$$\lim_{x \to 0} \frac{e^{-\frac{1}{x^2}}}{x^{100}} = \left[\frac{e^{-\infty}}{0^{100}} = \frac{\frac{1}{\infty}}{0} = \frac{0}{0} \right] \overset{\text{L'H}}{=}$$

a) $$= \lim_{x \to 0} \frac{e^{-\frac{1}{x^2}} \cdot 2 \cdot x^{-3}}{100 \cdot x^{99}} = 2 \cdot \lim_{x \to 0} \frac{e^{-\frac{1}{x^2}}}{100 \cdot x^{102}} = \times$$

b) $$= \lim_{x \to 0} \frac{\frac{1}{x^{100}}}{e^{\frac{1}{x^2}}} = \lim_{x \to 0} \frac{x^{-100}}{e^{\frac{1}{x^2}}} = \left[\frac{\infty}{\infty} \right]$$

$$\overset{\text{L'H}}{=} \lim_{x \to 0} \frac{-\cancel{100} \cdot x^{-101}}{e^{\frac{1}{x^2}} \cdot (-\cancel{2}) \cdot x^{-3}} = \lim_{x \to 0} \frac{50 \cdot x^{98}}{e^{\frac{1}{x^2}}} =$$

$$= \left[\frac{\infty}{\infty}\right] = \dots = C \cdot \lim_{x \to 0} \frac{x^0}{e^{\frac{1}{x^2}}} =$$

$$= c \cdot \lim_{x \to 0} \frac{1}{e^{\frac{1}{x^2}}} = \left[\frac{1}{\infty}\right] = c \cdot 0 =$$

$$= \underline{\underline{0}}$$

$$f(x) = x^2 + 1 + \sin x^3$$

$$\parallel$$

$$y$$

$$\underline{\underline{y}} \cdot \log_2 x + \sin \underline{\underline{y}} = 0$$

$y \in [1/2, 7/2]$

$$[x_0, y_0] = [1, ?]$$

$$\parallel$$

$$[x_0, f(x_0)]$$

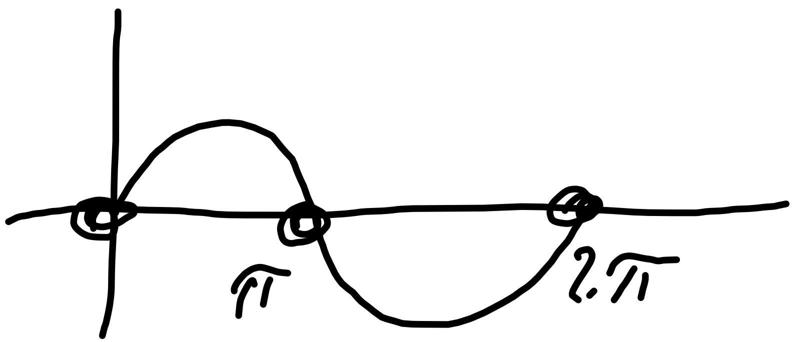
$f(x_0) = y_0$: $\quad y \cdot \underbrace{\log_2 1}_{=0} + \sin y = 0$

$$\sin y = 0 \iff$$

$$\iff y = k \cdot \pi, \; k \in \mathbb{Z}$$

$$\Downarrow$$

$$y = \pi$$



$[1, \pi]$ $\qquad y' = ?$

$$\overset{\shortparallel}{f'(x_0)}$$

$$y \cdot \log_2 x + \sin y = 0 \quad \Big/ \frac{d}{dx}, \quad y = y(x)$$

$$y' \cdot \log_2 x + y \cdot \frac{1}{x \cdot \ln 2} + \cos y \cdot y' = 0$$

$$y' \cdot (\log_2 x + \cos y) = -\frac{y}{x \cdot \ln 2}$$

$$y' = \frac{-y}{x \cdot \ln 2 \cdot (\underset{0}{\log_2 x} + \underset{-1}{\cos y})} = \Big|_{\substack{x=1 \\ y=\pi}} = +\frac{\pi}{\ln 2}$$

$$t: \quad y - y_0 = f'(x_0) \cdot (x - x_0)$$

$$y - \pi = \frac{\pi}{\ln 2} \cdot (x - 1)$$

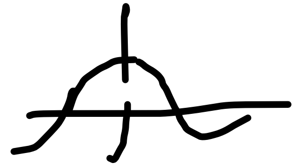
$$t: \quad \frac{\pi}{\ln 2} \cdot x - y + \pi - \frac{\pi}{\ln 2} = 0$$

$$F_x, \quad F_y \quad \Rightarrow \quad y' = -\frac{F_x}{F_y}, \quad F_y \neq 0$$

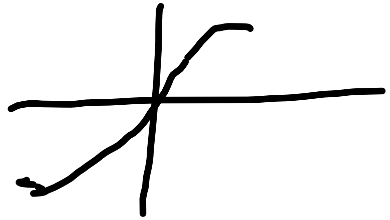
$$f(x) = x^2 \cdot e^{-x}$$

a) $Dom(f) = \mathbb{R}$, $H(f) = [0; \infty)$

b) SUDÁ $\Leftrightarrow$ $f(-x) = f(x)$



LICHÁ $\Leftrightarrow$ $f(-x) = -f(x)$



$$f(-x) = \underline{\underline{(-x)}}^2 \cdot e^{-(-x)} = x^2 \cdot e^{x} \neq f(x)$$

$$\neq -f(x)$$

c) s osou y $\Rightarrow$ $x = 0$

$$0^2 \cdot e^0 = y$$

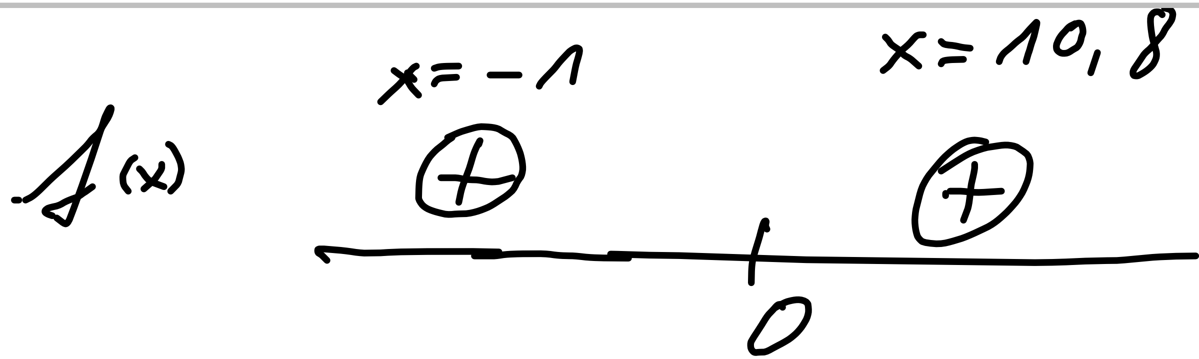
$$\begin{array}{c} \shortparallel \\ 0 \end{array} \Rightarrow [0, 0]$$

s osou x $\Rightarrow$ $\underline{\underline{y = 0}}$

$$x^2 \cdot e^{-x} = 0 \Leftrightarrow \underline{\underline{x = 0}}$$

$[0; 0]$

d)



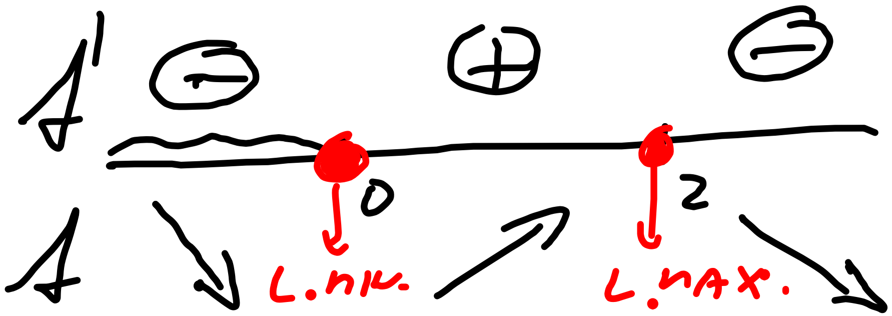
$$f(x) = 0 \Leftrightarrow x = 0$$

e) $$f'(x) = 2x \cdot e^{-x} + x^2 \cdot e^{-x} \cdot (-1) =$$

$$= \boxed{x \cdot e^{-x} \cdot (2 - x) = 0} \Leftrightarrow$$

$$\Leftrightarrow x = 0 \vee x = 2$$

STAC. BODY



f KLES. $\quad x \in (-\infty, 0] \cup [2; \infty)$

f ROST. $\quad x \in [0, 2]$

---

f $\quad f''(x) = \left[e^{-x} \cdot (2x - x^2)\right]' =$

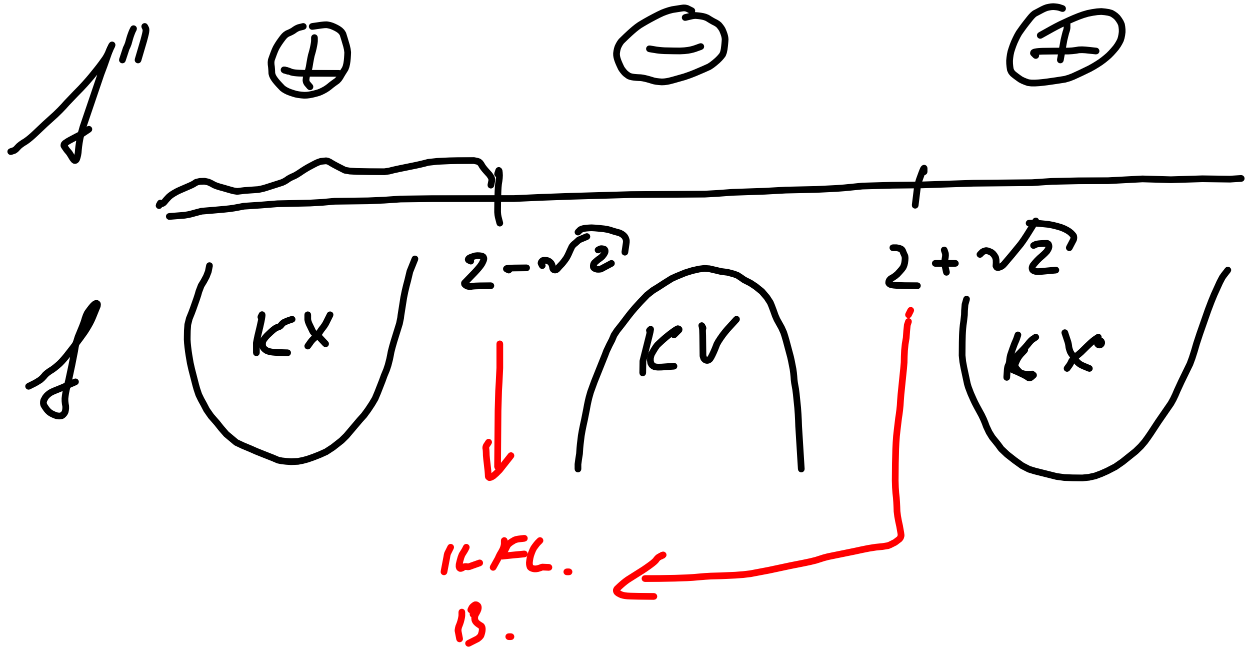
$$= e^{-x} \cdot (-1) \cdot (2x - x^2) + e^{-x} \cdot (2 - 2x) =$$

$$= e^{-x} \cdot (x^2 - 4x + 2) = 0 \iff$$

$x^2 - 4x + 2 = 0$

$x_1 = 2 + \sqrt{2}$

$x_2 = 2 - \sqrt{2}$



9) EXTR.: STAC. BODY $x = 0$, $x = 2$

$$f''(0) = 2 > 0 \Rightarrow \text{L. MIN.}$$

$$f''(2) = e^{-2} \cdot (-2) < 0 \Rightarrow \text{L. MAX.}$$

INFL. B. : $f''_{(x)} = 0$, $x = 2 \pm \sqrt{2}$

$$f''_{(x)} = -e^{-x} \cdot (-x^2 + 6x - 6) \neq 0$$

$$\uparrow$$

$2 \pm \sqrt{2}$

$\Downarrow$

JSOU

I. B.

4) SE šn.: $y = ax + b$, $a, b = ?$

$$a = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{x^2 \cdot e^{-x}}{x} =$$

$$= \lim_{x \to \infty} x \cdot e^{-x} = [\infty \cdot 0] =$$

$$= \lim_{x \to \infty} \frac{x}{e^x} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=} \lim_{x \to \infty} \frac{1}{e^x} = \underline{\underline{0}}$$

$$b = \lim_{x \to +\infty} [f(x) - a \cdot x] =$$

$$= \lim_{x \to \infty} (x^2 \cdot e^{-x} - 0 \cdot x) = \lim_{x \to \infty} \frac{x^2}{e^x} = \left[\frac{\infty}{\infty}\right]$$

$$= \left| 2 \times L'H \right| = 0$$

$$y = 0 \cdot x + 0 = 0 \Rightarrow$$ AS. SE SM. $(+\infty)$

JE OSA X

$\boxed{-\infty}$ $a = \lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \underbrace{x}_{-\infty} \cdot \underbrace{e^{-x}}_{\infty} = \underline{\underline{-\infty}}$

$(-\infty)$ – AS. SE. SM. NENÍ

AS. BEZ SM. – V BODECH, KDE

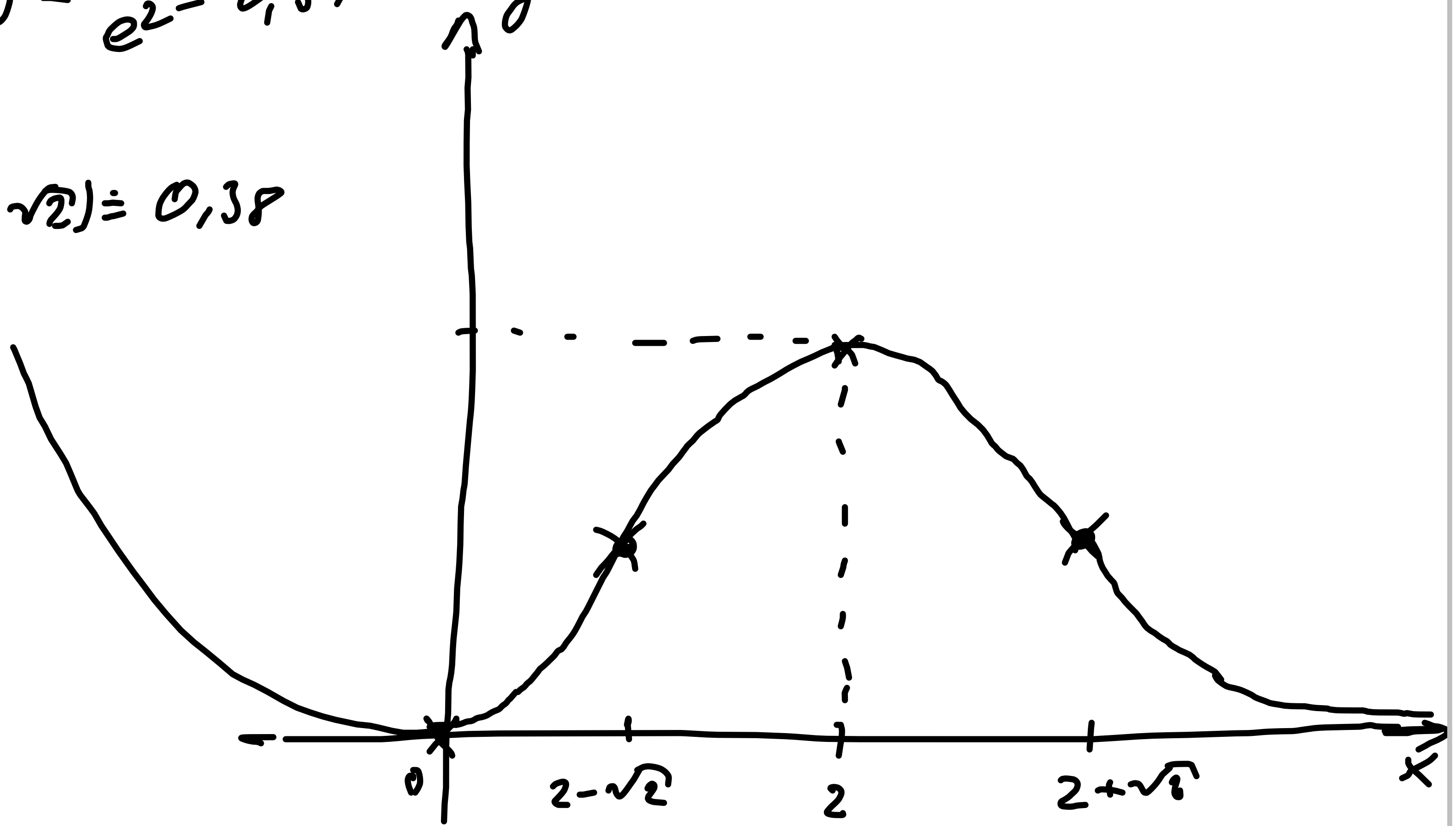
$f(x)$ NENÍ DEF.

i)

$f(0) = 0$

$f(2) = \frac{4}{e^2} \doteq 0{,}54$

$f(2 \pm \sqrt{2}) \doteq 0{,}38$



$$\mathrm{Dom}(f) = \mathbb{R} \setminus \{-1, +1\} \qquad f(x) = \frac{x}{x^2-1}$$

AS. BEZ. SM.: $x = \pm 1$

$$\lim_{x \to 1^+} \frac{x}{x^2-1} = \left| \frac{1}{0^+} \right| = \infty \checkmark$$

$$\lim_{x \to 1^-} \frac{x}{x^2-1} = \left| \frac{1}{0^-} \right| = -\infty \checkmark$$

$$\left.\right\} +1$$

(-1)

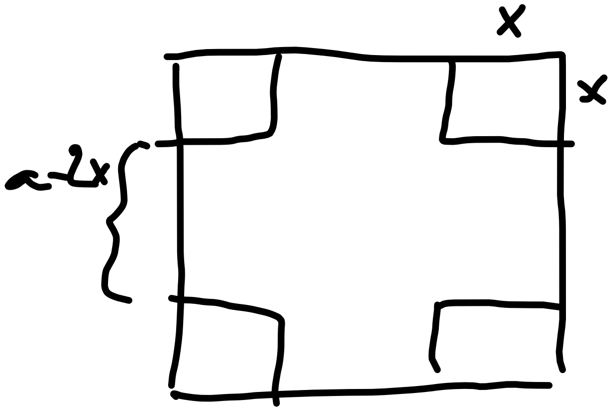
$$\lim_{x \to -1^+} \frac{x}{x^2-1} = \left| \frac{-1}{0^-} \right| = +\infty$$

$$\lim_{x \to -1^-} \frac{x}{x^2-1} = \left| \frac{-1}{0^+} \right| = -\infty$$

AS. SE SM. :

$$a = \lim_{x \to \pm\infty} \frac{\frac{x}{x^2-1}}{x} = \lim_{x \to \pm\infty} \frac{1}{x^2-1} = 0$$

$$b = \lim_{x \to \pm\infty} \left[\frac{x}{x^2-1} - 0 \cdot x \right] = \lim_{x \to \pm\infty} \frac{1}{2x} = 0$$



$$V = (a-2x)^2 \cdot x =$$

$$= a^2x - 4ax^2 + 4x^3$$

$$V'_{(x)} = a^2 - 8ax + 12x^2 = 0$$

$$D = 16a^2$$

$$x_{1,2} = \frac{8a \pm 4a}{24} = \begin{cases} \frac{a}{2} & \ldots \text{min.} \\ \frac{a}{6} & \ldots \text{max} \end{cases}$$

$$V\left(\frac{a}{6}\right) = \frac{2a^3}{27}$$