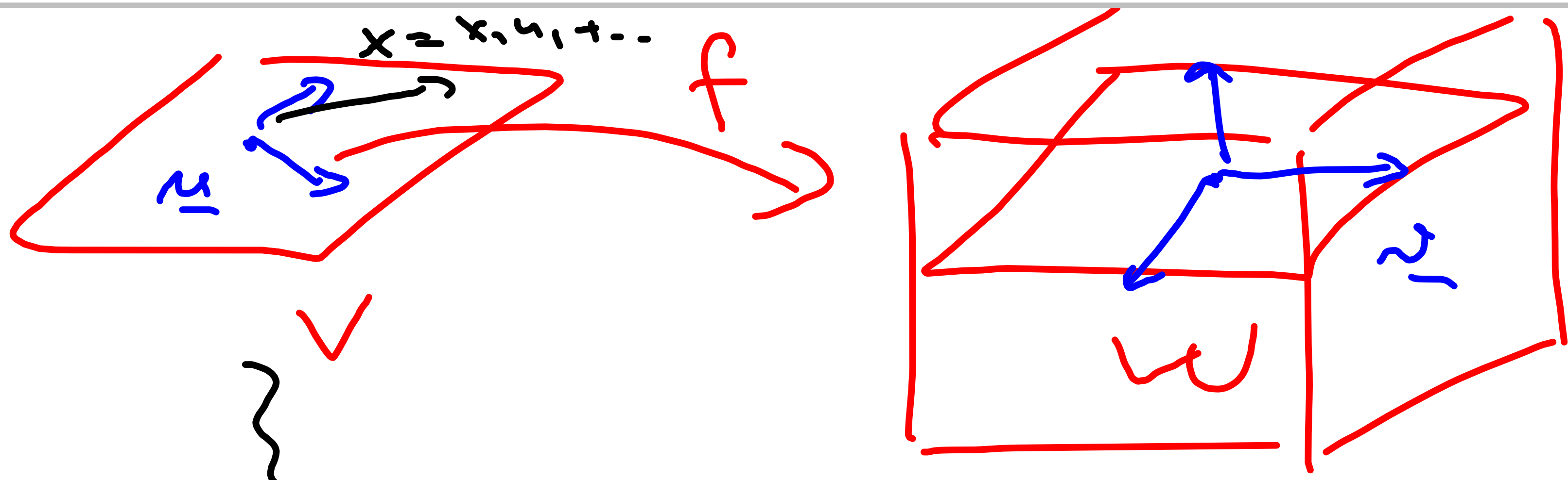


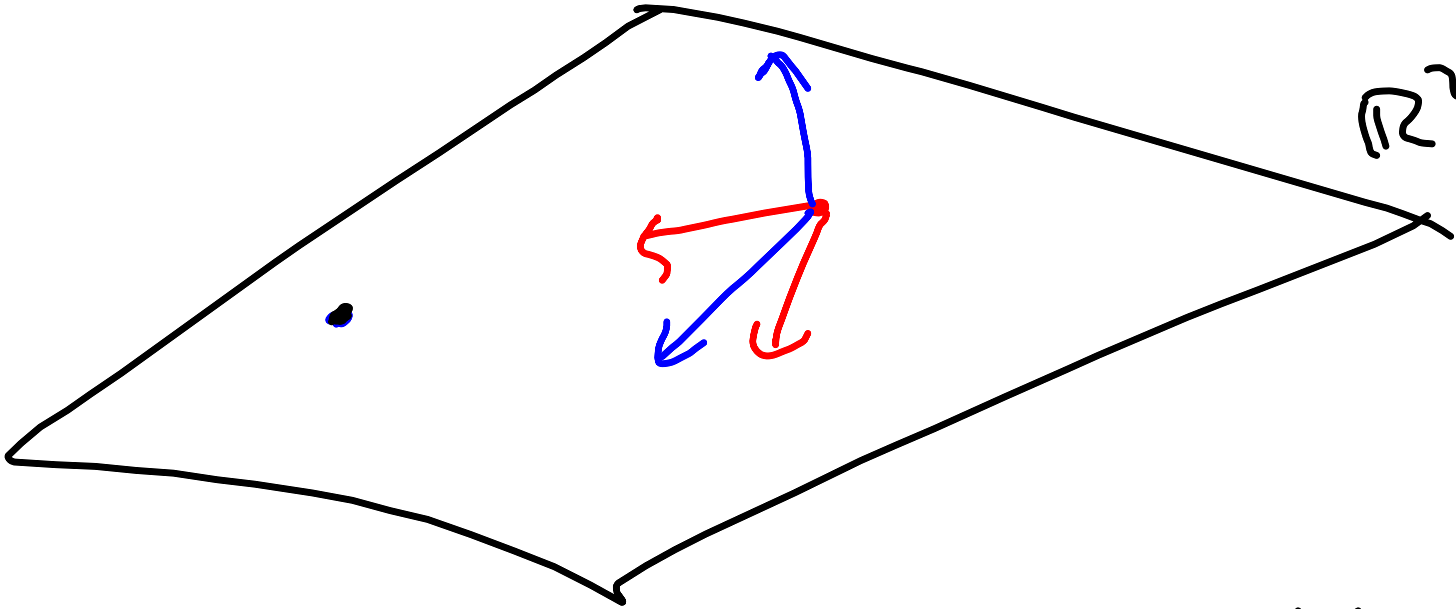
$$f(u_i) = a_{1i}\, v_1 + a_{2i}\, v_2 + \dots$$

$$(x^1, \dots, x^n) \leftarrow \qquad f(x) = A \cdot x = (a_{ij}) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\mathbb{K}^n \xrightarrow{f_{u,v}} \mathbb{K}^m$$

$$x \longmapsto A \cdot x$$

$$A(ax+by) = aA\cdot x + b \cdot A \cdot y$$



$\mathbb{R}^2 = V$

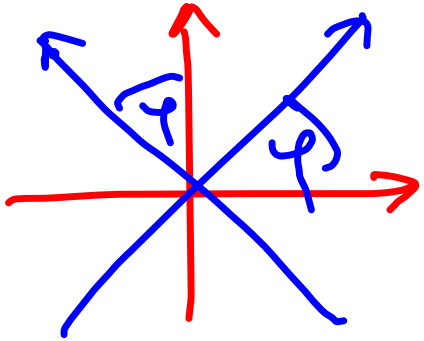
$$
\begin{array}{ccc}
X & \longmapsto & X \\
\downarrow & & \downarrow \\
\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} & \longmapsto & T \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}
\end{array}
$$

$$
\begin{tikzcd}
V \arrow[r, "f"] \arrow[d, "\underline{u}"] & W \arrow[r, "g"] \arrow[d, "\underline{v}"] & Z \arrow[d, "\underline{w}"] \\
K^n \arrow[r, dashed, "f_{\underline{u},\underline{v}}"] \arrow[rr, dashed, bend right] & K^m \arrow[r, dashed, "g_{\underline{v},\underline{w}}"] & K^s
\end{tikzcd}
$$

matice $g \circ f$ v bázích $\underline{u}$, $\underline{w}$ je rovna $g_{\underline{v},\underline{w}} \circ f_{\underline{u},\underline{v}}$, tj. součin matic

$$
x \longmapsto A \cdot x \longmapsto B \cdot (A \cdot x) = \underline{\underline{(B \cdot A) \cdot x}}
$$

$f: \mathbb{R}^2 \to \mathbb{R}^2$ otočení o φ



$f_{u,v} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$A \longrightarrow P \cdot A \cdot Q = \left(\begin{array}{ccc|c} 1 & & 0 & 0 \\ & \ddots & & \\ 0 & & 1 & \\ \hline & 0 & & 0 \end{array}\right)$$

$f: V \to W$ dim $f(V) = k$

$w = \langle f(u_1), \dots, f(u_k), w_{k+1}, \dots, w_m \rangle$ lin. nezávislé,

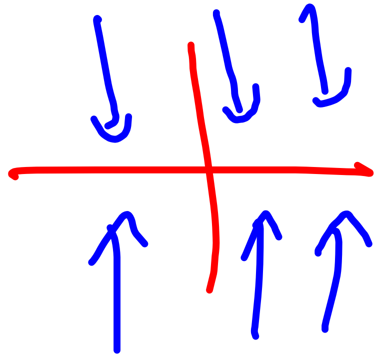
$\Rightarrow \exists\, u_1, \dots, u_k$

$f(\langle u_1, \dots, u_k \rangle) = f(V)$

$\exists\, u_1, \dots, u_k, v_{k+1}, \dots, v_n$ báze $f(v_i) = 0$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\quad C = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\longrightarrow$ otočení o 90° v $\mathbb{R}^2$ (v $\mathbb{C}^2$)



$f = f \circ f$

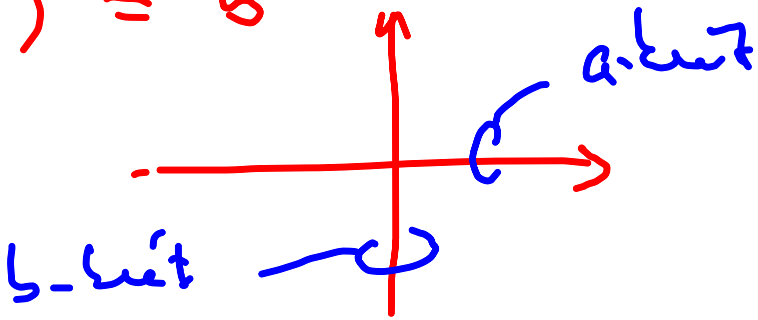
$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix}$

$V = \operatorname{Im} f \oplus \operatorname{Ker} f$

$B^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$B \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ 0 \end{pmatrix}$ $\qquad (a+bx)' = b$

$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto \begin{pmatrix} a x_1 \\ b x_2 \end{pmatrix}$



$f(u) = a u$ pro které u, a ?

$A \cdot x = a x \Leftrightarrow \underline{(A - a \cdot E) \cdot x = 0}$

$(A - aE) \cdot x = 0$, $x \neq 0 \Leftrightarrow$

$|A - \lambda E| = 0$ $\qquad D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

neznámý parametr

$|D - \lambda E| = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1 = 0$

$\Rightarrow \lambda_1 = i$, $\lambda_2 = -i$

$|C - \lambda E| = \begin{vmatrix} a-\lambda & 0 \\ 0 & b-\lambda \end{vmatrix} = (a-\lambda)(b-\lambda)$

$\boxed{|A - \lambda E|} = 0$ pro λ platí $\lambda = a$

pak $(A - aE) \cdot x = 0$ má řešení $u \neq 0$

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{vmatrix} = (a_{11}-\lambda)\cdots(a_{nn}-\lambda) + \cdots$$

$$= (-1)^n \lambda^n + (-1)^{n-1}(a_{11} + \cdots + a_{nn})\lambda^{n-1} + \cdots$$

$$\cdots + |A| \lambda^0 .$$

$$\boxed{C = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}} \qquad \begin{array}{ll} \lambda_1 = a & u_1 = (1,0) \\ \lambda_2 = b & u_2 = (0,1) \end{array}$$

báze z vlastních vektorů?

$\lambda_1, \dots, \lambda_n$ různé vlastní hodnoty pro f

$u_1, \dots, u_n$ příslušné vlastní vektory

$\Rightarrow$ jsou lin. nezávislé

dk. $u_1, \dots, u_{n-1}$ lin. nezávislé

$$0 = a_1 u_1 + \dots + a_n u_n \qquad u_n = \sum_{i=1}^{n-1} c_i u_i$$

$$f(u_n) = \lambda_n (u_n) = \underbrace{\sum_{1}^{n-1} c_i \lambda_i u_i}_{f\left(\sum c_i u_i\right)} = \lambda_n \sum_{1}^{n-1} c_i u_i$$

$$0 = \sum_{1}^{n-1} \underbrace{c_i \overset{\neq 0}{(\lambda_n - \lambda_i)}}_{= 0} u_i \longrightarrow \text{spor.}$$

$$J_{n+1} = \underbrace{\begin{pmatrix} 0 & 1 & & \\ & \ddots & 2 & \\ & 0 & \ddots & n \\ & & & 0 \end{pmatrix}}_{n+1} \qquad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} = J_3$$

$$(a_n x^n + a_{n-1} x^{n-1} + \dots + a_0)' \longmapsto n a_n x^{n-1} + \dots$$

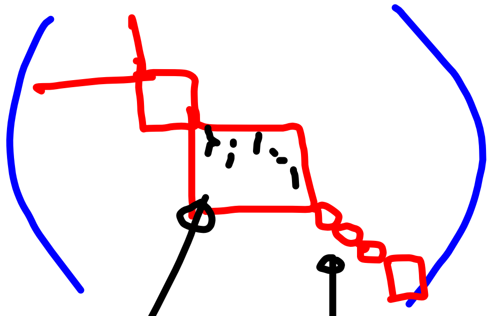
$(x^n, x^{n-1}, \dots, x^1, 1) \leftarrow$ báze (v opačném pořadí)

$$|J_{n+1} - \lambda E| = (-1)^{n+1} \lambda^{n+1}$$

$$J_{n+1} \cdot x = 0 \iff x = \begin{pmatrix} x_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

J_{n+1} .. nilpotentní, cyklická

$$J_2 = C = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$



$V = V_1 \oplus \dots \oplus V_n$

V_i odpovídá λ_i

$(f - \lambda_i \, id)|_{V_i}$ cyklické

$J_{n_i} + \lambda_i E$ $\quad$ λ_i s jinou délkou

pro $n = 3$ $\quad \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right)$

Pr. $\mathbb{K} = \mathbb{R}$ $\quad \lambda_i \in \mathbb{C}$ $\quad A \in Mat_n(\mathbb{R})$

$|A - xE|$ je reálný polynom

$\Rightarrow \overline{\lambda_i}$ také kořen

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \qquad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \qquad \boxed{x^T \cdot y}$$

T

násobení v [illegible]

lineární v každém argumentu

$S \in Mat_n(K)$

$$(x, y) \longmapsto \boxed{x^T S y}$$

lineární

další [illegible]

symetrická je to $\Leftrightarrow S = S^T$

$$(x^T S \cdot y)^T = y^T \cdot S^T \cdot x \quad ✓$$

$\langle u, v \rangle$ nebo $u \cdot v$

$e_1, \dots, e_n$ ortonormální báze

$$u = x_1 e_1 + \dots + x_n e_n \qquad v = y_1 e_1 + \dots + y_n e_n$$

$$\langle u, v \rangle = \sum_{i,j=1}^{n} x_i y_j \underbrace{\langle e_i, e_j \rangle}_{s_{ij}} \qquad \langle e_i, e_j \rangle = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$= \sum_{i=1}^{n} x_i y_i = x^T \cdot y$$

$$= x^T \cdot S \cdot y$$