Sčítání: $A = \{a, b, \dots\}$

$+ : (a, b) \xmapsto{+} a+b \in A$

$A \times A \to A$

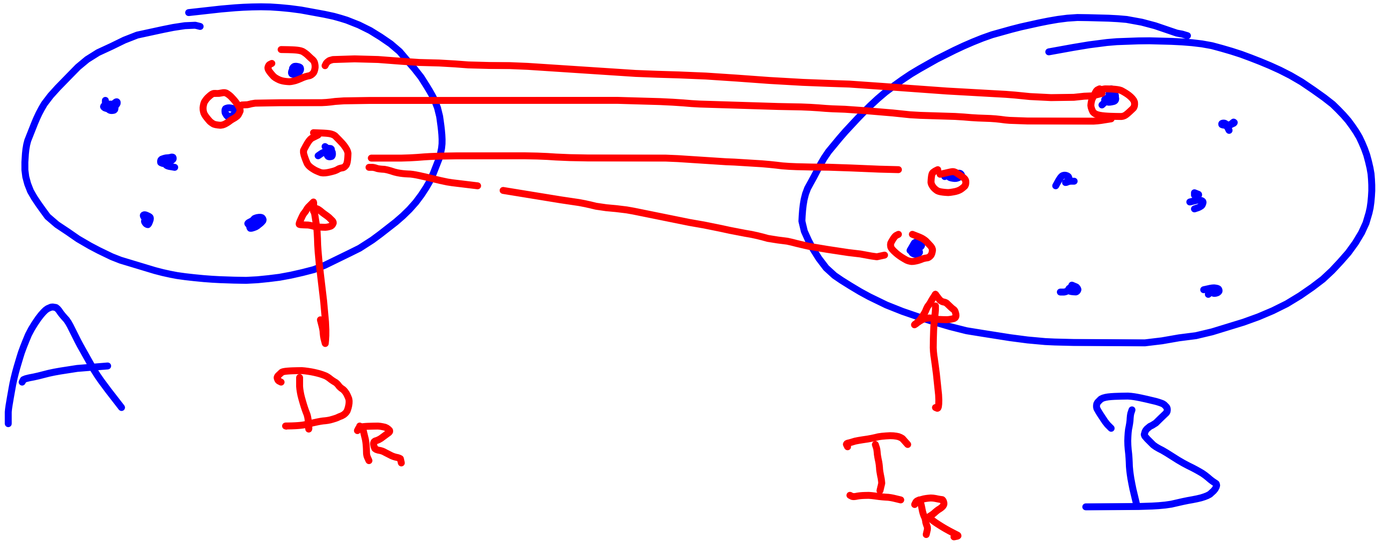
$\cdot : (a, b) \xmapsto{\cdot} a \cdot b \in A$

$A \times A \to A$

Zobrazení $f : A \longrightarrow B \qquad a \xmapsto{f} f(a)$

Relace: "práce s vztahy"

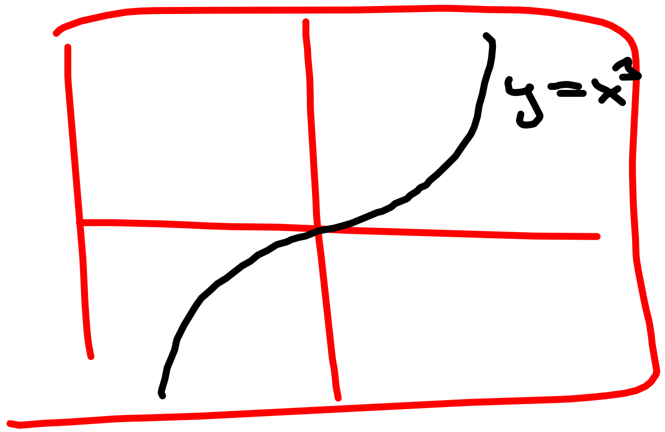
$R \subset A \times B$



$$f: \mathbb{R} \to \mathbb{R} \qquad x \mapsto x^3$$

$$R_f \subset \mathbb{R} \times \mathbb{R}$$

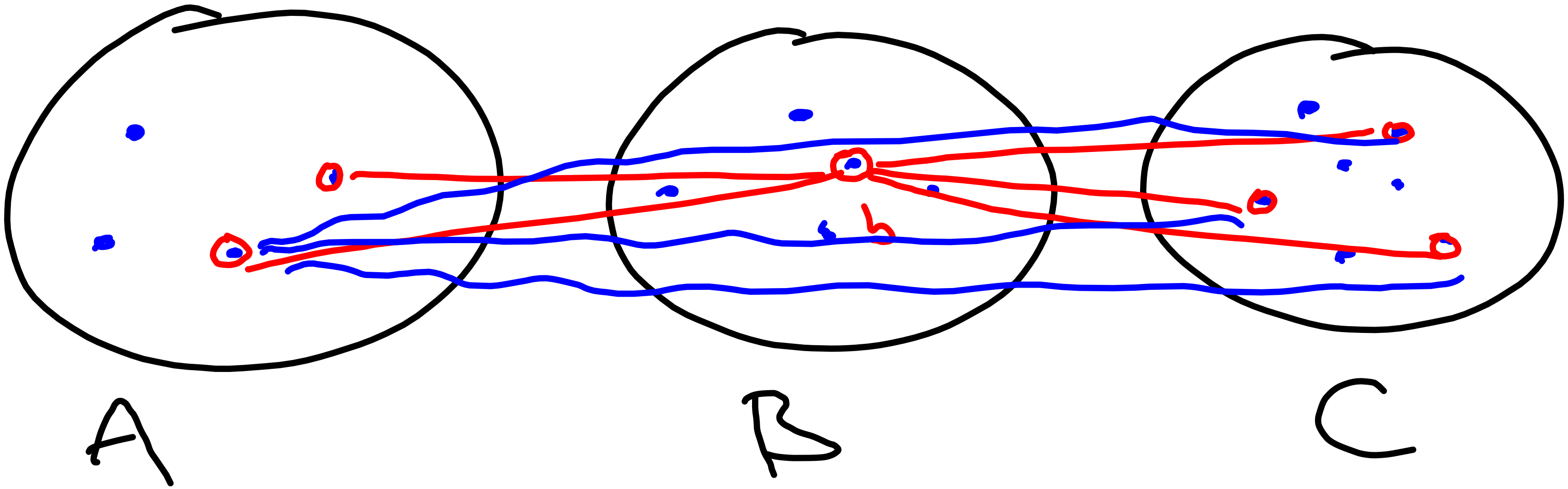
$$R_f = \{(x, x^3);\ x \in \mathbb{R}\}$$

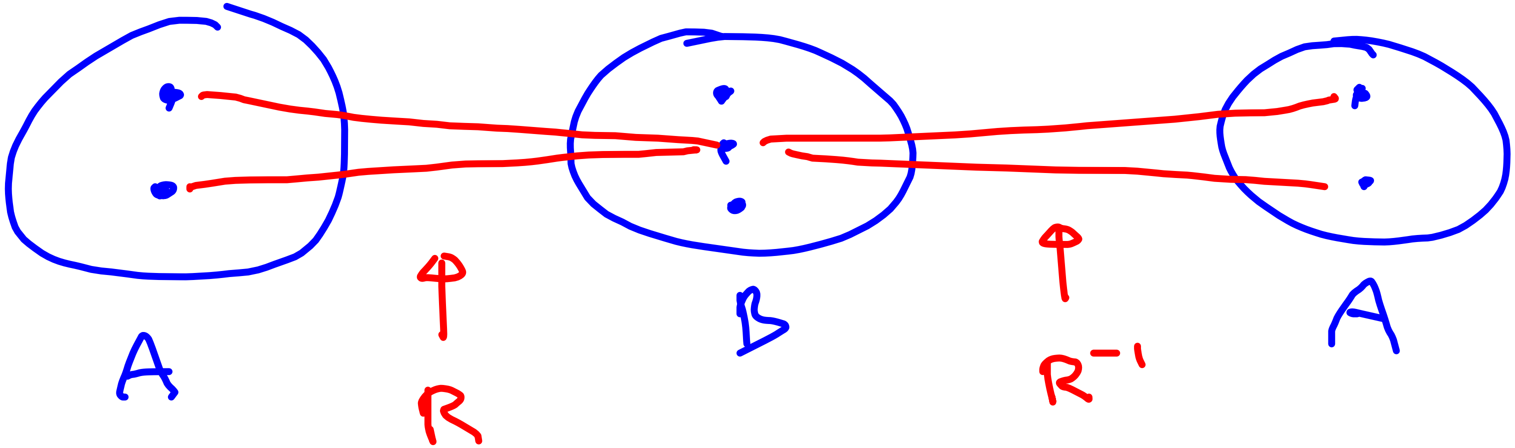


$\mathbb{R}^2$

$$A \xrightarrow{f} B \xrightarrow{g} C$$

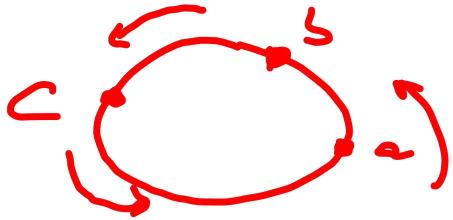
$$a \longmapsto f(a) \longmapsto g(f(a))$$

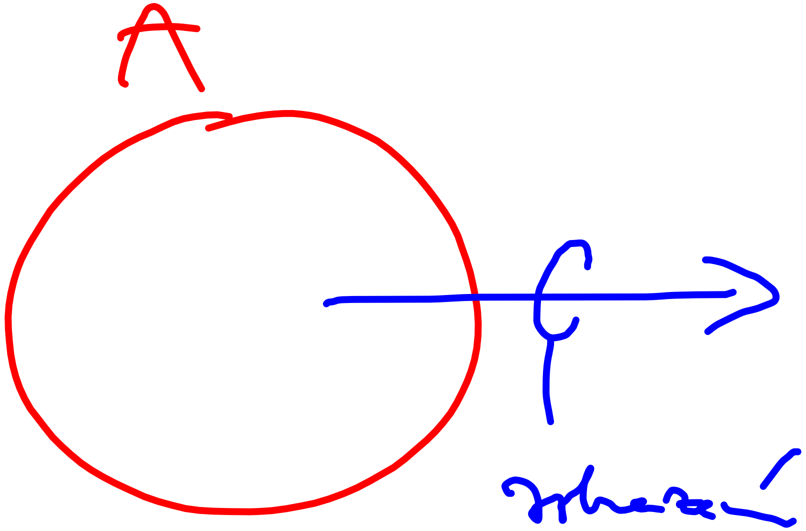




$R^{-1} \circ R$ je úplná relace $A \times A$

$a < b < c$ $\qquad$ $c < a$?





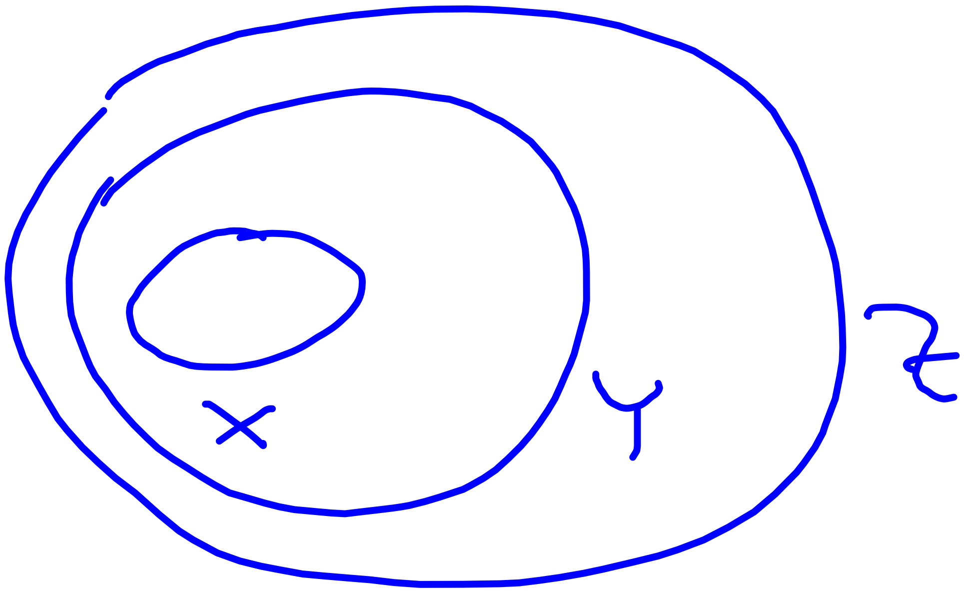
: 1

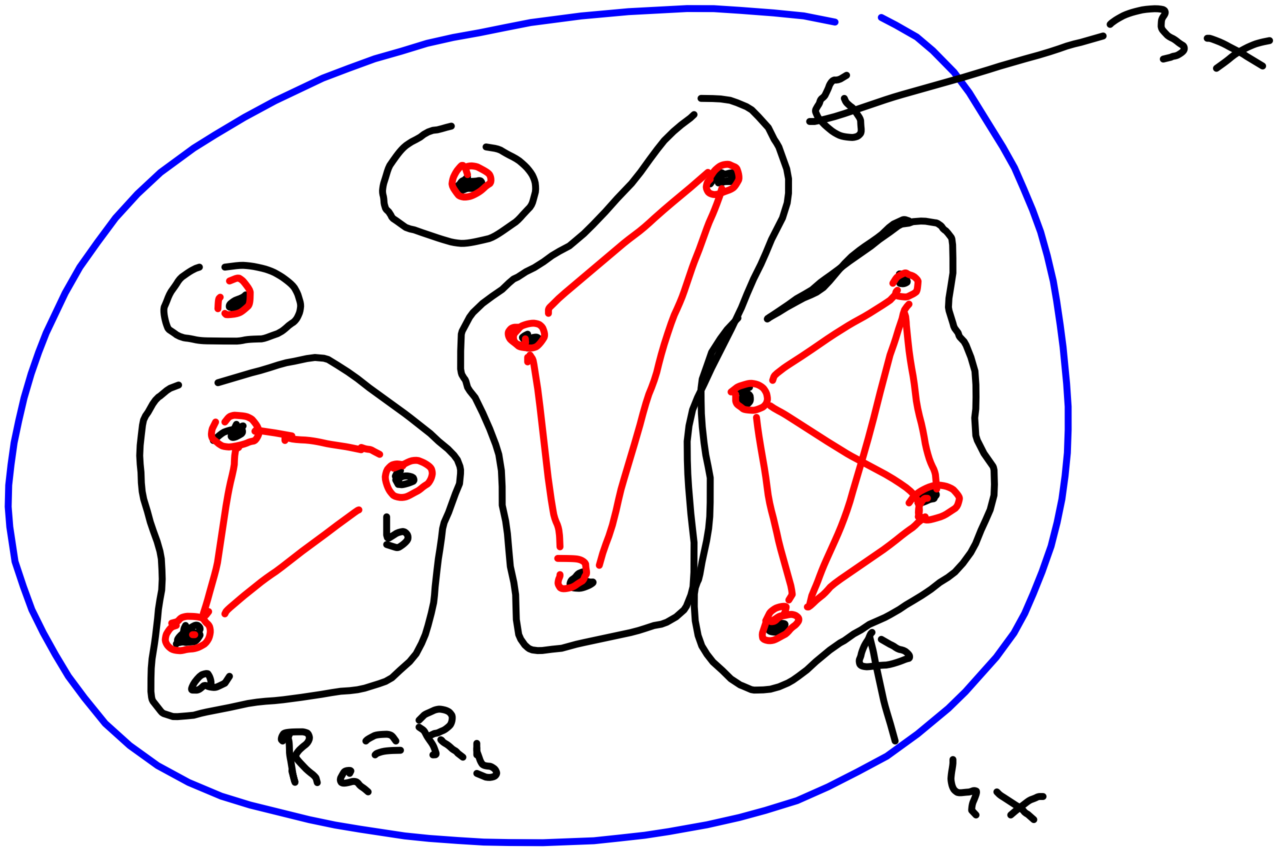
: 0

B^A

$\uparrow$

zobrazení z A do B





$$A = \bigcup_{a \in A} R_a$$

idea pro $\mathbb{Z}$: $\mathbb{N} \cup (-\mathbb{N})$

$0, 1, 2, \dots \qquad -1, -2, \dots,$

$(a,0) \qquad (0,a)$

→ rovnou $(a,b) \in \mathbb{N} \times \mathbb{N}$ (odpovídá $a-b$)

ekvivalence: $(a,b) \sim (a',b') \Leftrightarrow a+b' = a'+b$

- reflexivní $(a,b) \sim (a,b)$ $(a+b = a+b)$
- symetrické $(a,b) \sim (a',b')$ protože sčítání v $\mathbb{N}$ je symetrické
- tranzitivní $(a,b) \sim (a',b') \sim (a'',b'')$

$\Rightarrow a+b' = a'+b, \; a'+b'' = a''+b'$

$a+b'' \overset{?}{=} a''+b$

✓ sčítání: $a-b+a'-b' = a+a'-(b+b')$

$\mathbb{Z} = \{ [(a,b)] \}$

$\mathbb{N} = \{ [(a,0)] \}$, $-\mathbb{N} = \{ [(0,a)] \}$

idea pro $\mathbb{Q}$

$a-b = [(a,b)]$

násobení v $\mathbb{Z}$:

$$\left.\begin{aligned}&(a-b)(a'-b')\\ &= aa'+bb'-ab'-ba'\end{aligned}\right] \leftarrow \text{def.}$$

$$(a-0)\cdot(b-0) = ab-0$$

$$(a-0)\cdot(0-b) = 0-ab$$

$$\mathbb{Q} = \{\, p/q \,;\, (p,q) \in \mathbb{Z}\times\mathbb{Z} \,\}$$

$$(p,q) \sim (p',q') \iff p\cdot q' = q\cdot p'$$

vlastnosti:

- $p\cdot q = q\cdot p \Rightarrow$ reflexivní
- $(p,q)\sim(p',q') \Rightarrow p'\cdot q = q'\cdot p \Rightarrow (p',q')\sim(p,q) \Rightarrow$ symetrie
- $p/q = p'/q' = p''/q''$

$\oplus \quad p/q + p'/q' = q' \cdot p/qq' + q p'/qq'$

$\odot \quad = \dfrac{q' \cdot p + q p'}{qq'}$

$$p/q \cdot p'/q' = pp'/qq'$$

Zbytkové třídy:

$\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$ $\qquad a = \{a + n \cdot 5 ; n \in \mathbb{Z}\}$

$\mathbb{Z}_k = \{0, 1, 2, \dots, k-1\}$

$a + b = ?$

$$\underbrace{a + n \cdot k}_{a'} + \underbrace{b + m \cdot k}_{b'} = a + b + (m+n) \cdot k$$

v $\mathbb{Z}_5$: $\boxed{2+3=0 \quad 2+4=1 \quad 2+2=4}$

$$(a + n \cdot k)(b + m \cdot k) = a \cdot b + \underbrace{(bn + am + mn)k}$$

$4 \cdot 4 = 1$, $\quad 4 \cdot 3 = 2$, $\dots$

$3 \cdot 2 = 1$, $\quad$ (v $\mathbb{Z}_6$; $3 \cdot 2 = 0$)

$$\mathbb{Z}_2 = \{0, 1\}$$

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1