$$\int_0^1 1 \cdot \operatorname{arctg} x \, dx = \left| \begin{matrix} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \dfrac{1}{1+x^2} \end{matrix} \right| =$$

$$= \Big[x \cdot \operatorname{arctg} x \Big]_0^1 - \frac{1}{2} \int_0^1 \frac{2x}{1+x^2} \, dx =$$

$$= \left(1 \cdot \frac{\pi}{4} - 0\right) - \frac{1}{2} \Big[\ln \underbrace{|1+x^2|}_{(1+x^2)} \Big]_0^1 =$$

$$= \frac{\pi}{4} - \frac{1}{2} \left(\ln 2 - \underbrace{\ln 1}_{=0} \right) = \frac{\pi}{4} - \frac{1}{2} \ln 2$$

$$\int_1^2 x\cdot\sqrt{1+x^2}\,dx = \left|\begin{array}{l} t = x^2+1 \\ dt = 2x\,dx \\ \frac{1}{2}\,dt = x\,dx \end{array}\right|\begin{array}{l} x=1 \Rightarrow t=2 \\ x=2 \Rightarrow t=5 \end{array}\Bigg| =$$

$$= \int_2^5 \sqrt{t}\ \frac{1}{2}\,dt = \frac{1}{2}\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^5 =$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot\left(5^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) = \frac{1}{3}\cdot\left(\sqrt{5^2\cdot 5} - \sqrt{2^2\cdot 2}\right) =$$

$$= \underline{\underline{\frac{1}{3}\cdot\left(5\sqrt{5} - 2\sqrt{2}\right)}}$$

$$\int \sqrt{t}\,dt = \int t^{\frac{1}{2}}\,dt =$$

$$= \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

VZOREC: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$t^2 = 1 + x^2$ ... TUTO SUBST. ZKUSIT DOMA

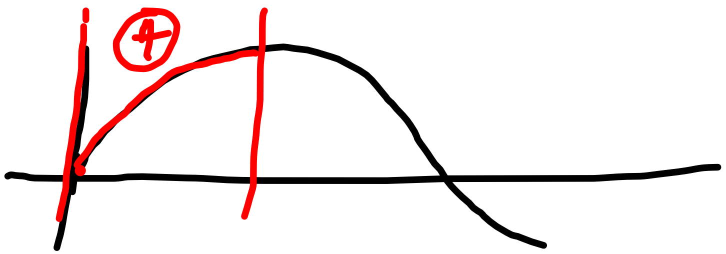
$$\int_0^1 \sqrt{2-x^2}\,dx = \left| \begin{array}{l} x=\sqrt{2}\cdot\cos t \\ dx = \sqrt{2}\cdot(-\sin t)\,dt \\ \hline x=0 \Rightarrow t=\frac{\pi}{2} \\ 0=\sqrt{2}\cdot\cos t \\ \cos t = \frac{\pi}{2} \\ \hline x=1 \Rightarrow t=\frac{\pi}{4} \\ 1=\sqrt{2}\cos t \\ \cos t = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \end{array} \right| =$$

$$= \boxed{\int_{\frac{\pi}{2}}^{\frac{\pi}{4}}} \sqrt{2-2\cdot\cos^2 t}\cdot\sqrt{2}\cdot(-1)\cdot\sin t\,dt =$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{2\cdot(1-\cos^2 t)} \cdot \sin t \cdot \sqrt{2}\, dt =$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{2}\cdot\sqrt{1-\cos^2 t} \cdot \sin t \cdot \sqrt{2}\, dt =$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\cdot \sin t \cdot \sin t \, dt = 2\cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 t \, dt =$$

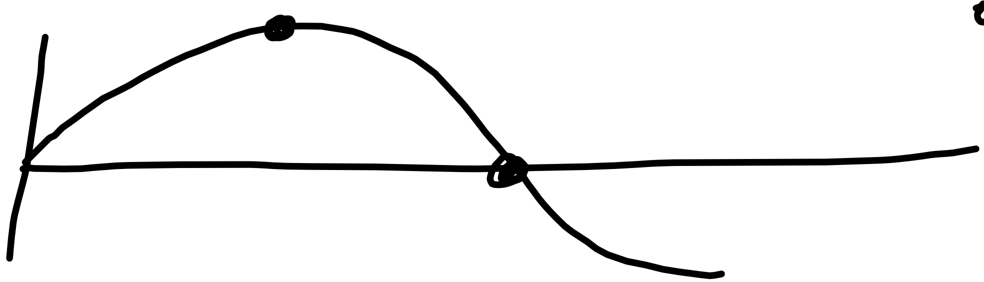


$$= \cancel{2} \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2t}{\cancel{2}} \, dt =$$

$$= \left[t - \frac{\sin 2t}{2} \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \left(\frac{\pi}{2} - \frac{\sin \pi}{2} \right) -$$

$$- \left(\frac{\pi}{4} - \frac{\sin \frac{\pi}{2}}{2} \right) = \frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{1}{2} \right) =$$

$$= \frac{\pi}{4} + \frac{1}{2} =$$



$$= \underline{\underline{\frac{\pi + 2}{4}}}$$

$$\int \sin^3 x \cdot \cos^4 x \, dx =$$

$$= \int \sin^2 x \cdot \cos^4 x \cdot \sin x \, dx =$$

$$= \int (1-\cos^2 x) \cdot \cos^4 x \cdot \underbrace{\sin x \, dx} =$$

$$= \left| \begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \\ -dt = \sin x \, dx \end{array} \right| = \int (1-t^2) \cdot t^4 \cdot (-1) \, dt =$$

$$= \int \underbrace{t^6} - t^4 \, dt = \underbrace{\frac{t^7}{7}} - \frac{t^5}{5} + C =$$

$$= \frac{\cos^7 x}{7} + \frac{\cos^5 x}{5} + C$$

$$\int (1-\cos^2 x)^2 \cdot \sin x \, dx = \left| \begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \end{array} \right|$$

$$= \int (1-t^2)^2 \cdot (-1) \, dt =$$

$$= -\int (1-t^2)^2 \, dt = -\int (1-2t^2+t^4) \, dt =$$

$$= -\left(t - 2\cdot\frac{t^3}{3} + \frac{t^5}{5} \right) + c =$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$$

$$\int e^{-x^3} \cdot \underbrace{x^2\, dx}_{} = \left| \begin{array}{l} t = -x^3 \\ dt = -3x^2\, dx \\ x^2\, dx = -\frac{1}{3}\, dt \end{array} \right| =$$

$$= -\frac{1}{3} \cdot \int e^t\, dt = -\frac{1}{3} e^t + C =$$

$$= -\frac{1}{3} e^{-x^3} + C$$

$$\int \cos^3 x \cdot \underbrace{\sin x \quad dx} = \left| \begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \end{array} \right| =$$

$$= -\int t^3 \, dt = -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$$

$$(*) \int \frac{1+\cos^2 x}{1+\cos^2 x - \sin^2 x} \, dx = \int \frac{1+\cos^2 x}{1+\cos^2 x - (1-\cos^2 x)} \, dx =$$

$$= \int \frac{1+\cos^2 x}{2\cos^2 x} \, dx = \frac{1}{2} \cdot \int \frac{1}{\cos^2 x} \, dx + \frac{1}{2} \int \frac{\cancel{\cos^2 x}^{\,1}}{\cancel{\cos^2 x}_{\,1}} \, dx$$

$$= \frac{1}{2} \cdot \operatorname{tg} x + \frac{1}{2} \cdot x + C$$

$$(*) = \int \frac{1+\cos^2 x}{1+\cos 2x} \, dx$$

$$\int 2\cdot \sin^2\frac{x}{2}\,dx = \int\left(\sin^2\frac{x}{2} + \sin^2\frac{x}{2}\right)dx =$$

$$= \int\left(\sin^2\frac{x}{2} + 1 - \cos^2\frac{x}{2}\right)dx =$$

$$= \int\left[1 - \left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)\right]dx =$$

$$= \int(1 - \cos x)\,dx = \int 1\,dx - \int \cos x\,dx =$$

$$= \underline{x - \sin x + c}$$

$$\boxed{\cos 2\alpha = \cos^2\alpha - \sin^2\alpha}$$

$$\int_0^{\frac{\pi}{2}} \cos x \, dx = \left[\sin x\right]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 =$$

$$= 1 - 0 = \underline{\underline{1}}$$

$$\int_0^{\ln 2} \underbrace{\frac{x}{e^x}}_{= x \cdot e^{-x}} \, dx = \left| \begin{matrix} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{matrix} \right| =$$

$$= \left[-x \cdot e^{-x}\right]_0^{\ln 2} + \int_0^{\ln 2} 1 \cdot (\cancel{-}e^{-x}) \, dx =$$

$$= \underline{\underline{\left(-\ln 2 \cdot e^{-\ln 2}\right.}} - 0 \cdot 1\Big) - \left[-e^{-x}\right]_0^{\ln 2} =$$

$$= -\frac{\ln 2}{2} - \frac{1}{2} + 1 = \frac{1}{2}(1 - \ln 2)$$

$$\int_0^1 x\cdot(2-x^2)^5\,dx = \left|\begin{array}{l} t=2-x^2 \\ dt=-2x\,dx \\ x\,dx=-\frac{1}{2}\,dt \end{array}\right| \begin{array}{l} x=1 \Rightarrow \\ \Rightarrow t=1 \\ \hline x=0 \Rightarrow \\ \Rightarrow t=2 \end{array}$$

$$\left\{\begin{array}{l} = -\frac{1}{2}\cdot\int_2^1 t^5\,dt = -\frac{1}{2}\cdot\left[\frac{t^6}{6}\right]_2^1 = \\ = \frac{1}{2}\int_1^2 t^5\,dt = \frac{1}{2}\cdot\left[\frac{t^6}{6}\right]_1^2 = \end{array}\right\} = \frac{64}{12}-\frac{1}{12} = \underline{\underline{\frac{63}{12}}}$$

$$\int_1^{e^8} \frac{dx}{x\cdot\sqrt{\ln x+1}} = \left| \begin{array}{l} t=\ln x+1 \\ dt=\frac{1}{x}\,dx \end{array} \right| \begin{array}{l} x=e^8 \Rightarrow \\ \Rightarrow t=\ln e^8+1= \\ =8+1=9 \\ \hline x=1 \Rightarrow \\ \Rightarrow t=\ln 1+1= \\ =0+1=1 \end{array}$$

$$= \int_1^9 \frac{1}{\sqrt{t}}\,dt = \int_1^9 t^{-\frac{1}{2}}\,dt =$$

$$= \left[\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^9 = 2\cdot\left[\sqrt{t}\right]_1^9 = 2\cdot(3-1) = \underline{\underline{4}}$$