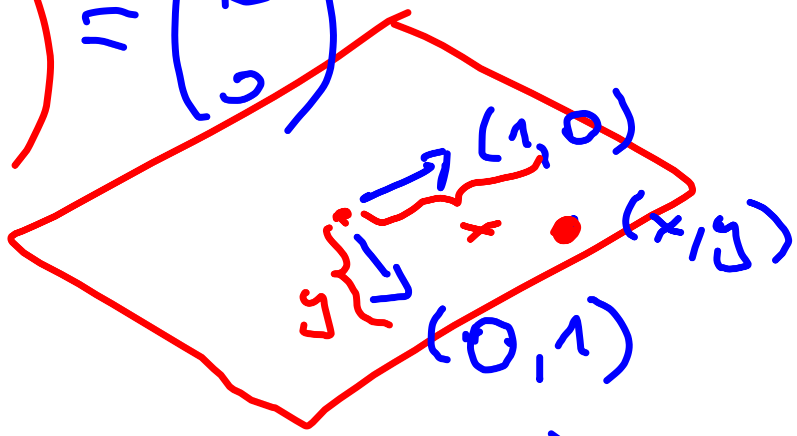
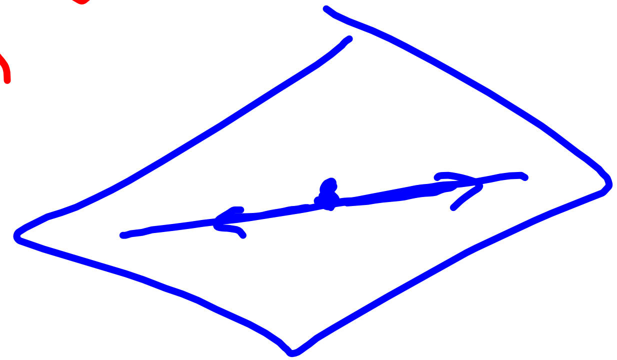
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$



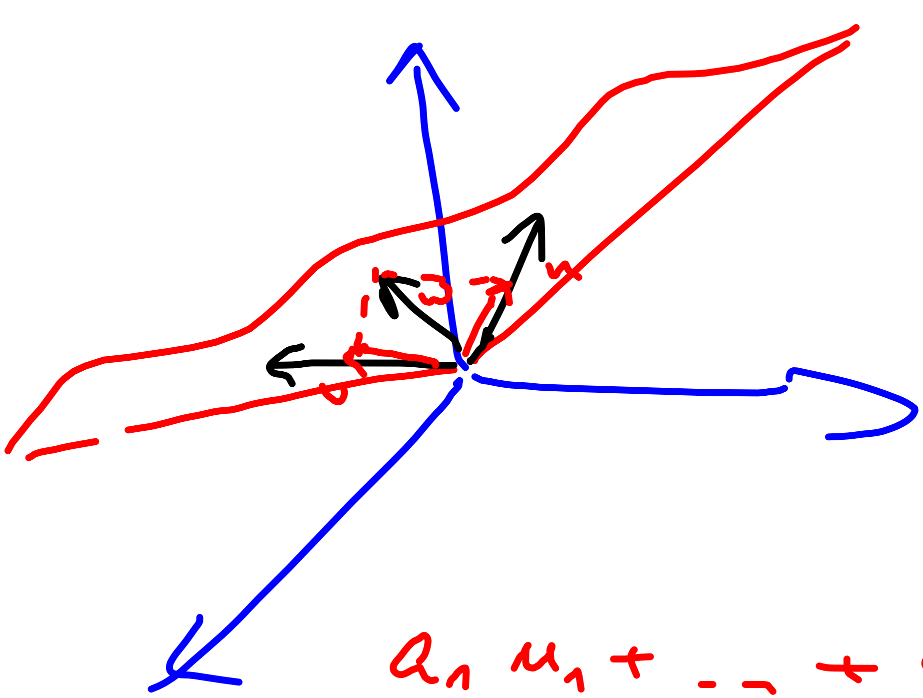
$A \in \mathrm{Mat}_{m,n}(\mathbb{K}) \qquad m \leq n$

$x, y \in \mathbb{K}^n$



$A \cdot x = 0 \quad , \quad A \cdot y$

$A \cdot (cx) = (c \cdot A) \cdot x = c \cdot (A \cdot x) = c \cdot 0 = 0$



$\mathbb{R}^3$

$au + bv + cw = 0$

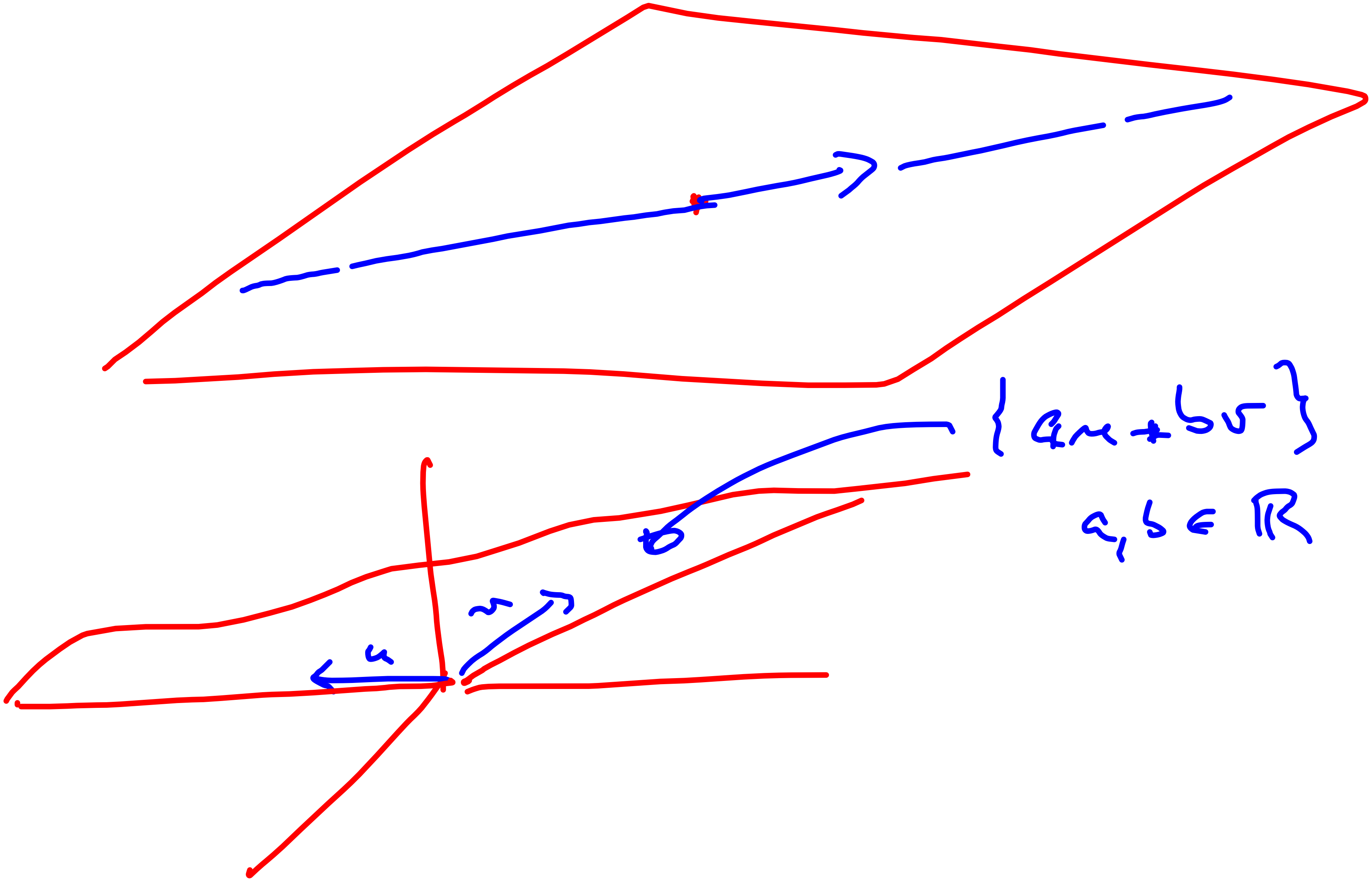
$a_i \neq 0$

$\Downarrow$

$a_1 u_1 + \ldots + a_i u_i + \ldots + a_k u_k = 0$

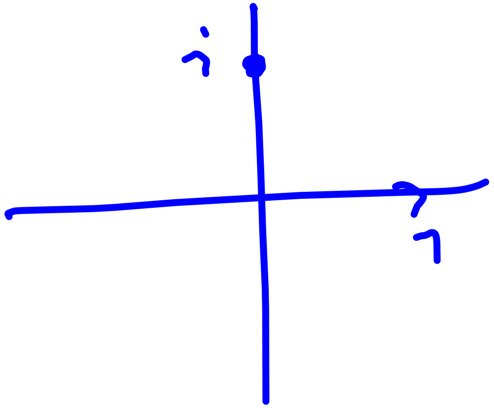
$\underset{\neq 0}{a_i} u_i = -(a_1 u_1 + a_2 u_2 + \ldots \overset{i}{\vee} + \ldots + a_k u_k)$

$u_i = \frac{-1}{a_i}(a_1 u_1 + \ldots \overset{i}{\vee} + a_k u_k)$


u
v
$\{au + bv\}$
$a, b \in \mathbb{R}$

$\mathbb{C}$ nad $\mathbb{R}$ je $\mathbb{R}^2$

$\{1, i\}$ je nezávislé nad $\mathbb{R}$
závislé nad $\mathbb{C}$



$\{f : \mathbb{R} \to \mathbb{R}\}$ $\quad (f+g)(t) = f(t) + g(t)$

$(cf)(t) = c \cdot (f(t))$

$\{f : M \to \mathbb{K}\}$ $\quad$ — " —

(lib. množina)

je vekt. prostor nad
těl. $\mathbb{R}$, $\mathbb{K}$

počítají též, když jsou polynomy

$$\mathbb{K}_n[x] = \{ \underbrace{a_0 + a_1 x + \dots + a_n x^n}_{f} \}$$

$$g = b_0 + b_1 x + \dots + b_n x^n$$

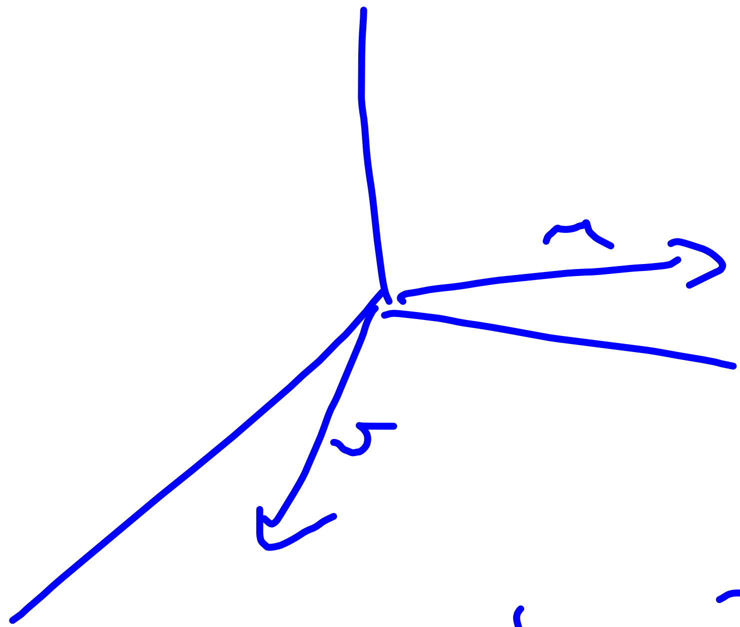
$$f + g = (a_0 + b_0) + (a_1 + b_1)x + \dots + (a_n + b_n)x^n$$

nad $\mathbb{Z}_2$ $f = x + x^2$ je 0 vždy

zobrazení!

$M \subset V$

$\langle M \rangle = U \subset V$

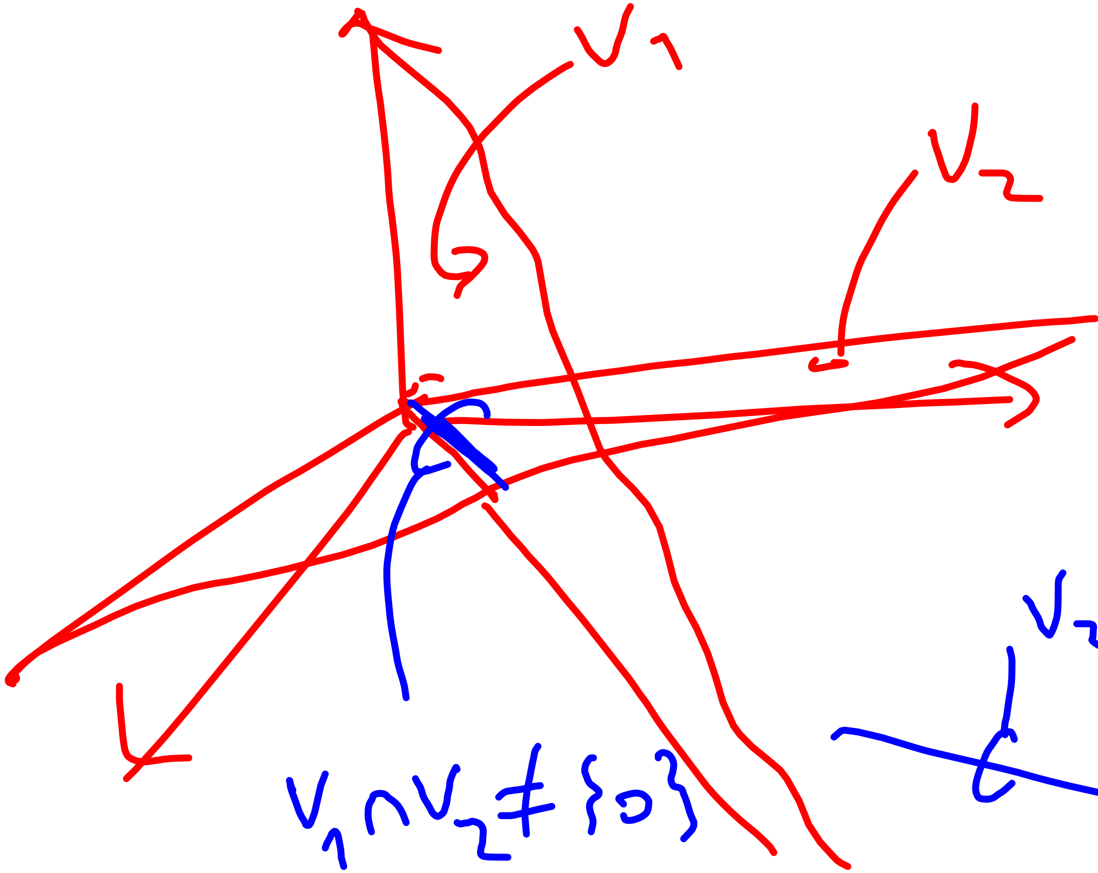


$M = \{u, v\}$

$\in V_1$ $\in V_2$

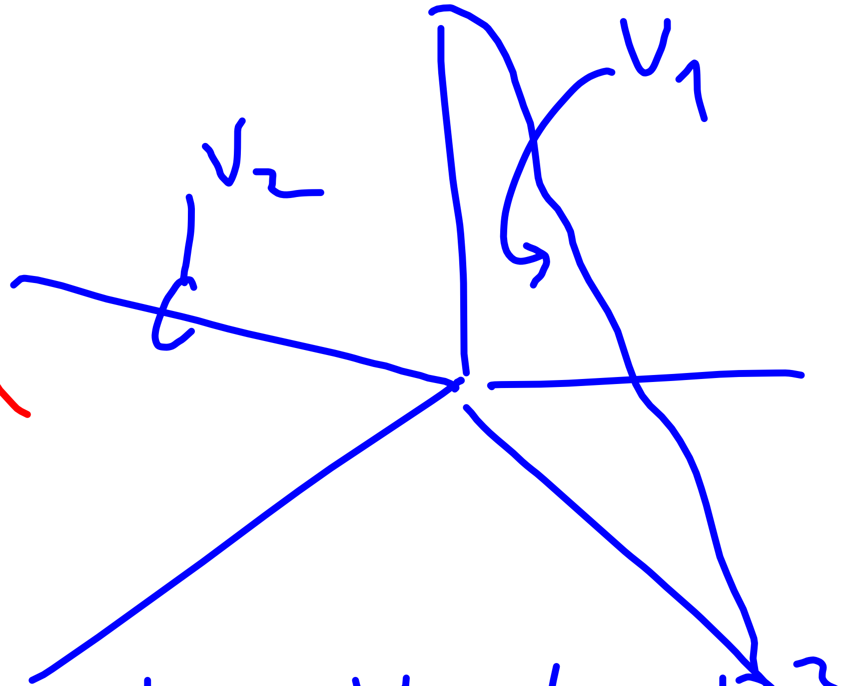
$(a_1 u_1 + \dots + a_{k_1} u_{k_1}) + (a_{k_1+1} u_{k_1+1} + \dots + a_{k_1+k_2} u_{k_1+k_2}) + \dots$

$= v_1 + \dots + v_k$



$V_1 + V_2 = \mathbb{R}^3$

$V_1 \cap V_2 \neq \{0\}$



$V_1 \cap V_2 = \{0\}$ $V_1 + V_2 = \mathbb{R}^3$

$\mathbb{K}^n$ i báze std.

$u_1 = (1, 0, \dots, 0)$

$u_2 = (0, 1, \dots, 0)$

$\vdots$

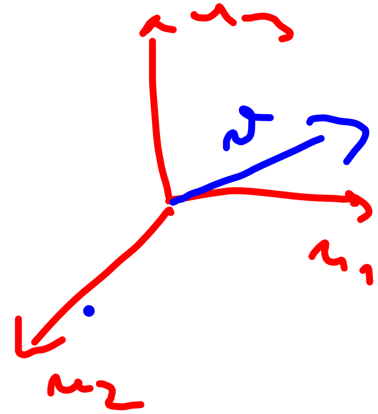
$u_n = (0, 0, \dots, 1)$

$a_1 u_1 + \dots + a_n u_n = \underbrace{(a_1, \dots, a_n)} = 0$

$\Rightarrow a_1 = a_2 = \dots = a_n = 0$

$u_1, \dots, u_k$ ... báze

$v_1, \dots, v_l$ ... lineárně nezávislé



$$\Downarrow$$

l vektorů mezi $u_1, \dots, u_k$ lze nahradit v_i

a opět získáme bázi

Př. $V = \langle u_1, \dots, u_n \rangle$

lin. závislé $\Rightarrow u_j = a_1 u_1 + \dots \hat{j} + \dots + a_n u_n$

$\Rightarrow V = \langle u_1, \dots \hat{j}, \dots, u_n \rangle$

$v_1 = a_1 u_1 + \dots + a_k u_k \Rightarrow u_i = \frac{1}{a_i} v_1 - \frac{a_1}{a_i} u_1 - \dots$

důkaz Steinitzovy věty:

1) $\ell < k$

$\vdots$

$$\underset{\underset{k}{\parallel}}{\dim W_1} + \underset{\underset{n}{\parallel}}{\dim W_2} = \dim(W_1 + W_2) + \underset{\underset{\ell}{\parallel}}{\dim(W_1 \cap W_2)}$$

$$W_1 \cap W_2 = \langle u_1, \dots, u_\ell \rangle$$

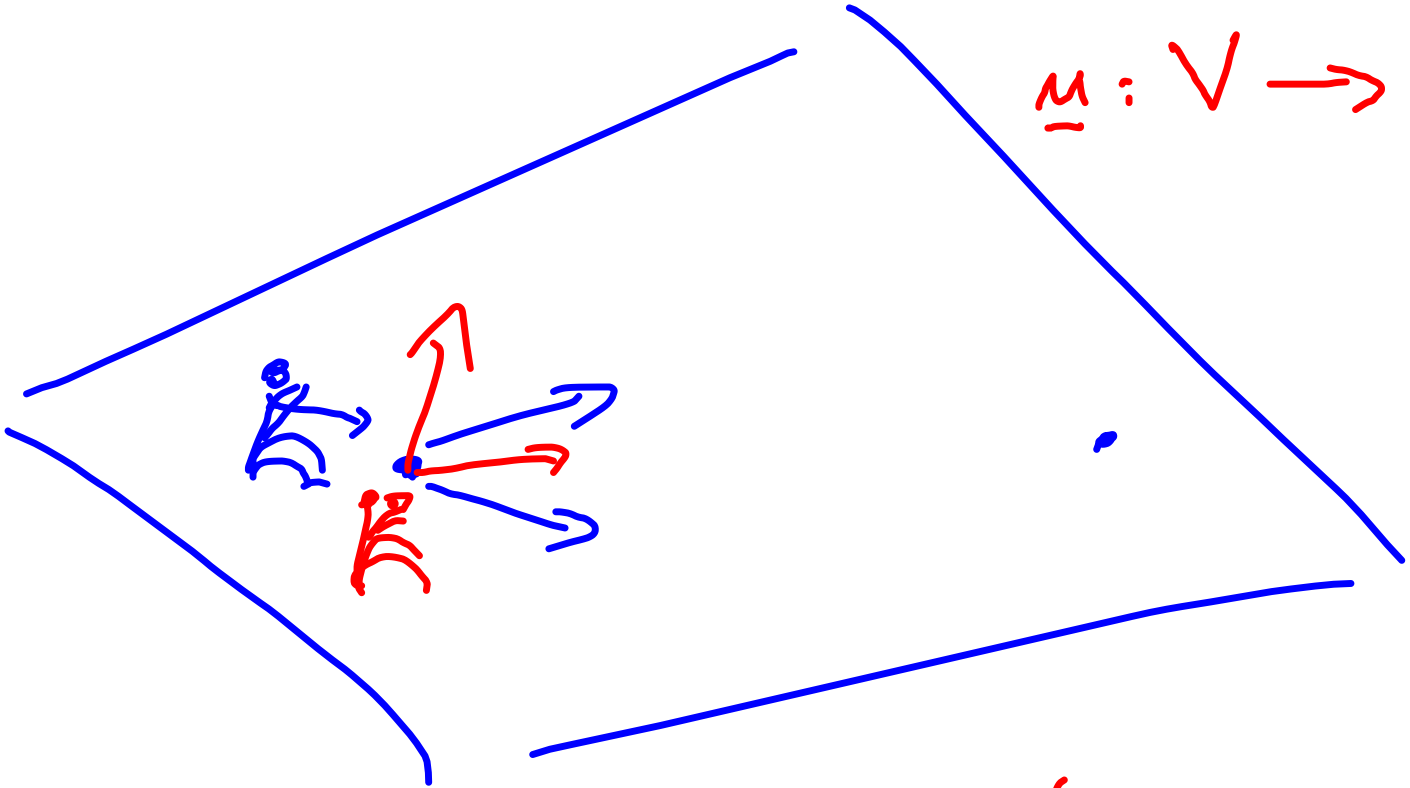
$$W_1 = \langle u_1, \dots, u_\ell, v_1, \dots, v_{k-\ell} \rangle$$

$$W_2 = \langle u_1, \dots, u_\ell, w_1, \dots, w_{n-\ell} \rangle$$

$$W_1 + W_2 = \langle u_1, \dots, u_\ell, v_1, \dots, v_{k-\ell}, w_1, \dots, w_{n-\ell} \rangle$$

$(u_1, \dots, u_n) = \underline{u}$ báze

$\underline{u} : V \to \mathbb{K}^n$



$u = a_1 v_1 + \dots + a_n v_n \qquad w = b_1 v_1 + \dots + b_n v_n$

$u + w \underset{\varphi}{=} (a_1 + b_1) v_1 + \dots + (a_n + b_n) v_n$

axiom $v_1, v_2, \dots$

$$0 = f(u) + f(v) = f(u+v) \in \operatorname{Ker} f$$

$$\in \operatorname{Ker} f$$

$$0 = a\, f(u) = f(a \cdot u) \in \operatorname{Ker} f$$

$$\in \operatorname{Ker} f$$

$$\operatorname{Ker} f = f^{-1}(0)$$

$$\underline{f(0) = f(u-u)} = f(u) - f(u) = 0$$

$$(g \circ f)(u+v) = g(f(u) + f(v)) = g(f(u)) + g(f(v))$$

$$f(u) = f(v) \iff f(u-v) = 0 \implies u - v = 0$$

$$V \xrightarrow{\underline{u}} K^n \xleftarrow{\underline{v}} W, \qquad \underline{v}^{-1} \circ \underline{u}$$

$\underline{u}$ $\underline{v}$

pro $f = id_V$

získáme matici přechodu

$$v = x_1 u_1 + \dots + x_n u_n \overset{f}{\longmapsto} x_1 f(u_1) + \dots + x_n f(u_n)$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \xmapsto{f_{\underline{u},\underline{v}}} \begin{pmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & & \\ \vdots & & \\ a_{k1} & \dots & a_{kn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$\uparrow \underline{u}^{-1}$ $\downarrow \underline{v}$

$f(u_1) = a_{11} v_1 + a_{21} v_2 + \dots + a_{k1} v_k$

$\vdots$

$f(u_n) = a_{1n} v_1 + \dots +$