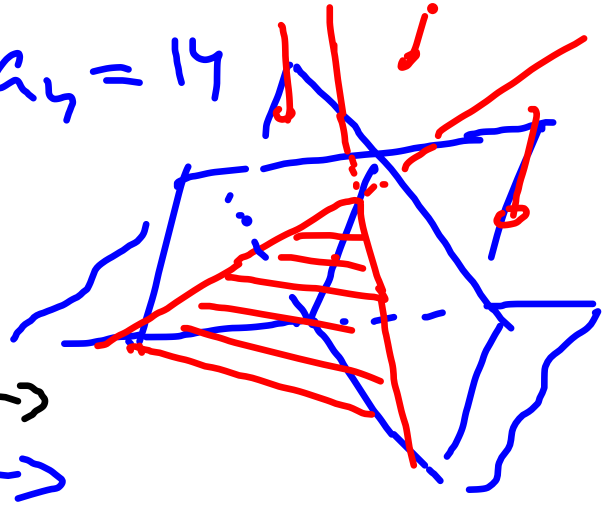


$\boxed{a_1 = 2}$

$a_2 = 4 \qquad a_2 = 3$

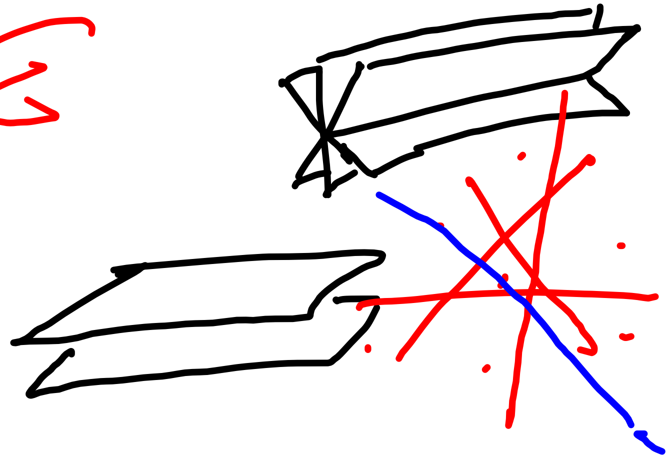
$a_3 = 8 \qquad a_3 = 4$

$a_4 = 14$



$\boxed{\cancel{a_n = a_{n-1} + 2(n-1)}}$

NE



$a_0 = 1$ $\qquad$ $a_n = a_{n-1} + a_{n-2}$

$a_1 = 1$ $\qquad$ vždy řešení s $\mathbb{N}$

$a_2 = 2$

$a_n = c\, a_{n-1} \Rightarrow \boxed{a_n = c^n a_0}$

dosazením: $(c^n - c^{n-1} - c^{n-2})\, a_0 = 0$

$\Rightarrow$ rovnice $\lambda^2 - \lambda - 1 = 0$ $\quad \lambda_{1,2} = \frac{1}{2}(1 \pm \sqrt{5})$

$x_n = \alpha x_{n-1} + \beta x_{n-2}$ $\qquad$ TOTÉŽ pro vektory

$y_n = \alpha y_{n-1} + \beta y_{n-2}$ $\qquad$ $z_n = x_n + y_n$

OPĚT ŘEŠENÍ!

$$z_n = \alpha(\underbrace{x_{n-1} + y_{n-1}}_{= z_{n-1}}) + \beta(\underbrace{x_{n-2} + y_{n-2}}_{= z_{n-2}})$$

$$\Rightarrow \underbrace{C_1 \frac{1}{2^n}\left(1+\sqrt{5}\right)^n + C_2 \frac{1}{2^n}\left(1-\sqrt{5}\right)^n}_{= a_n} \quad \text{je řešení } \forall C_1, C_2$$

$$a_0 = 1 : \quad C_1 + C_2 = 1$$

$$a_1 = 1 : \quad \frac{1}{2}\left(1+\sqrt{5}\right) C_1 + \frac{1}{2}\left(1-\sqrt{5}\right) C_2 = 1$$

$$\Rightarrow \quad a_n = \frac{\left(1+\sqrt{5}\right)^n - \left(1-\sqrt{5}\right)^n}{2^n\left(\sqrt{5}\right)}$$

$$a_{n+2} = a(1+b)\,a_{n+1} - ab\,a_n + 1$$

$$= 1 \cdot a_{n+1} - \frac{1}{4} a_n + 1$$

abs. člen

$$y_{t+2} = \alpha y_{t+1} + \beta y_t + \gamma$$

$$z_{t+2} = \alpha z_{t+1} + \beta z_t$$

$$\boxed{w_t = y_t + z_t}$$

HLEDANÉ

$$w_{t+2} = \alpha w_{t+1} + \beta w_t + \gamma$$

"STEJNÉ"

$$a_{n+2} - a_{n+1} + \frac{1}{4} a_n = \begin{cases} 1 \\ 0 \end{cases}$$

$$\lambda^2 - \lambda + \frac{1}{4} = 0 \Rightarrow \lambda_{1,2} = 1/2$$

$$\left(\lambda - \frac{1}{2}\right)^2$$

$$\boxed{C_1 \underbrace{\lambda^n} + C_2 n \underbrace{\lambda^n}}$$

homogenní

Hledáme konstantní řešení:

$$C - C + \frac{1}{4} C = 1 \Rightarrow C = 4$$

Obecné řešení $a_n = C_1 \left(\frac{1}{2}\right)^n + C_2 n \left(\frac{1}{2}\right)^n + 4$