$$\int \operatorname{arctg} x \, dx = \int 1 \cdot \operatorname{arctg} x \, dx =$$

$$= \left| \begin{array}{ll} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right| =$$

$$= x \cdot \operatorname{arctg} x - \int \frac{x}{1+x^2} \, dx =$$

$$= x \cdot \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx = x \cdot \operatorname{arctg} x -$$

$$- \frac{1}{2} \ln \underline{\underline{|1+x^2|}} + c = \underline{\underline{x \cdot \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2)}} + c$$

$$\int \frac{\ln x}{x}\,dx = \left| \begin{array}{ll} u' = \frac{1}{x} & u = \ln x \\ v = \ln x & v' = \frac{1}{x} \end{array} \right| =$$

$$= \ln^2 x - \underbrace{\int \frac{\ln x}{x}\,dx}_{=:\, I}$$

$$I = \ln^2 x - I \quad \Rightarrow \quad 2I = \ln^2 x$$

$$\int \frac{\ln x}{x}\,dx = I = \underline{\underline{\frac{\ln^2 x}{2} + c}}$$

$$\int f(x)\,dx = \left| \begin{array}{l} x = \varphi(t) \\ dx = \varphi'(t)\,dt \end{array} \right| =$$

$$= \underline{\underline{\int f(\varphi(t)) \cdot \varphi'(t)\,dt}}$$

$$\int (2x+5)^{10}\,dx = \left| \begin{array}{l} t = 2x+5 \\ dt = 2\,dx \\ dx = \frac{1}{2}\,dt \end{array} \right| =$$

$$= \int t^{10}\,\frac{1}{2}\,dt = \frac{1}{2}\int t^{10}\,dt =$$

$$= \frac{1}{2}\cdot\frac{t^{11}}{11} + c = \underline{\underline{\frac{1}{22}\cdot(2x+5)^{11} + c}}$$

$$\int \frac{\ln x}{x}\,dx = \int \underbrace{\ln x}_{\varphi} \cdot \underbrace{\frac{1}{x}}_{\varphi'}\,dx =$$

$$= \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \end{array} \right| = \int t\,dt = \frac{t^2}{2} + c =$$

$$= \frac{\ln^2 x}{2} + c$$

$$\int \sin\sqrt{x}\,dx = \left| \begin{array}{l} t = \sqrt{x} \\ t^2 = x \\ 2t\,dt = dx \end{array} \right| =$$

$$= \int \sin t \cdot 2t\,dt = 2 \cdot \int t \cdot \sin t\,dt =$$

$$= \left| \begin{array}{ll} u' = \sin t & u = -\cos t \\ v = t & v' = 1 \end{array} \right| = 2 \cdot \left[-t \cdot \cos t + \int +1 \cdot \cos t\,dt \right] =$$

$$= 2 \cdot (\sin t - t \cdot \cos t) + C =$$

$$= 2 \cdot (\sin \sqrt{x} - \sqrt{x} \cdot \cos \sqrt{x}) + C$$

$$\int \frac{1}{x \cdot \ln^2 x}\,dx = \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \end{array} \right| =$$

$$= \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = \frac{t^{-1}}{-1} + c =$$

$$= -\frac{1}{t} + c = -\frac{1}{\ln x} + c$$

$$\int \frac{\operatorname{arctg}\sqrt{x}}{\sqrt{x}\cdot(1+x)}\,dx =$$

$$= \left| \begin{array}{l} t = \operatorname{arctg}\sqrt{x} \\ dt = \dfrac{1}{1+x}\cdot\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{x}}\,dx = \dfrac{1}{2}\cdot\dfrac{1}{\sqrt{x}\cdot(1+x)}\,dx \\ \dfrac{1}{\sqrt{x}\,(1+x)}\,dx = 2\,dt \end{array} \right| =$$

$$= \int 2t\, dt = \cancel{2} \cdot \frac{t^2}{\cancel{2}} + c =$$

$$= \underline{\underline{\left(\operatorname{arctg} \sqrt{x}\right)^2 + c}}$$

$$\int x^n \cdot \ln x \, dx = \qquad , n \neq -1$$

$$= \left| \begin{matrix} u' = x^n & u = \dfrac{x^{n+1}}{n+1} \\ v = \ln x & v' = \dfrac{1}{x} \end{matrix} \right| =$$

$$= \frac{x^{n+1}}{n+1} \cdot \ln x - \int \frac{x^n}{n+1} \, dx =$$

$$= \frac{x^{n+1}}{n+1} \cdot \ln x - \frac{x^{n+1}}{(n+1)^2} + C$$

$$\int x \cdot e^{-x}\,dx = \begin{vmatrix} u' = e^{-x} & u = -e^{-x} \\ v = x & v' = 1 \end{vmatrix} =$$

$$= -x \cdot e^{-x} - \int 1 \cdot (-e^{-x})\,dx =$$

$$= -x \cdot e^{-x} + \int e^{-x}\,dx = -x \cdot e^{-x} - e^{-x} + C =$$

$$= -e^{-x} \cdot (x+1) + C$$

$$\int \frac{x}{1+x^4}\,dx = \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \\ \frac{1}{2}\,dt = x\,dx \end{array} \right| =$$

$$= \int \frac{1}{1+t^2} \cdot \frac{1}{2}\,dt = \frac{1}{2} \cdot \operatorname{arctg} t + C =$$

$$= \frac{\operatorname{arctg} x^2}{2} + C$$

$$\int \cos^5 x \cdot \sqrt{\sin x}\, dx = \left| \begin{array}{l} t = \sin x \\ dt = \cos x\, dx \end{array} \right| =$$

$$= \int \cos^4 x \cdot \sqrt{t}\, dt = \int (1 - \sin^2 x)^2 \cdot \sqrt{t}\, dt =$$

$$= \int (1 - t^2)^2 \cdot \sqrt{t}\, dt =$$

$$= \int \left(t^{\frac{1}{2}} - 2t^{\frac{5}{2}} + t^{\frac{9}{2}} \right) dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2 \cdot \frac{t^{\frac{7}{2}}}{\frac{7}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}}$$

$$= \sin^{\frac{3}{2}} x \cdot \left(\frac{2}{3} - \frac{4}{7} \cdot \sin^2 x + \frac{2}{11} \cdot \sin^4 x \right) + C$$

I.) $\dfrac{A}{x-a}$

$$\int \frac{3}{x-2}\,dx = \left| \begin{matrix} t = x-2 \\ dt = dx \end{matrix} \right| = \int \frac{3}{t}\,dt =$$

$$= 3 \cdot \int \frac{1}{t}\,dt = 3 \cdot \ln|t| + c =$$

$$= 3 \cdot \ln|x-2| + c$$

II.) $\dfrac{A}{(x-a)^n}$, $n \geq 2$

$$\int \frac{3}{(x-2)^3}\,dx = \left| \begin{array}{l} t = x-2 \\ dt = dt \end{array} \right| = \int \frac{3}{t^3}\,dt =$$

$$= 3 \cdot \int t^{-3}\,dt = 3 \cdot \frac{t^{-2}}{-2} + C =$$

$$\underline{\underline{= -\frac{3}{2 \cdot (x-2)^2} + C}}$$

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$$\int \frac{1}{x^2-1}\,dx = \left[-\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + c\right]$$

III. $\dfrac{Ax+B}{x^2+px+q}$ $\left(p^2 < 4q \;\ldots\; \text{DISKR.} < 0\right)$

$$\int \frac{3x+5}{x^2+4x+8}\,dx = \left| \to \text{"} \frac{f'}{f} \text{"} \right| =$$

$$= \frac{3}{2} \underbrace{\int \frac{2x+4}{x^2+4x+8}\,dx}_{=:I_1} - \underbrace{\int \frac{1}{x^2+4x+8}\,dx}_{=:I_2} =$$

$$\left(\frac{3}{2}\cdot 4 + ?\right) = 5 \Rightarrow 6 + ? = 5 \Rightarrow ? = -1$$

$$I_1 = \int \frac{2x+4}{x^2+4x+8}\,dx = \ln \underbrace{|x^2+4x+8|}_{\text{vždy} > 0} + c_1 =$$

$$= \ln(x^2+4x+8) + c_1$$

$$I_2 = \int \frac{1}{\underbrace{x^2+4x+8}_{\to \text{ČTVEREC}}}\,dx = \int \frac{1}{(x+2)^2-4+8}\,dx =$$

$$x^2+4x+4$$

$$= \int \frac{1}{(x+2)^2+4}\,dx = \left| \to \text{"}\frac{1}{t^2+1}\text{"} \right| =$$

$$= \int \frac{1}{4\cdot\left[\frac{(x+2)^2}{4}+1\right]}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}\,dx =$$

$$= \left| \begin{array}{l} t = \frac{x+2}{2} \\ 2\,dt = \end{array} \right.$$

V této chvíli přestala spolupracovat technika, pročež jak zbytek tohoto příkladu, tak i celého cvičení byl psán křídou - viz videozáznam.