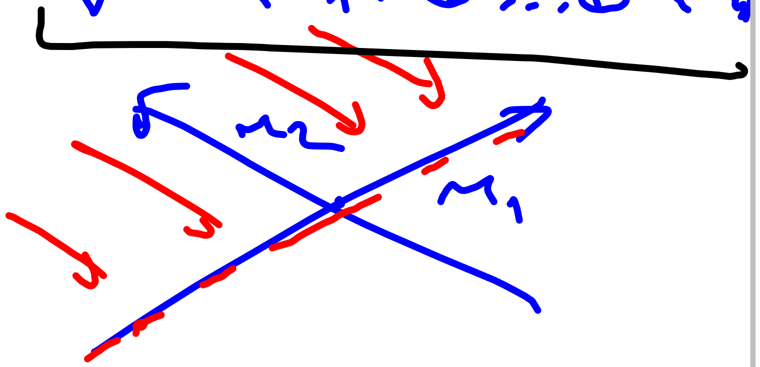
$\underline{u} = (u_1, \dots, u_n)$

$v = x^1 u_1 + \dots + x^n u_n$

$V = \langle u_1 \rangle \oplus \dots \oplus \langle u_n \rangle$

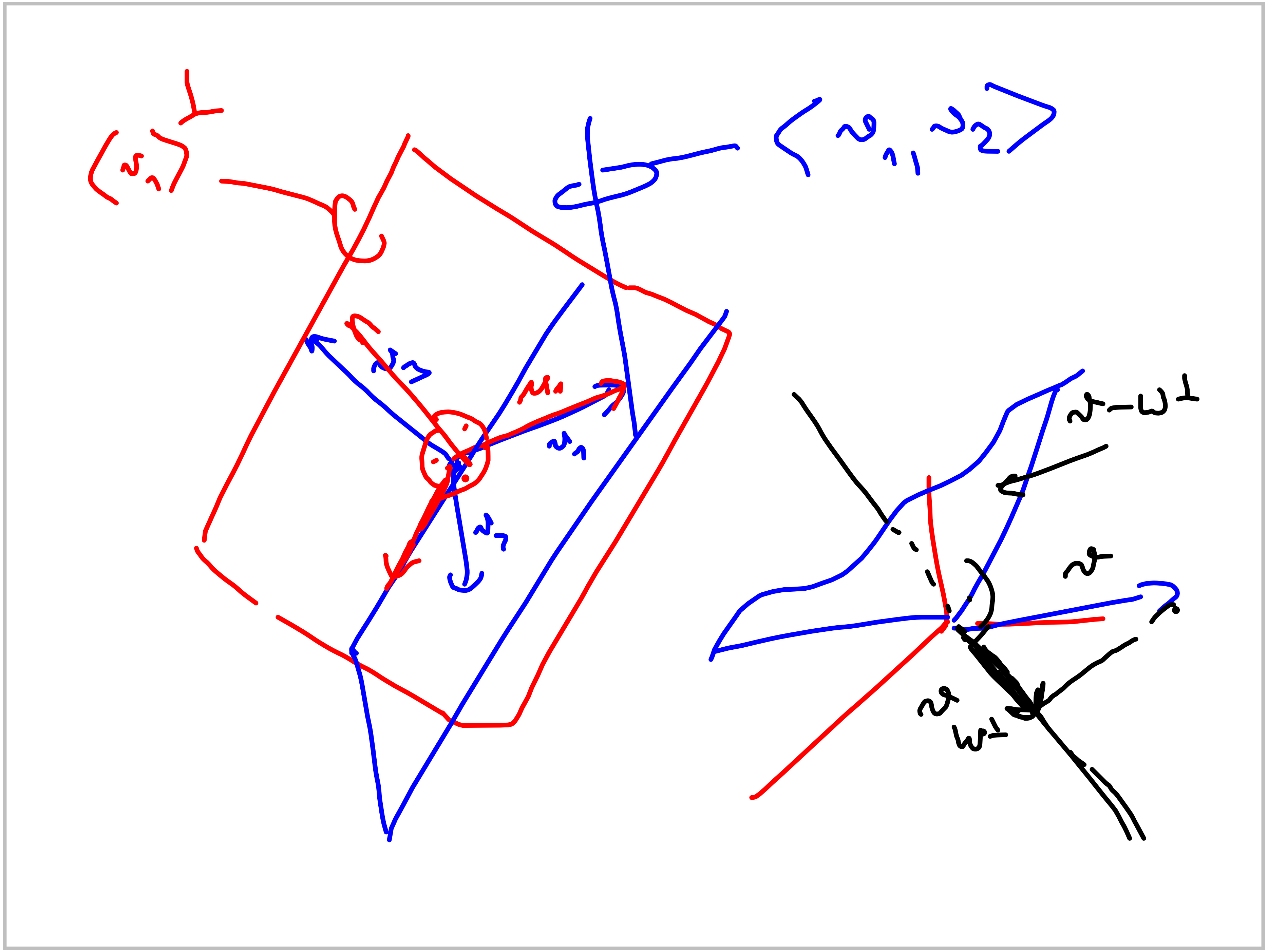


$$\left.\begin{array}{l} f|_{\mathrm{Im}\, f} = \mathrm{id}|_{\mathrm{Im}\, f} \\ \mathrm{Ker}\, f \end{array}\right\} \Rightarrow f \circ f = f$$

$V = W \oplus U \Rightarrow$ projekce na W podél U

na U podél W

skalární součin $\Rightarrow W, W^{\perp}$


⟨v1⟩⊥
⟨v1, v2⟩
u1
v1
v2
v3
v − w⊥
v
w⊥

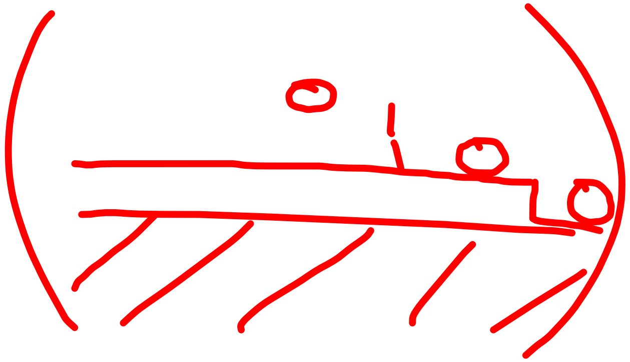
$$f : V \to W$$

$$\underline{e} \qquad \underline{e}' = (f(e_1), \dots, f(e_n), e_{n+1}, \dots$$

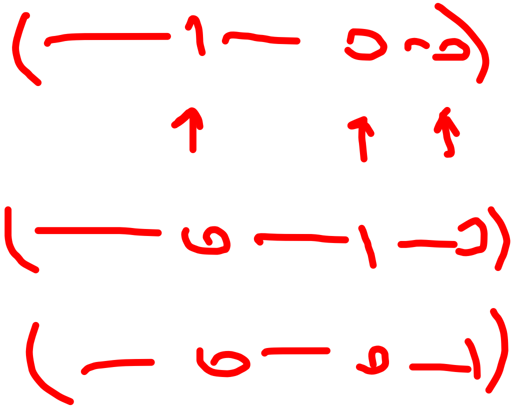
$$A = \begin{pmatrix} \Box & \Box & \cdots & \Box \\ \hline 0 & 0 & & 0 \end{pmatrix} \qquad \langle x, y \rangle = x^T \cdot y$$

$$A' = S^{-1} A S$$

$$S^{-1} = S^T$$



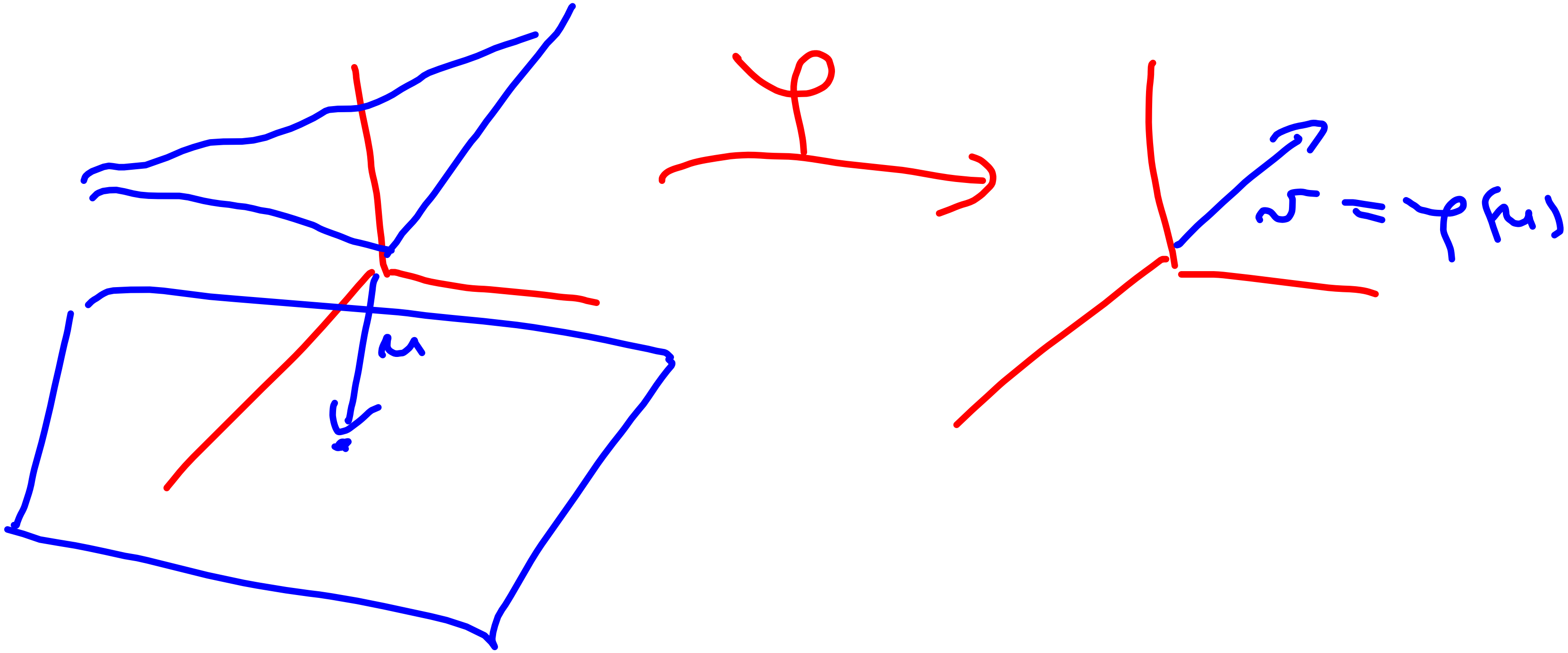
A



$$\begin{pmatrix} 1 & 2 & \cdots & n \end{pmatrix} \begin{pmatrix} x^1 \\ \vdots \\ x^n \end{pmatrix} = x^1 \begin{bmatrix} 1 \end{bmatrix} + x^2 \begin{bmatrix} 2 \end{bmatrix} \cdots$$

$$= \begin{bmatrix} b \end{bmatrix}$$

$$0 \stackrel{?}{=} b \qquad y \neq 0$$


u
φ
v = φ(u)

$$A = \begin{pmatrix} f_1 & f_2 & f_3 & f_4 & f_5 \\ \tau_1 & 0 & 0 & 0 & 0 \\ & \tau_2 & 0 & 0 & 0 \\ & & \tau_3 & 0 & 0 \\ & 0 & & \tau_4 & 0 \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} f_1-\lambda & f_2 & f_3 & f_4 & f_5 \\ \tau_1 & -\lambda & & 0 & \\ & \tau_2 & -\lambda & & \\ & & \tau_3 & -\lambda & \\ & 0 & & \tau_4 & -\lambda \end{vmatrix}$$

$$= -\lambda^5 + f_1\lambda^4 + \dots$$

$$x_{n+k} = F(n, x_n, x_{n+1}, \dots, x_{n+k-1})$$

$$x_n = b_{n-1} x_{n-1} + \dots + b_{n-k} x_{n-k}$$

$+ b$

$$\left.\begin{aligned} x &= (x_0, x_1, \dots) \\ y &= (y_0, y_1, \dots) \end{aligned}\right\} \in K^\infty$$

$u_1 = (1, 0, \dots)$

$u_2 = (0, 1, 0, \dots)$

$u_3 = (0, 0, 1, \dots)$

$K^\infty \overset{?}{=} \langle u_1, u_2, u_3, \dots \rangle$

NE

x, y řešení $\Rightarrow$ $x_n + y_n = b_{n-1}(x_{n-1} + y_{n-1}) + \dots$

opět řešení

$\lambda_1, \dots, \lambda_k$ kořeny $a_0\lambda^k + a_1\lambda^{k-1} + \dots + a_k = 0$

$x(\lambda_1) = (\lambda_1^0, \lambda_1^1, \lambda_1^2, \dots, \lambda_1^n, \dots)$

$x(\lambda_2) = (\lambda_2^0, \lambda_2^1, \dots, \lambda_2^n, \dots)$

$\vdots$

$x = c_1 x(\lambda_1) + c_2 x(\lambda_2) + \dots + c_k x(\lambda_k)$

dosazení $x_0 = b_0, x_1 = b_1, \dots, x_{k-1} = b_{k-1}$

$\Rightarrow$ lineární rovnice pro $c_1, \dots, c_k$

pro $\lambda_i \neq \lambda_j$ pro $i \neq j$

$$\begin{vmatrix} \lambda_1^0 & \lambda_1^1 & \lambda_1^2 & \dots & \lambda_1^{k-1} \\ \lambda_2 & \lambda_2^1 & \lambda_2^2 & \dots & \lambda_2^{k-1} \\ \vdots & & & & \\ \lambda_k^0 & \lambda_k^1 & \lambda_k^2 & \dots & \lambda_k^{k-1} \end{vmatrix} \overset{?}{=} \prod_{i \neq j} (\lambda_i - \lambda_j) \neq 0$$

Vandermonde

$$x_{n+2} = -x_n \qquad x_0 = 0,\ x_1 = 1$$

$$\lambda^2 + 1 = 0 \qquad \lambda = \pm i = \cos \pi/2 \pm i \sin \pi/2$$

$$\lambda^n = \cos \tfrac{n}{2}\pi \pm i \sin \tfrac{n}{2}\pi$$

$$\sim \cancel{A} \cos \tfrac{n}{2}\pi \qquad \sin \tfrac{n}{2}\pi$$