$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n(n+1)} = \sum_{n=1}^{\infty} \left[(-1)^{n+1} \cdot \frac{1}{n} \int x^{n}\, dx \right] =$$

$$= \int \sum_{n=1}^{\infty} \left[(-1)^{n+1} \frac{x^{n}}{n} \right] dx = \int \sum_{n=1}^{\infty} \left[(-1)^{n+1} \cdot \int x^{n-1}\, dx \right] dx =$$

$$= \int \left[\int \sum_{n=1}^{\infty} (-1)^{n+1} \cdot x^{n-1}\, dx \right] dx =$$

$$= \int \left[\int (1 - x + x^{2} - x^{3} + \dots)\, dx \right] dx = \left| \begin{matrix} a_1 = 1 \\ q = -x \end{matrix} \right| =$$

$$= \int \left[\int \frac{1}{1-(-x)}\, dx \right] dx = \int \left[\int \frac{1}{1+x}\, dx \right] dx =$$

$$= \int \ln(\underbrace{1+x}_{>0})\,dx = \left| \begin{matrix} u = \ln(1+x) & u' = \dfrac{1}{1+x} \\ v' = 1 & v = x \end{matrix} \right| =$$

$$= x \cdot \ln(1+x) - \int \frac{x}{1+x}\,dx =$$

$$= x \cdot \ln(1+x) - \int \left(\frac{x+1}{1+x} - \frac{1}{1+x} \right) dx =$$

$$= x \cdot \ln(1+x) - \int 1\,dx + \int \frac{1}{1+x}\,dx =$$

$$= x \cdot \ln(1+x) - x + \ln(1+x) + C =$$

$$= \ln(1+x) \cdot (x+1) - x + C \;\; = ?$$

ŘADA KONV. PRO $x = 0$ $(0 \in I)$

SE SOUČTEM $= 0$ $(\sum 0 = 0 + 0 + 0 + \dots)$

$$\sum 0 = 0 \underline{\underline{=}} \ln 1 \cdot 1 - 0 + C$$

$$\underline{\underline{0 = C}}$$

$\Rightarrow$ NA I:
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n \cdot (n+1)} =$$

$$= \underline{\underline{\ln(1+x) \cdot (x+1) - x}}$$

$$\frac{1}{x^2-x-12} = \frac{1}{(x-4)\cdot(x+3)} = \frac{A}{x-4} + \frac{B}{x+3} =$$

$$= \frac{\frac{1}{7}}{x-4} - \frac{\frac{1}{7}}{x+3}$$

$$\frac{a_1}{1-q}$$

(i) $$\frac{\frac{1}{7}}{x-4} = \frac{-\frac{1}{7}}{4-x} = \left| \cdot \frac{\frac{1}{4}}{\frac{1}{4}} \right| = \frac{-\frac{1}{28}}{1-\frac{x}{4}} =$$

$$= \left| \begin{array}{l} a_1 = -\frac{1}{28} \\ q = \frac{x}{4} \end{array} \right| = -\frac{1}{28} - \frac{x}{4\cdot 28} - \frac{x^2}{4^2\cdot 28} - \frac{x^3}{4^3\cdot 28} - \ldots$$

$$\ldots - \frac{x^{n-1}}{28\cdot 4^{n-1}} - \ldots$$

(iii) $$\frac{\frac{1}{7}}{3+x} = \left| \cdot \frac{\frac{1}{3}}{\frac{1}{3}} \right| = \frac{\frac{1}{21}}{1+\frac{x}{3}} \cdot$$

$$= \frac{\frac{1}{21}}{1-\left(-\frac{x}{3}\right)} = \left| \begin{array}{l} a_1 = \frac{1}{21} \\ q = -\frac{x}{3} \end{array} \right| =$$

$$= \frac{1}{21} - \frac{x}{21\cdot 3} + \frac{x^2}{21\cdot 3^2} - \dots + (-1)^{n-1}\cdot \frac{x^{n-1}}{21\cdot 3^{n-1}} + \dots$$

$$\frac{1}{x^2-x-12} = -\frac{1}{28}-\frac{1}{21}-\left(\frac{1}{28\cdot 4}-\frac{1}{21\cdot 3}\right)\cdot x+\ldots=$$

$$=\sum_{n=1}^{\infty}\left(-\frac{x^{n-1}}{28\cdot 4^{n-1}}-\underline{(-1)^{n-1}}\cdot\frac{x^{n-1}}{21\cdot 3^{n-1}}\right)=$$

$$=\sum_{n=1}^{\infty}\left(\left[\underline{(-1)^{n}}\,\frac{1}{21\cdot 3^{n-1}}-\frac{1}{28\cdot 4^{n-1}}\right]x^{n-1}\right)=$$

$$=\sum_{n=0}^{\infty}\left[(-1)^{n+1}\cdot\frac{1}{21\cdot 3^{n}}-\frac{1}{28\cdot 4^{n}}\right]x^{n}$$

$$\sqrt[3]{70} = \sqrt[3]{64+6} = \sqrt[3]{4^3+6} =$$

$$= \sqrt[3]{4^3\cdot\left(1+\frac{6}{64}\right)} = 4\cdot\sqrt[3]{1+\frac{3}{32}} =$$

$$= 4\cdot\left(1+\frac{3}{32}\right)^{\frac{1}{3}}$$

$$\underline{\underline{f(x) = (1+x)^{\frac{1}{3}}}} \quad , \quad x_0 = 0$$

$$\underline{f(0) = 1}$$

$$f'(x) = \frac{1}{3}\cdot(1+x)^{-\frac{2}{3}}$$

$$\underline{\underline{f'(0) = \frac{1}{3}}}$$

$$(1+x)^{\frac{1}{3}} \approx \underline{\underline{1 + \frac{1}{3}\cdot x}}$$

$$\left(f(0) + \frac{f'(0)}{1!}\cdot x^1 \right)$$

$$\sqrt[3]{70} \approx 4\cdot\left(1 + \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{32}\right) = 4 + \frac{1}{8} =$$

$$= \underline{\underline{4{,}125}}$$

(PŘESNÁ HODN. $\doteq$ 4,12129)

$$\cos x = \underline{\underline{1}} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$10^\circ \dots \underline{\underline{\frac{\pi}{18}}}$$

$$\underline{1 > 10^{-5}}$$

$$\frac{x^2}{2!} \doteq \frac{\left(\frac{\pi}{18}\right)^2}{2} \doteq 0{,}015 > 10^{-5}$$

$$\frac{x^4}{4!} = \frac{\left(\frac{\pi}{18}\right)^4}{4!} \doteq 3{,}866 \cdot 10^{-5} > 10^{-5}$$

$$\frac{x^6}{6!} = \frac{\left(\frac{\pi}{18}\right)^6}{6!} \doteq 3{,}926 \cdot 10^{-8} < 10^{-5}$$

$$\cos\frac{\pi}{18} \approx 1 - \frac{\left(\frac{\pi}{18}\right)^2}{2!} + \frac{\left(\frac{\pi}{18}\right)^4}{4!} \doteq 0{,}985$$

$$y' = (2-y)\cdot \operatorname{tg} x$$

$$\frac{dy}{dx} = (2-y)\cdot \operatorname{tg} x \qquad \Big/ \cdot \frac{1}{2-y}\cdot dx, \quad y \neq 2$$

$$\int \frac{1}{2-y}\,dy = \int \operatorname{tg} x\; dx \qquad -\int \frac{-\sin x}{\cos x}\,dx = -\ln|\cos x| + c$$

$$-\ln|2-y| = -\ln|\cos x| + c_0$$

$$\ln|2-y| = \ln|\cos x| + c_1 \qquad \Big/ \exp$$

$$|2-y| = e^{\ln|\cos x| + c_1} = e^{\ln|\cos x|} \cdot e^{c_1} =$$

$$= |\cos x| \cdot e^{c_1}$$

$$|2-y| = |\cos x| \cdot e^{c_1} \qquad \Big/ \; K := e^{c_1} \text{ e}$$

PŘÍPADNĚ ZNAMÉNKO

$$2-y = \cos x \cdot K$$

$$\underline{\underline{y = 2 - K \cdot \cos x}}$$

($y = 2$ JE ŘEŠ., DOSTANEM HO VOLBOU $K = 0$)

$$1 + y^2 - xy\,(1 + x^2)\, y' = 0$$

$$xy\,(1+x^2)\, y' = 1 + y^2$$

$$\frac{dy}{dx} = y' = \frac{1+y^2}{xy\,(1+x^2)} \qquad (1 + m^2 \neq 0)$$

$$\int \frac{y}{1+y^2}\, dy = \int \frac{1}{x \cdot (1+x^2)}\, dx$$

$$\int \frac{y}{1+y^2}\,dy = \frac{1}{2}\int \frac{2y}{1+y^2}\,dy =$$

$$= \frac{1}{2}\cdot \ln(1+y^2) + c_1 =$$

$$= \underline{\underline{\ln\sqrt{1+y^2} + c_1}}$$

$$\int \frac{1}{x\cdot(1+x^2)}\,dx = \int \frac{A}{x}\,dx + \int \frac{Bx+C}{1+x^2}\,dx =$$

$$= \int \frac{1}{x}\,dx - \int \frac{x}{1+x^2}\,dx = \ln|x| - \frac{1}{2}\cdot\int \frac{2x}{1+x^2}\,dx =$$

$$= \ln|x| - \frac{1}{2}\cdot \ln(x^2+1) + c_2 =$$

$$= \ln \frac{|x|}{\sqrt{x^2+1}} + c_2$$

Celkem:
$$\ln \frac{|x|}{\sqrt{x^2+1}} = \ln \sqrt{1+y^2} + c_3$$

$$\frac{|x|}{\sqrt{x^2+1}} = \sqrt{1+y^2}\cdot e^{c_3}$$

$$\frac{x}{\sqrt{x^2+1}} = \sqrt{1+y^2}\cdot K$$

$$x \cdot y' = y \cdot \ln \frac{y}{x}$$

$$y' = \frac{y}{x} \cdot \ln \frac{y}{x}$$

SUBST.: $u = \frac{y}{x}$ $\quad (y = y(x),\ u = u(x))$

$$y = u \cdot x \Rightarrow y' = u + u' \cdot x$$

$$u + u' \cdot x = u \cdot \ln u$$

$$u' \cdot x = u \cdot \ln u - u$$

$$\frac{du}{dx} = u' = \frac{u \cdot (\ln u - 1)}{x}$$

$$\int \frac{1}{x\cdot(\ln x - 1)}\,dx = \int \frac{1}{x}\,dx$$

$x \cdot \ln(x-1) \neq 0$

$$\int \frac{1}{x\cdot(\ln x - 1)}\,dx = \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \end{array} \right| = \int \frac{1}{t-1}\,dt =$$

$$= \ln|t-1| + c_1 = \ln|\ln x - 1| + c_1$$

$$\int \frac{1}{x}\,dx = \ln|x| + c_2$$

$$\ln|\ln u - 1| = \ln|x| + c_3 \qquad / \exp$$

$$|\ln u - 1| = |x| \cdot e^{c_3}$$

$$\ln u - 1 = x \cdot K$$

$$\ln u = x \cdot K + 1 \qquad / \exp$$

$$\frac{y}{x} = u = e^{x \cdot K + 1}$$

$$\underline{\underline{y = x \cdot e^{xK+1}}}$$

$$u \cdot (\ln u - 1) \neq 0$$

$$u \cdot (\ln u - 1) = 0 \iff u = 0 \lor u = e$$

$$\frac{y}{x} = 0 \qquad\qquad \frac{y}{x} = e$$

$$y = 0 \qquad\qquad y = ex$$

NELZE DOS. DO ZADÁNÍ

JE ŘEŠ.

(VOLBA $k = 0$)

$$y' = a(x)\,y + b(x)$$

$$\mu(x) = e^{-\int a(x)\,dx}$$

$$y' + 2xy = x \cdot e^{-x^2} \quad \dots \text{RCE}$$

$$y' = -2x\,y + x \cdot e^{-x^2}$$

$$\mu(x) = e^{-\int -2x\,dx} = e^{2 \cdot \frac{x^2}{2}} = e^{x^2}$$

$$\text{RCE} \quad / \cdot e^{x^2}$$

$$\underbrace{y' \cdot e^{x^2} + 2xy\,e^{x^2}}_{(y \cdot e^{x^2})'} = x$$

$$\left(y \cdot e^{x^2}\right)' = x \qquad / \int$$

$$y e^{x^2} = \int x \, dx + c_1$$

$$y e^{x^2} = \frac{x^2}{2} + c$$

$$y = e^{-x^2} \cdot \left(\frac{x^2}{2} + c\right)$$

zk:

$$y' = e^{-x^2} \cdot (-2x) \cdot \left(\frac{x^2}{2} + c\right) + \underbrace{e^{-x^2} \cdot x}$$

$$-2xy + \underbrace{x \cdot e^{-x^2}} = -2x \cdot e^{-x^2} \cdot \left(\frac{x^2}{2} + c\right) + x \cdot e^{-x^2}$$

$$y' = a(x)y + b(x)\cdot y^r$$

$$u = y^{1-r}$$

$$\frac{y'}{y^r} = a(x)\frac{y}{y^r} + b(x)$$

$$\frac{y}{y^r} = y^{1-r}$$

$$y' + y = x\cdot\sqrt{y} \quad \dots \quad a(x) = -1,\ b(x) = x$$

$$r = \frac{1}{2}$$

$$y' = -y + x\sqrt{y}$$

SUBST.: $u = y^{1-\frac{1}{2}} = y^{\frac{1}{2}} = \sqrt{y}$

$$u = \sqrt{y}$$

$$u' = \frac{1}{2} \cdot y^{-\frac{1}{2}} \cdot y' = \frac{1}{2} \cdot \frac{y'}{\sqrt{y}} \Rightarrow \frac{y'}{\sqrt{y}} = 2u'$$

$$y' + y = x\sqrt{y} \quad / \cdot \frac{1}{\sqrt{y}}$$

$$\frac{y'}{\sqrt{y}} + \sqrt{y} = x$$ (APL. SUBST.)

$$\Rightarrow 2 \cdot u' + u = x$$ ... LIN. D. RCE 1. ŘÁDU

$$u' + \frac{u}{2} = \frac{x}{2} \qquad , \quad a(x) = -\frac{1}{2}$$

$$\mu(x) = e^{-\int -\frac{1}{2}\,dx} = e^{\int \frac{1}{2}\,dx} = \underline{\underline{e^{\frac{x}{2}}}}$$

$$u' + \frac{u}{2} = \frac{x}{2} \quad \Big/ \cdot e^{\frac{x}{2}}$$

$$u' \cdot e^{\frac{x}{2}} + \frac{u}{2} \cdot e^{\frac{x}{2}} = \frac{x}{2} \cdot e^{\frac{x}{2}}$$

$$\left(u \cdot e^{\frac{x}{2}}\right)' = \frac{x}{2} \cdot e^{\frac{x}{2}} \quad \Big/ \int$$

$$u \cdot e^{\frac{x}{2}} = \int \frac{x}{2} \cdot e^{\frac{x}{2}}\,dx = \left| \begin{array}{l} t = \frac{x}{2} \\ dt = \frac{1}{2}\,dx \\ 2\,dt = dx \end{array} \right| =$$

$$= 2 \cdot \int t \cdot e^{t}\,dt =$$

$$= \left| \begin{matrix} u = t & u' = 1 \\ v' = e^t & v = e^t \end{matrix} \right| = 2 \cdot \left[t \cdot e^t - \int e^t \, dt \right] =$$

$$= 2 \cdot \left(t \cdot e^t - e^t \right) + C =$$

$$= 2 \cdot e^{\frac{x}{2}} \cdot \left(\frac{x}{2} - 1 \right) + C =$$

$$= \underline{\underline{e^{\frac{x}{2}} \cdot (x - 2) + C}}$$

$$\Rightarrow u \cdot e^{\frac{x}{2}} = e^{\frac{x}{2}} \cdot (x-2) + C$$

$$\sqrt{y} = u = x - 2 + C \cdot e^{-\frac{x}{2}} \Rightarrow y = \underline{\underline{\left(x - 2 + C \cdot e^{-\frac{x}{2}} \right)^2}}$$