$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A \cdot x \in \mathbb{R}^m \qquad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$A : \mathbb{R}^n \longrightarrow \mathbb{R}^m \qquad A \cdot x = b \quad , \quad \exists A^{-1} ?$$

$$A^{-1} A \cdot x = E \cdot x = x = A^{-1} \cdot b$$

$$(\mathrm{Mat}_n \mathbb{K}, +, \cdot)$$ okruh ale ne komutativní

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$(1,2)$ $(2,1)$

$$= \underline{a_{11} a_{22}} - a_{12} a_{21}$$

$n = 3$

+	−
1 2 3	3 2 1
3 1 2	1 3 2
2 3 1	2 1 3

3 2 1 — 3 inverze

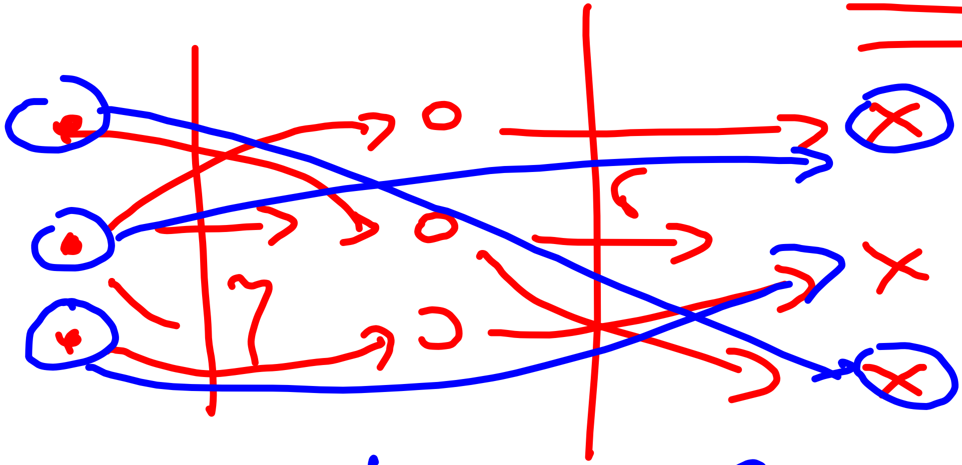
1 3 2 — 1 inverze

3 1 2 — 2 inverze

1 2 3

3 2 1

2 1 3



$$sgn : \Sigma_n \longrightarrow \{+1, -1\}$$

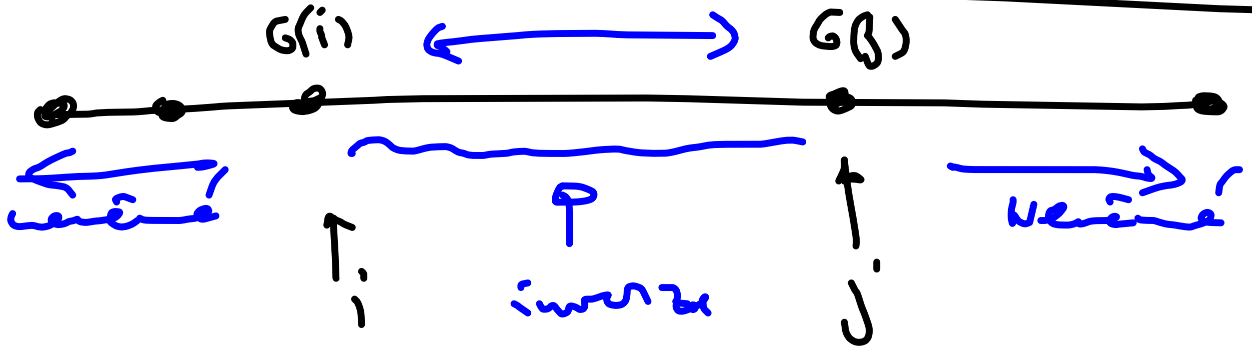
Dk. indukcí přes n.

$$\begin{pmatrix} 1 & 2 & \dots & n \\ \sigma(1) & \sigma(2) & \dots & \sigma(n) \end{pmatrix}$$

n možností a po zafixování $\sigma(1)$ zbývá volnost pro před! na $n-1$ prvků.

$\Rightarrow n\cdot(n-1)!$ možností

jediná transpozice realizuje jiný začátek $\sigma(1)$.



$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} = A \qquad A^T = \begin{pmatrix} a_{11} & \cdots & a_{m1} \\ a_{12} & \cdots & a_{m2} \\ \vdots & & \vdots \\ a_{1n} & & a_{mn} \end{pmatrix}$$

$$B \in \mathrm{Mat}_n(\mathbb{K})$$

$$B = \underbrace{\frac{1}{2}\left(B + B^T\right)}_{\text{symetrická}} + \underbrace{\frac{1}{2}\left(B - B^T\right)}_{\text{antisymetrická}}$$

$$|A| = \sum_{\sigma \in \Sigma_n} \boxed{\operatorname{sgn} \sigma \; a_{1\sigma(1)} a_{2\sigma(2)} \cdots a_{n\sigma(n)}}$$

$$= \sum_{\sigma \in \Sigma_n} \operatorname{sgn} \sigma^{-1} a_{\sigma^{-1}(1)\,1} \, a_{\sigma^{-1}(2)\,2} \cdots a_{\sigma^{-1}(n)\,n}$$

$$= \sum_{\sigma^{-1} \in \Sigma_n} \operatorname{sgn} \sigma^{-1} a'_{1\,\sigma^{-1}(1)} \cdots a'_{n\,\sigma^{-1}(n)}$$

$$= |A^T|$$

$$|B| = \sum_{\sigma \in \Sigma_n} \operatorname{sgn} \sigma \; b_{1\sigma(1)} \cdots b_{n\sigma(n)}$$

transp. $i \leftrightarrow k$, záměn řádků

$$\sigma' = \underline{\underline{\tau \circ \sigma}}$$

$$= \sum_{\sigma \in \Sigma_n} \operatorname{sgn} \sigma \; a_{1\sigma'(1)} \cdots a_{n\sigma'(n)}$$

$$= \sum_{\sigma' \in \Sigma_n} - \operatorname{sgn} \sigma' \; a_{1\sigma'(1)} \cdots a_{n\sigma'(n)}$$

$$= -|A|$$

$$\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ (b_{k1}+c_{k1}) & \cdots & (b_{kn}+c_{kn}) \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ b_{k1} & \cdots & b_{kn} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ c_{k1} & \cdots & c_{kn} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

$$|C| = \sum_{\sigma \in S_n} \operatorname{sgn}\sigma \; a_{1\sigma(1)} a_{2\sigma(2)} \cdots (b_{k\sigma(k)} + c_{k\sigma(k)}) \cdots a_{n\sigma(n)}$$

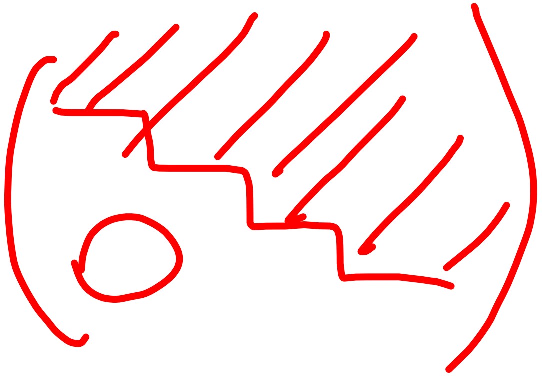
$$= \sum \operatorname{sgn}\sigma \left(a_{1\sigma(1)} \cdots b_{k\sigma(k)} \cdots a_{n\sigma(n)} + \cdots \right)$$

$$|A| = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{21} + ? & \cdots & a_{2n} + ? \\ \vdots & & \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = |A| + ?$$

$$\begin{pmatrix} 2 & - & - & - \\ 1 & - & \cdot & - \end{pmatrix}$$

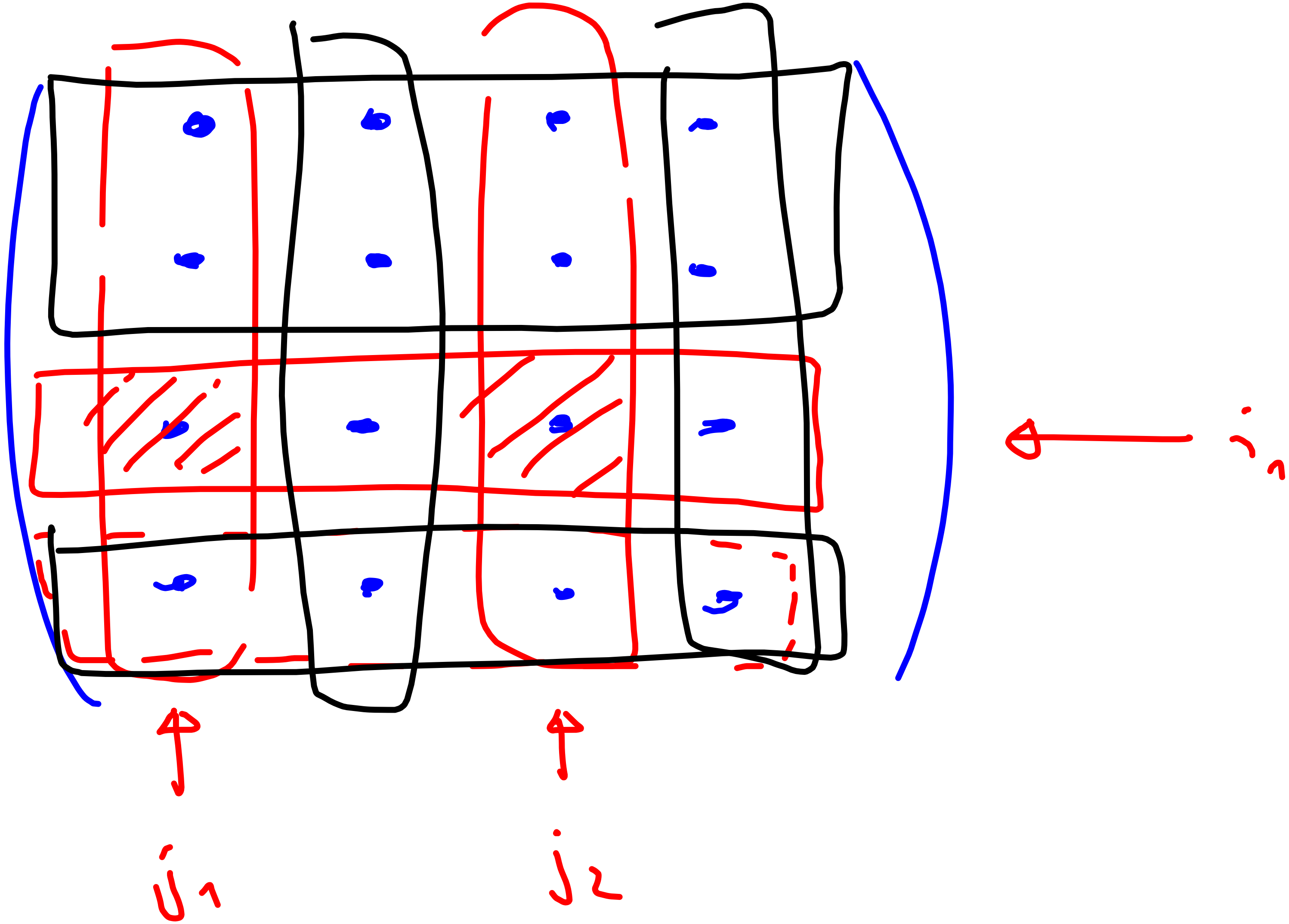
ošetřete 1/2 předložka !



schodovitý tvar

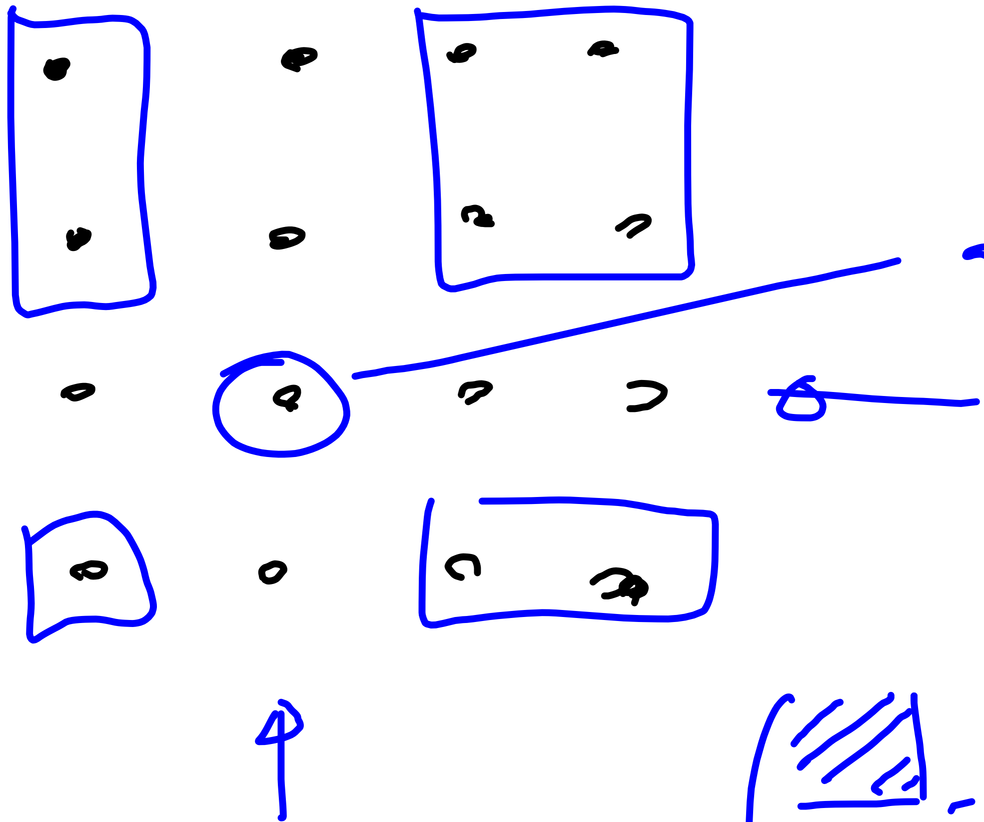
$\Downarrow$

$$|A| = a_{11} a_{22} \cdots a_{nn}$$



i
j1
j2

$$(-1)^{i+j}\,|M^*| = A_{ij}$$



a_{ij}

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & -1 \\ 0 & 0 & 1 \end{vmatrix}$$

← řádek (jediný)

alg. adj. prvky

$$= \underbrace{(-1)^{3+1}\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix}}_{=A_{31}} \underset{=0}{a_{31}} + \underbrace{(-1)^{3+2}\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}}_{=A_{32}} \underset{=0}{a_{32}} + \underbrace{(-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix}}_{=A_{33}} \underset{=1}{a_{33}}$$

$$= 1 \cdot \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} = \underline{\underline{0}}$$

$$|A^{-1}\cdot A| = |E| = 1$$

$$= |A^{-1}|\cdot|A|$$

$$\Rightarrow \quad |A^{-1}| = (|A|)^{-1}$$

$$A = \begin{pmatrix} a & b \\ \boxed{c} & d \end{pmatrix} \qquad A^{*} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A\cdot A^{*} = \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = |A|\cdot E$$