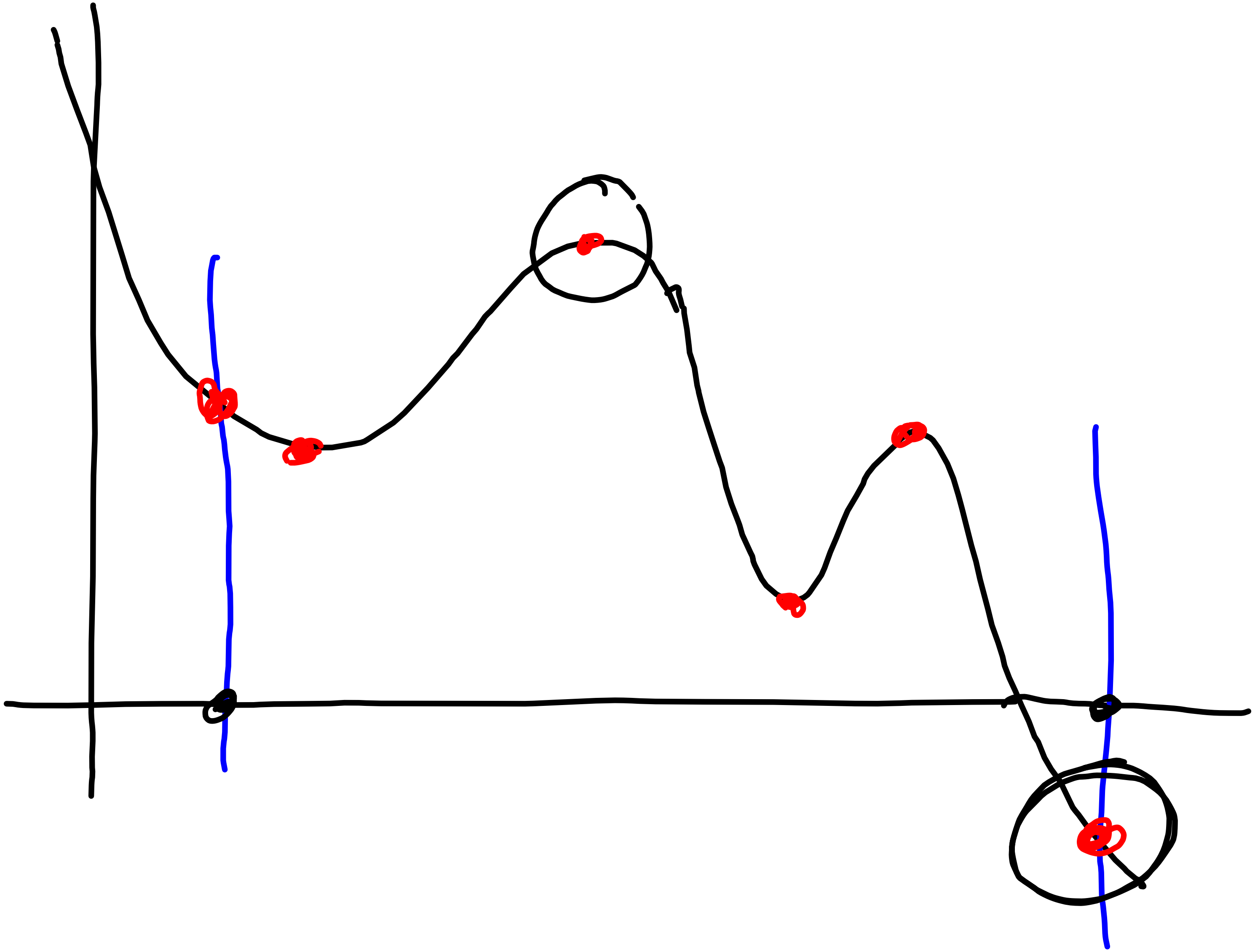


$$28 = a + b \quad , \; a \geq 0 , \; b \geq 0$$

$$a^2 + b^3 \to \text{min.}$$

$$\boxed{a = 28 - b}$$

$$f(b) = (28-b)^2 + b^3 \quad , \quad b \in [0, 28] = I$$

1. STAC. BODY.

$$f'(b) = 2 \cdot (28-b)^1 \cdot (0-1) + 3b^2 =$$

$$= 3b^2 + 2b - 56 = 0$$

$$3b^2+2b-56=0$$

$$D = 4 - 4\cdot(-56)\cdot 3 =$$

$$= 672$$

$$\sqrt{D} = 26$$

$$b_{1,2} = \frac{-2 \pm 26}{6} = \begin{cases} 4 \in I \\ -\cancel{\frac{14}{6}} \notin I \end{cases}$$

$$ax^2+bx+c=0$$

$$D = b^2 - 4ac$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

② HZ. B. : $b = 0$, $b = 28$

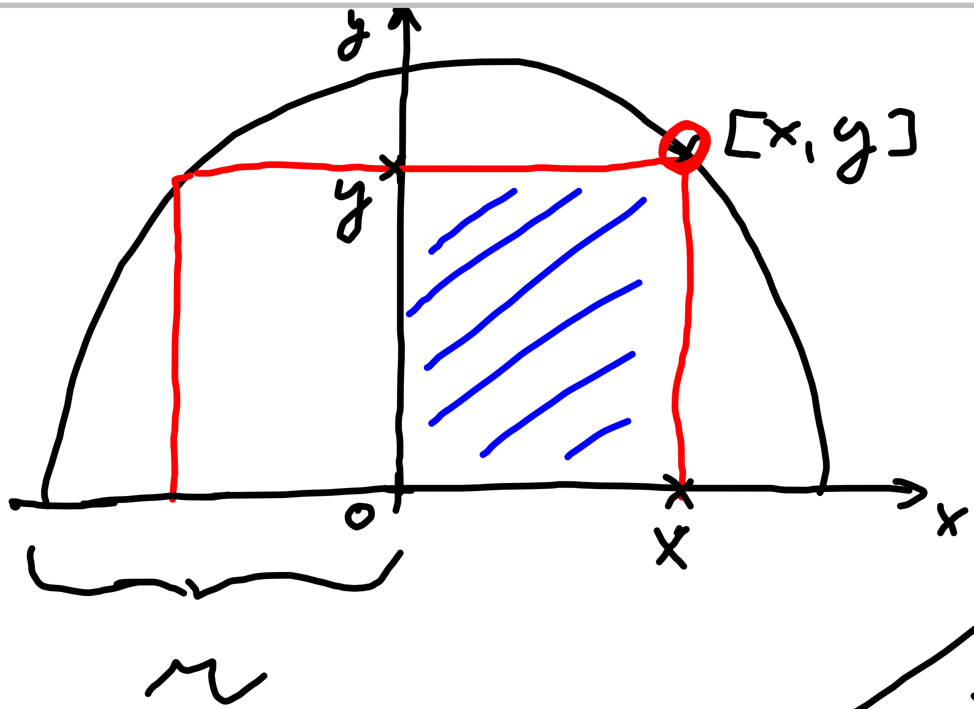
GL. EXT. :

$f(0) = 28^2 = 784$

$f(28) = 28^3 = $ moc

$f(4) = 24^2 + 4^3 = 640$ ... G. min.

$$28 = 24 + 4$$



$[x, y]$ … $x^2 + y^2 = r^2$

$$S = 2 \cdot x \cdot y$$

$$y = +\sqrt{r^2 - x^2}$$

$$\Downarrow$$

$$S(x) = 2 \cdot x \cdot \sqrt{r^2 - x^2}$$

$$I = [0, r]$$

$$S'(x) = 2\cdot\sqrt{r^2-x^2} + \frac{1}{2}\,2x\cdot(r^2-x^2)^{-\frac{1}{2}}\cdot(-2x) =$$

$$= 2\cdot\sqrt{r^2-x^2} - \frac{2\cdot x^2}{\sqrt{r^2-x^2}} =$$

$$= \frac{2\cdot(r^2-x^2) - 2\cancel{x}^2}{\sqrt{r^2-x^2}} = \frac{2r^2 - 4x^2}{\sqrt{r^2-x^2}} = 0$$

$$\Leftrightarrow \quad 2\cdot(r^2-2\cancel{x}^2) = 0 \quad \Leftrightarrow \quad r^2 = 2x^2$$

$$x^2 = \frac{r^2}{2}$$

$$x = \pm\frac{r}{\sqrt{2}}$$

$x = \frac{r}{\sqrt{2}}$, $\cancel{x = -\frac{r}{\sqrt{2}}}$

$y = \sqrt{r^2 - x^2} = \sqrt{r^2 - \frac{r^2}{\sqrt{2}}} =$

$= \frac{\sqrt{2}}{2} r$

$S(0) = 0$, $S(r) = 0$

$S\left(\frac{r}{\sqrt{2}}\right) = \ldots = \underline{\underline{r^2}}$

$$\sigma = 2\cdot(2x + y) \wedge y = \sqrt{r^2 - x^2}$$

$$\Rightarrow \sigma(x) = 2\cdot\left(2x + \sqrt{r^2 - x^2}\right)$$

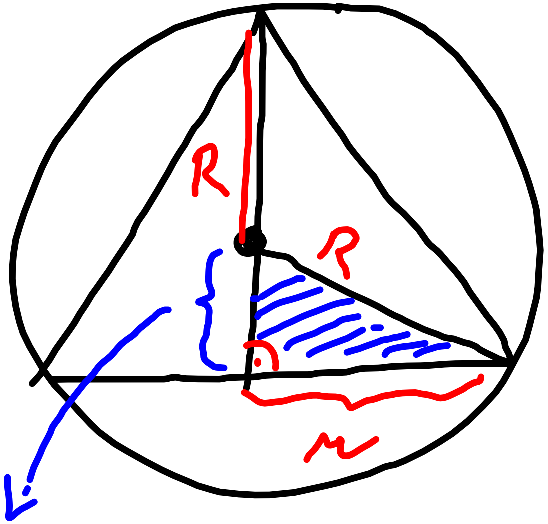
$$\sigma'(x) = \frac{4\cdot\sqrt{r^2-x^2} - 2x}{\sqrt{r^2-x^2}} = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\cdot\sqrt{r^2-x^2} = x$$

$$x^2 = 4\cdot(r^2 - x^2)$$

$$x^2 = \frac{4}{5} r^2$$

$$\boxed{\begin{array}{l} x = \frac{2}{\sqrt{5}} r \\ y = \frac{r}{\sqrt{5}} \\ \hline \sigma_{max} = 2\sqrt{5}\, r \end{array}}$$



$$x = v - R$$

$$V = \frac{1}{3}\underbrace{\pi r^2}\,\underbrace{v} \longrightarrow \max$$

$$x = \sqrt{R^2 + r^2}$$

$$v = x + R =$$

$$= \pm\sqrt{R^2 - r^2} + R$$

$$V(r) = \frac{1}{3}\pi r^2 \cdot \left(R + \sqrt{R^2 - r^2}\right), \quad r \in [0, R]$$

$$V(r) = \frac{1}{3}\pi r^2 \cdot R + \frac{1}{3}\pi r^2 \cdot \sqrt{R^2 - r^2}$$

$$V'(r) = \frac{2}{3}\pi r R + \frac{2}{3}\pi r \cdot \sqrt{R^2 - r^2}$$

$$+ \frac{1}{3}\pi r^2 \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{R^2 - r^2}} \cdot (-\cancel{2} r) = 0 \quad / \cdot \frac{1}{r}$$

$r \neq 0$

$$\frac{2}{3}\cancel{\pi} R + \frac{2}{3}\cancel{\pi} \cdot \sqrt{R^2 - r^2} - \frac{\cancel{\pi} r^2}{3\sqrt{R^2 - r^2}} = 0 \quad / \cdot \text{un}$$

$$2R \cdot \sqrt{R^2 - r^2} + 2R^2 - 3r^2 = 0$$

$$2R\sqrt{R^2 - r^2} = 3r^2 - 2R^2 \quad /^2$$

$$4R^2\cdot(R^2 - r^2) = 9r^4 - 12r^2R^2 + 4R^4$$

$$-4R^2r^2 = 9r^4 - 12r^2R^2$$

$$9r^4 - 8r^2R^2 = 0$$

$$r^2\cdot(9r^2 - 8R^2) = 0 \Leftrightarrow$$

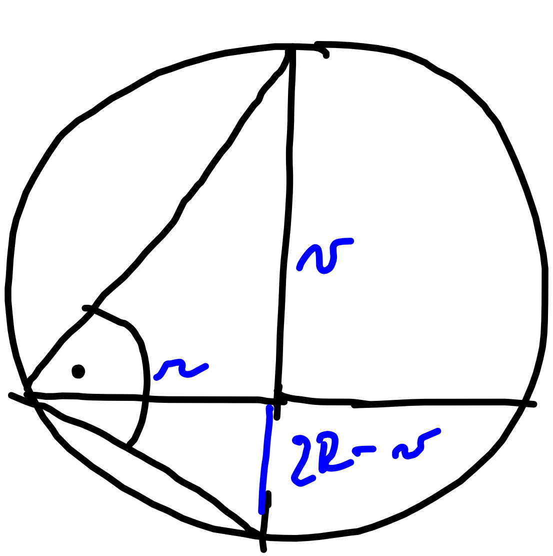
$$\Leftrightarrow \cancel{r = 0} \vee r = \pm\frac{2\sqrt{2}}{3}R$$

$$\Rightarrow v = \frac{4}{3}R$$

$$V(0) = 0$$

$$V(R) = \frac{1}{3}\pi R^3$$

$$V\left(\frac{2\sqrt{2}}{3}\right) = \frac{32}{81}\pi R^3 \ \dots \ \text{max}$$



$$r^2 = v \cdot (2R - v)$$

$$V = \frac{1}{3} \pi r^2 \cdot v$$

$$V(v) = \frac{1}{3} \pi \, v \cdot v \cdot (2R - v)$$

$$v \in [0, 2R]$$

$$T_4(x) = f(x_0) + f'(x_0)\cdot(x-x_0) + \frac{f''(x_0)}{2!}\cdot(x-x_0)^2 + \frac{f'''(x_0)}{3!}\cdot(x-x_0)^3 + \frac{f^{(4)}(x_0)}{4!}\cdot(x-x_0)^4$$

$$f(x) = \frac{1}{x}, \quad x_0 = 1 \quad \Rightarrow \quad f(1) = \underline{\underline{1}}$$

$$f'(x) = -\frac{1}{x^2} \quad \Rightarrow \quad f'(1) = \underline{\underline{-1}}$$

$$f''(x) = \left(-x^{-2}\right)' = -\left(x^{-2}\right)' = -\left(-2\cdot x^{-3}\right) = \frac{2}{x^3}$$

$$\Rightarrow f''(1) = \underline{\underline{2}}$$

$$f'''_{(x)} = \left(\frac{2}{x^3}\right)' = 2 \cdot \frac{-3}{x^4} = -\frac{6}{x^4} \Rightarrow f'''_{(1)} = -6$$

$$f^{(4)}_{(x)} = \left(-\frac{6}{x^4}\right)' = -6 \cdot \frac{-4}{x^5} = \frac{24}{x^5} \Rightarrow f^{(4)}_{(1)} = 24$$

$$T_4(x) = 1 - 1 \cdot (x-1) + \frac{\cancel{2}}{\cancel{2}!} \cdot (x-1)^2 + \frac{-\cancel{6}}{\cancel{3}!} \cdot (x-1)^3 +$$

$$+ \frac{24}{4!} \cdot (x-1)^4 = \ldots = x^4 - 5x^3 + 10x^2 - 10x + 5$$

$$R_4(x) = \frac{f^{(5)}(c)}{5!} \cdot (x - x_0)^5 =$$

$$= \frac{\frac{-120}{c^6}}{5!} \cdot (x-1)^5 = -\frac{1}{c^6} \cdot (x-1)^5$$

$$f^{(5)}(x) = \left(\frac{24}{x^5}\right)' = 24 \cdot \frac{-5}{x^6} = \frac{-120}{x^6}$$

$$\int x^2 \cdot (5-x)^3 \, dx = \int x^2 \cdot (125 - 3\cdot 5^2 \cdot x + 3\cdot 5 \cdot x^2 - x^3) \, dx$$

$$\int (125x^2 - 75x^3 + 15x^4 - x^5) \, dx =$$

$$= 125 \cdot \int x^2 \, dx - 75 \cdot \int x^3 \, dx + 15 \int x^4 \, dx - \int x^5 \, dx =$$

$$= 125 \cdot \frac{x^3}{3} - 75 \cdot \frac{x^4}{4} + 15 \cdot \frac{x^5}{5} - \frac{x^6}{6} \boxed{+ C}$$

$$\int \frac{x+1}{\sqrt{x}}\,dx = \int\left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right)dx =$$

$$= \int \sqrt{x} + \frac{1}{\sqrt{x}}\,dx = \int x^{\frac{1}{2}}\,dx + \int x^{-\frac{1}{2}}\,dx =$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C =$$

$$= \frac{2}{3}\cdot x\cdot\sqrt{x} + 2\cdot\sqrt{x} + C$$

$$\int (2^x + 3^x)^2 \, dx = \int (2^{2x} + 2 \cdot 2^x \cdot 3^x + 3^{2x}) \, dx =$$

$$= \int (2^2)^x \, dx + 2 \cdot \int (2 \cdot 3)^x \, dx + \int (3^2)^x \, dx =$$

$$= \frac{4^x}{\ln 4} + 2 \cdot \frac{6^x}{\ln 6} + \frac{9^x}{\ln 9} + C$$

$$\int (1 + \sin x + \cos x)\, dx =$$

$$= \int 1\, dx + \int \sin x\, dx + \int \cos x\, dx =$$

$$= \underline{x - \cos x + \sin x + C}$$

$$(u \cdot v)' = u' \cdot v + u \cdot v' \quad / \int$$

$$\underbrace{\int (u \cdot v)'}_{uv \, + c_0} = \int u'v + \int uv'$$

$$\Rightarrow \boxed{\int u'v = uv \underbrace{+c_0 - \underbrace{\int uv'}_{c_1}}_{c}}$$

$$\int x \cdot \sin x \, dx = \left| \begin{array}{l|l} u' = \sin x & u = -\cos x \\ v = x & v' = 1 \end{array} \right| =$$

$$= -x \cdot \cos x - \int -1 \cdot \cos x \, dx =$$

$$= -x \cdot \cos x + \int \cos x \, dx =$$

$$= -x \cdot \cos x + \sin x + C$$

$$\int x^2 \cdot e^x \, dx = \left| \begin{array}{ll} u' = e^x & u = e^x \\ v = x^2 & v' = 2x \end{array} \right| =$$

$$= x^2 \cdot e^x - \int 2x e^x \, dx =$$

$$= \underline{x^2 \cdot e^x - 2 \cdot} \int x \cdot e^x \, dx = \left| \begin{array}{ll} u' = e^x & u = e^x \\ v = x & v' = 1 \end{array} \right| =$$

$$= x^2 \cdot e^x - 2 \cdot \left[x \cdot e^x - \int e^x \, dx \right] =$$

$$= x^2 e^x - 2 \cdot x e^x + 2 \cdot e^x + C =$$

$$= e^x \cdot (x^2 - 2x + 2) + C$$