$$\sum_{n=1}^{\infty} \frac{1}{\ln^n(n+1)}$$

$$\sqrt[n]{a_n} = \frac{1}{\ln(n+1)}$$

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0 < 1$$

$\Rightarrow$ KONV.

$$\sum_{n=1}^{\infty} \frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}$$

$$\sqrt[n]{a_n} = \frac{\left(\frac{n+1}{n}\right)^{n}}{3} = \frac{1}{3}\cdot\left(\frac{n+1}{n}\right)^{n} \xrightarrow{n\to\infty} ?$$

$$\frac{1}{3}\cdot\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{n} = \left|1^{\infty}\right| = \frac{1}{3}\cdot\lim_{n\to\infty} e^{\ln\left(\frac{n+1}{n}\right)^{n}} = *$$

$$\lim_{n\to\infty} \ln\left(\frac{n+1}{n}\right)^{n} = \lim_{n\to\infty} n\cdot\ln\left(\frac{n+1}{n}\right) = \left|\infty\cdot 0\right| =$$

$$= \lim_{n\to\infty} \frac{\ln\left(\frac{n+1}{n}\right)}{\frac{1}{n}} = \left|\frac{0}{0}\right| =$$

$$= \lim_{n\to\infty} \frac{\frac{n}{n+1}\cdot \cancel{\left(-\frac{1}{n^2}\right)}}{\cancel{-\frac{1}{n^2}}} = \lim_{n\to\infty} \frac{n}{n+1} = \left(\cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right) = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = 1$$

$$\boxed{\left(\frac{n+1}{n}\right)' = \frac{1\cdot n - (n+1)\cdot 1}{n^2} = -\frac{1}{n^2}}$$

$$\circledast = \frac{1}{3}\cdot e^{1} = \frac{e}{3} < 1 \Rightarrow \underline{\underline{\text{KONV.}}}$$

$$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{3n-1}$$

(ALT. ŘADA $\Rightarrow$ DOST. PODM.)

$$\lim_{n\to\infty} \frac{1}{3n-1} = \left|\frac{1}{\infty}\right| = \underline{\underline{0}} \Rightarrow \text{KONV.}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{\sqrt{n}}{n+100}$$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{n+100} = \left|\frac{\infty}{\infty}\right| = \dots$$

$$= \left|\cdot \frac{\frac{1}{n}}{\frac{1}{n}}\right| = \lim_{n\to\infty} \frac{\sqrt{\frac{1}{n}}}{1 + \frac{100}{n}} = \underline{\underline{0}}$$

$$\Downarrow$$

KONV

S_8

$$a_9 = (-1)^9 \cdot \frac{\sqrt{9}}{109} = -\frac{3}{109} \Rightarrow |\text{ZBYTEK}| < \frac{3}{109}$$

ZNAM. ... $\ominus$

$$a_{10\,000} = (-1)^{10\,000} \cdot \frac{\sqrt{10\,000}}{10\,100} = \frac{100}{10\,100} = \frac{1}{101} \Rightarrow$$

$\Rightarrow$ CHYBA $< \frac{1}{101}$, ZN. $\oplus$

$$\sum_{n=0}^{\infty} (-1)^n \cdot \left(\frac{2n-1}{3n+2}\right)^n$$

$|a_n|$ ... ODM. KRIT. : $\lim\limits_{n\to\infty} \sqrt[n]{|a_n|} =$

$$= \lim_{n\to\infty} \sqrt[n]{\left(\frac{2n-1}{3n+2}\right)^n} = \lim_{n\to\infty} \frac{2n-1}{3n+2} = \left| \begin{matrix} \cdot \frac{1}{n} \\ \cdot \frac{1}{n} \end{matrix} \right| =$$

$$= \lim_{n\to\infty} \frac{2-\frac{1}{n}}{3+\frac{2}{n}} = \frac{2}{3} < 1 \Rightarrow \text{KONV. ABS.}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1}$$

$$|a_n| = \frac{1}{n+1} \quad \dots \quad \sum \frac{1}{n+1} \quad \dots \quad \text{DIV.}$$

$$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{n+1} = 0 \Rightarrow \text{ALT. ŘADA KONV.}$$

$$\Rightarrow \text{Ř. KONV. RELATIVNĚ}$$

$$\sum_{n=0}^{\infty} \frac{n\,x}{e^{nx}}$$

$$\frac{\dfrac{(n+1)\cancel{x}}{e^{(n+1)x}}}{\dfrac{n\cancel{x}}{e^{nx}}} = \frac{n+1}{n} \cdot \frac{\cancel{e^{nx}}}{\cancel{e^{nx}} \cdot e^{x}} =$$

$$= \underbrace{\frac{n+1}{n}}_{\to 1} \cdot \underbrace{\frac{1}{e^{x}}}_{\text{konst. vůči } n} \xrightarrow{n \to \infty} \frac{1}{e^{x}} > 0, \forall x$$

$< 1 \quad \left(\frac{1}{e^x}\right) \Leftrightarrow x > 0 \Rightarrow$ KONV. ABS.

$> 1 \quad \left(\frac{1}{e^x}\right) \Leftrightarrow x < 0 \Rightarrow$ DIV.

$= 1 \quad \left(\frac{1}{e^x} = 1\right) \Leftrightarrow x = 0 \Rightarrow \sum 0 = 0$

$\Rightarrow$ KONV. ABS.

$$\sum_{n=0}^{\infty} \frac{x^n}{(n+1)\cdot 8^n}$$

$$\boxed{\sum \underline{\underline{a_n}} \cdot (x-S)^n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{1}{(n+2)\cdot 8^{n+1}}}{\frac{1}{(n+1)\cdot 8^n}} =$$

$$= \lim_{n\to\infty} \frac{(n+1)\cdot \cancel{8^n}}{(n+2)\cdot \cancel{8^n}\cdot 8} = \lim_{n\to\infty} \frac{n+1}{8n+16} = \underline{\underline{\frac{1}{8}}}$$

$$\Rightarrow \quad R = \frac{1}{\frac{1}{8}} = \underline{\underline{8}}$$

$$(-8, 8)$$

$$\boxed{x^n = (x-0)^n}$$

$\underline{\underline{v\ -8\ \text{a}\ 8\ ?}}$

$\underline{x=-8}$: $\sum_{0}^{\infty} \frac{(-8)^n}{(n+1)\cdot 8^n} = \sum_{0}^{\infty} \frac{(-1)^n \cdot \cancel{8^n}}{(n+1)\cdot \cancel{8^n}} =$

$= \sum_{0}^{\infty} (-1)^n \cdot \frac{1}{n+1}$ | $\lim_{n\to\infty} \frac{1}{n+1} = 0$

$\Rightarrow$ konv.

$\underline{\underline{x=8}}$: $\sum_{0}^{\infty} \frac{\cancel{8^n}}{(n+1)\cdot \cancel{8^n}} = \sum_{0}^{\infty} \frac{1}{n+1}$ DIV.

$\Rightarrow$ INT. KONV. : $\underline{\underline{[-8, 8)}}$

řádek	r	N	S
1.	$\frac{1}{2}$	2	$\pi \cdot \left(\frac{1}{2}\right)^2 = \pi \cdot \frac{1}{2^2}$
2.	$\frac{1}{2^2}$	2^2	$\pi \cdot \left(\frac{1}{2^2}\right)^2 \cdot 2 = \pi \cdot \frac{1}{2^3}$
3.	$\frac{1}{2^3}$	2^3	$\pi \cdot \left(\frac{1}{2^3}\right)^2 \cdot 2^2 = \pi \frac{1}{2^4}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n.	$\frac{1}{2^n}$	2^n	$\pi \cdot \left(\frac{1}{2^n}\right)^2 \cdot 2^{n-1} = \pi \cdot \frac{1}{2^{n+1}}$

celkem: $\sum_{n=1}^{\infty} \pi \cdot \frac{1}{2^{n+1}} = \pi \cdot \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n+1} = \left| \begin{matrix} q = \frac{1}{2} \\ a = \frac{1}{4} \end{matrix} \right| =$

$$= \pi \cdot \frac{\frac{1}{4}}{1-\frac{1}{2}} = \pi \cdot \frac{\frac{1}{4}}{\frac{1}{2}} = \pi \cdot \frac{1}{2} = \underline{\underline{\frac{\pi}{2}}}$$

$$\left(\frac{a}{1-q}\right)$$

NEJVĚTŠÍ PŮLKRUHY (1. Ř.) $\rightarrow$ PLOCHA $\frac{\pi}{4}$

$\Rightarrow$ CELKEM 2x PLOCHA 1. ŘÁDKU

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!}$$

$$\cos x = (\sin x)' =$$

$$= 1 - \frac{3 \cdot x^2}{3!} + \frac{5 \cdot x^4}{5!} - \frac{7 \cdot x^6}{7!} + \dots =$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^{2n}}{(2n)!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$(e^x)' = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \dots =$$

$$= 1 + x + \frac{x^2}{2!} + \dots \quad \checkmark$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \dots$$

<u>přímo:</u> $f(x) = e^{-x}$, $f'(x) = -e^{-x}$, $f''(x) = e^{-x}$,

$f'''(x) = -e^{-x}$, ...

$$e^{-x} = f(x_0) + \frac{f'(x_0)}{1!} \cdot (x - x_0) + \frac{f''(x_0)}{2!} \cdot (x - x_0)^2 + \dots$$

$$\left| x_0 = 0 \right| = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots$$

<u>přímo</u>: $f(x) = e^{x^2}$, $f'(x) = e^{x^2} \cdot 2x$,

$$f''(x) = \underbrace{e^{x^2} \cdot 2x} \cdot 2x + \underbrace{e^{x^2}} \cdot 2 =$$

$$= (4x^2 + 2) \cdot e^{x^2} = |x=0| = (0+2) \cdot 1 = 2$$

$$f'''(x) = 8x \cdot e^{x^2} + (4x^2+2) \cdot e^{x^2} \cdot 2x =$$

$$= 4x e^{x^2} \cdot (2 + 2x^2 + 1) = 4x e^{x^2} \cdot (2x^2 + 3)$$

$\vdots$

$x_0 = 0$

$$e^{x^2} = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0)^1 + \dots =$$

$$= 1 + 0 + \frac{2}{2!} \cdot x^2 + 0 + \dots =$$

$$= 1 + x^2 + \dots$$

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \dots = ?$$

$$f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \quad (-1 \le x \le 1)$$

$$f'(x) = 1 - x^2 + x^4 - \dots = \quad (-1 < x < 1)$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n} = \left| \begin{array}{l} q = -x^2 \\ a = 1 \end{array} \right| =$$

$$= \frac{1}{1 + x^2}$$

$$f'(x) = \frac{1}{1+x^2} \quad \Big/ \int$$

$$f(x) = \operatorname{arctg} x + \underline{\underline{c}}$$

$= ?$

DO PLIV. ŘADY DOS. $\underline{\underline{0}}$ $\Rightarrow$ $f(0) = 0$

$$\operatorname{arctg} 0 + c = 0$$

$$0 + c = 0 \Rightarrow \underline{\underline{c = 0}}$$

$$\underline{\underline{f(x) = \operatorname{arctg} x}}$$

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2} = \frac{\sin 1}{1^2} + \frac{\sin 2}{2^2} + \frac{\sin 3}{3^2} + \dots$$

$$\sum_{n=1}^{\infty} \left| \frac{\sin n}{n^2} \right| = \frac{|\sin 1|}{1} + \frac{|\sin 2|}{4} + \frac{|\sin 3|}{9} + \dots$$

$$\leq \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

→ KONV. MAJORANTA ⇒ PŮV. Ř. KONV. ABS.

$$I = (-1, 1) \quad , \quad \sum_{n=1}^{\infty} n \cdot (n+1) \cdot x^{n}$$

$$\sum_{n=1}^{\infty} n \cdot (n+1) \cdot x^{n} = \sum_{n=1}^{\infty} n \cdot \left(x^{n+1}\right)' =$$

$$= \sum_{n=1}^{\infty} \left(n \cdot x^{n+1}\right)' = \left(\sum_{n=1}^{\infty} n \cdot x^{n+1}\right)' =$$

$$= \left(\sum_{n=1}^{\infty} n \cdot x^{n-1} \cdot x^{2}\right)' = \left(x^{2} \cdot \sum_{n=1}^{\infty} \left(x^{n}\right)'\right)' =$$

$$= \left(x^{2} \cdot \left(\sum_{n=1}^{\infty} x^{n}\right)'\right)' = \left(x^{2} \cdot \left(\frac{x}{1-x}\right)'\right)' =$$

$$= \left(x^2 \cdot \frac{1\cdot(1-x) - x\cdot(-1)}{(1-x)^2} \right)' = \left(x^2 \cdot \frac{1}{(1-x)^2} \right)' =$$

$$= \left(\frac{x^2}{(1-x)^2} \right)' = \frac{2x\cdot(1-x)^2 - x^2\cdot 2\cdot(1-x)\cdot(-1)}{(1-x)^4} =$$

$$= \frac{\cancel{(1-x)}\cdot\left[2x\cdot(1-x) + 2x^2\right]}{(1-x)^{\cancel{4}\ 3}} = \frac{2x - 2x^2 + 2x^2}{(1-x)^3} =$$

$$= \underline{\underline{\frac{2x}{(1-x)^3}}} \quad , \ x \in (-1, 1)$$