$$\int_2^\infty \frac{1}{x^2+x-2}\,dx = \int = ⊛$$

$$x^2+x-2=0 \Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{9}}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

$$\frac{1}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} \quad / \cdot (x^2+x-2)$$

$$1 = A\cdot(x+2) + B\cdot(x-1)$$

$$x=1 \ \ldots \ 1 = A\cdot 3 \Rightarrow A = \frac{1}{3}$$

$$x=-2 \ \ldots \ 1 = B\cdot(-3) \Rightarrow B = -\frac{1}{3}$$

$$\circledast = \int_2^{\infty} \frac{\frac{1}{3}}{x-1}\,dx + \int_2^{\infty} \frac{-\frac{1}{3}}{x+2}\,dx =$$

$$= \frac{1}{3}\int_2^{\infty} \frac{1}{x-1}\,dx - \frac{1}{3}\int_2^{\infty} \frac{1}{x+2}\,dx =$$

$$= \frac{1}{3}\cdot\left[\ln(x-1)\right]_2^{\infty} - \frac{1}{3}\left[\ln(x+2)\right]_2^{\infty} =$$

$$= \frac{1}{3}\cdot\Big(\lim_{x\to\infty} \ln(x-1) - \underbrace{\ln(2-1)}_{=0}\Big)$$

$$- \frac{1}{3}\cdot\Big(\lim_{x\to\infty} \ln(x+2) - \underbrace{\ln(2+2)}_{2\cdot\ln 2}\Big) =$$

$$= \frac{1}{3} \cdot \underbrace{\left(\lim_{x\to\infty} \ln(x-1) - \lim_{x\to\infty} \ln(x+2) \right)} + \frac{2}{3} \ln 2 =$$

$$\lim_{x\to\infty} \ln \frac{x-1}{x+2} = \lim_{x\to\infty} \ln \frac{1-\frac{1}{x}}{1+\frac{2}{x}} = \ln 1 = \underline{\underline{0}}$$

$$= \underline{\underline{\frac{2}{3} \ln 2}}$$

$[2,\infty)$, $f(x) = (x+1)\cdot \dfrac{x^2-x+1}{x^2+x-2}$

$$g(x) = \frac{x^3}{x^2+x-2}$$

$f(x) = \dfrac{x^3+1}{x^2+x-2}$ na $[2,\infty)$ $>$ $g(x)$

$$\Rightarrow S = \int_2^\infty f(x) - g(x)\, dx = \int_2^\infty \frac{1}{x^2+x-2}\, dx =$$

$$= \underline{\underline{\frac{2}{3}\ln 2}}$$

$$\int_1^\infty \frac{dx}{x^\alpha}$$

$\underline{\underline{\alpha = 1}}$ $\Rightarrow$ $\displaystyle\int_1^\infty \frac{dx}{x} = [\ln x]_1^\infty = \lim_{x\to\infty} \ln x - \ln 1 =$

$= \underline{\underline{\infty}}$ $\underline{\underline{\text{DIV.}}}$

$\underline{\underline{\alpha < 1}}$ $\Rightarrow$ $x > x^\alpha$ $\Rightarrow$ $\dfrac{1}{x^\alpha} > \dfrac{1}{x}$

$\Rightarrow$ $\displaystyle\int_1^\infty \frac{dx}{x^\alpha} = \underline{\underline{\infty}}$ $\underline{\underline{\text{DIV.}}}$

$\underline{\underline{\alpha > 1}} \Rightarrow \int_1^\infty \frac{dx}{x^\alpha} = \left[\frac{x^{1-\alpha}}{1-\alpha}\right]_1^\infty =$

$\int_1^\infty \frac{dx}{x^\alpha} = \int_1^\infty x^{-\alpha}\,dx$

$= \frac{1}{1-\alpha} \cdot \left(\lim_{x\to\infty} x^{1-\alpha} - x^0\right) =$

$= \frac{1}{1-\alpha} \cdot \Big(\underbrace{\lim_{x\to\infty} \frac{1}{x^{\alpha-1}}}_{x^{\oplus} \to \infty,\ \frac{1}{\infty} = 0} - 1\Big) = \frac{1}{1-\alpha} \cdot (0-1) =$

$= \underline{\underline{\frac{1}{\alpha-1}}}$

$\underline{\underline{\text{konv.}}}$

$$\int_1^\infty \frac{dx}{x^\alpha} = \begin{cases} \infty, & \alpha \leq 1 \\ < \infty, & \alpha > 1 \end{cases}$$

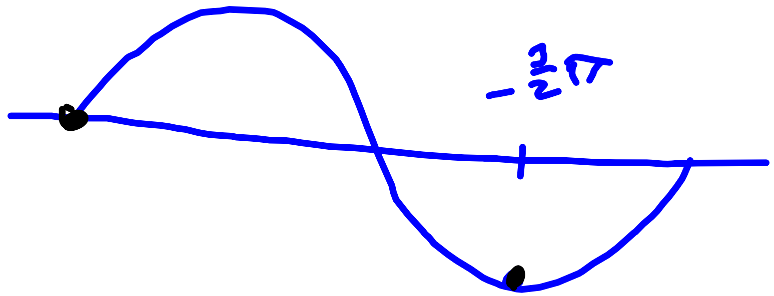
$$\int_0^4 \frac{2x^2+\sqrt{x}}{x}\,dx = \int_0^4 \frac{2x^2}{x}\,dx + \int_0^4 \frac{\sqrt{x}}{x}\,dx =$$

$$= 2\cdot\int_0^4 x\,dx + \int_0^4 \frac{1}{\sqrt{x}}\,dx =$$

$$= 2\cdot\int_0^4 x\,dx + \int_0^4 x^{-\frac{1}{2}}\,dx = \cancel{2}\cdot\left[\frac{x^2}{\cancel{2}}\right]_0^4 + \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^4 =$$

$$= (16-0) + 2\cdot(2-0) = \underline{\underline{20}}$$

$$\int_0^{\frac{3}{2}\pi} \frac{2\cdot\cos x}{1+\sin x}\,dx = \left|\begin{array}{l} t = \sin x \\ dt = \cos x\,dx \\ \hline x = \frac{3}{2}\pi \Rightarrow t = -1 \\ x = 0 \Rightarrow t = 0 \end{array}\right| =$$

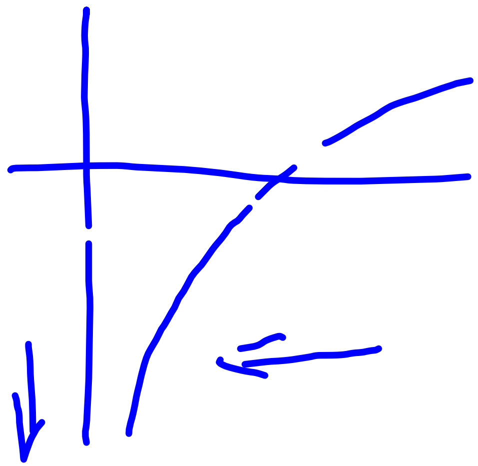


$$= \int_0^{-1} \frac{2}{1+t}\,dt = -2\int_{-1}^{0} \frac{1}{1+t}\,dt =$$

$$= -2 \cdot \Big[\ln(1+t) \Big]_{-1}^{0} =$$

$$= -2 \cdot \left(\ln 1 - \lim_{t \to -1^+} \ln(1+t) \right) =$$

$$= 2 \cdot \lim_{t \to -1^+} \ln(1+t) = 2 \cdot (-\infty) = \underline{\underline{-\infty}}$$



$$\int_0^1 \frac{1}{x^\alpha}\,dx$$

$\alpha \le 0 \Rightarrow \beta := -\alpha \Rightarrow \beta \ge 0$

$$\int_0^1 \frac{1}{x^\alpha}\,dx = \int_0^1 x^{-\alpha}\,dx = \int_0^1 x^\beta\,dx =$$

$$= \left[\frac{x^{\beta+1}}{\beta+1}\right]_0^1 = \frac{1}{\beta+1} = \frac{1}{1-\alpha}$$ KONV.

$\underline{\underline{\alpha = 1}} \Rightarrow \int_0^1 \frac{dx}{x} = [\ln x]_0^1 =$

$= \ln 1 - \lim_{x \to 0^+} \ln x = 0 - (-\infty) = \underline{\underline{\infty}}$

$\underline{\underline{\text{Div.}}}$

$\alpha > 0 \wedge \alpha \neq 1 \Rightarrow$

$$\int_0^1 \frac{dx}{x^\alpha} = \int_0^1 x^{-\alpha}\,dx = \left[\frac{x^{1-\alpha}}{1-\alpha}\right]_0^1 =$$

$$= \frac{1}{1-\alpha}\left(1 - \lim_{x \to 0^+} x^{1-\alpha}\right) =$$

$$\lim_{x \to 0^+} x^{1-\alpha}$$

$\alpha < 1$

$$\lim_{x \to 0} x^{\text{kl.}} = 0$$

$\alpha > 1$

$$\lim_{x \to 0} x^{\text{záp.}} = \lim_{x \to 0} \frac{1}{x^{\text{kl.}}} = \infty$$

$$= \begin{cases} \dfrac{1}{1-\alpha}, & \alpha < 1 \\[2ex] \dfrac{1}{1-\alpha} - \overbrace{\dfrac{1}{1-\alpha}}^{<0} \cdot \infty = +\infty, & \alpha > 1 \end{cases}$$

$$\int_0^1 \frac{dx}{x^\alpha} = \begin{cases} \text{konv.}, & \alpha < 1 \\ \infty, & \alpha \geq 1 \end{cases}$$

$$\sum_{n=0}^{\infty} \frac{n+1}{3^n} = ?$$

$$s_n = a_0 + a_1 + \dots + a_n$$

$$\ominus \begin{cases} s_n = \dfrac{1}{3^0} + \dfrac{2}{3^1} + \dfrac{3}{3^2} + \dots + \dfrac{n}{3^{n-1}} + \dfrac{n+1}{3^n} \quad \Big/ \cdot \dfrac{1}{3} \\[2ex] \dfrac{1}{3} s_n = \dfrac{1}{3} + \dfrac{2}{3^2} + \dfrac{3}{3^3} + \dots + \dfrac{n}{3^n} + \dfrac{n+1}{3^{n+1}} \end{cases}$$

$$\frac{2}{3} s_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \dots + \frac{1}{3^n} - \frac{n+1}{3^{n+1}} \quad \Big/ \cdot \frac{3}{2}$$

$$s_n = \frac{3}{2}\cdot\Big(\underbrace{1+\frac{1}{3}+\frac{1}{3^2}+\dots+\frac{1}{3^n}} - \frac{n+1}{3^{n+1}}\Big)$$

$1+\frac{1}{3}+\frac{1}{3^2}+\dots \leftarrow$ GEOM. ŘADA s $q=\frac{1}{3}$ ($<1 \Rightarrow$ konv.)

$$\Rightarrow \quad = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

$$\sum_{n=0}^{\infty}\frac{n+1}{3^n} = \lim_{n\to\infty} s_n \qquad \Bigg|\quad \lim_{n\to\infty}\frac{n+1}{3^{n+1}} = 0$$

$$\sum_{n=0}^{\infty}\frac{n+1}{3^n} = \frac{3}{2}\cdot\left(\frac{3}{2}-0\right) = \underline{\underline{\frac{9}{4}}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3+3n^2+2n} = ?$$

$$\frac{1}{n^3+3n^2+2n} = \frac{\frac{1}{2}}{n} - \frac{1}{n+1} + \frac{\frac{1}{2}}{n+2}$$

$$s_n = a_1 + a_2 + a_3 + \dots + a_n =$$

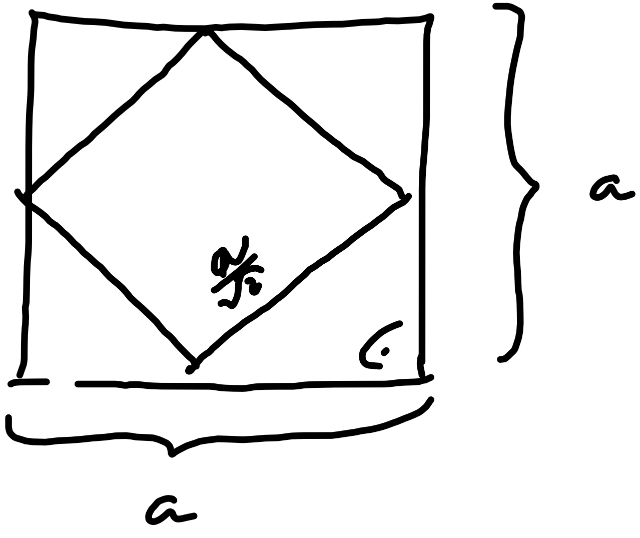
$$= \underbrace{\frac{1}{2} - \frac{1}{2} + \frac{1}{6}}_{a_1} + \underbrace{\frac{1}{4} - \frac{1}{3} + \frac{1}{8}}_{a_2} + \underbrace{\frac{1}{6} - \frac{1}{4} + \frac{1}{10}}_{a_3} + \underbrace{\frac{1}{8} - \frac{1}{5} + \frac{1}{12}}_{a_4} + \dots$$

$$\dots + \underbrace{\frac{1}{2(n-1)} - \frac{1}{n} + \frac{1}{2\cdot(n+1)}}_{a_{n-1}} + \underbrace{\frac{1}{2n} - \frac{1}{n+1} + \frac{1}{2\cdot(n+2)}}_{a_n} =$$

$$= \frac{1}{4} + \frac{1}{2(n+1)} - \frac{1}{n+1} + \frac{1}{2(n+2)}$$

$$\lim_{n \to \infty} \text{— " —} = \frac{1}{4} + 0 - 0 + 0 = \underline{\underline{\frac{1}{4}}}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3 + 3n^2 + 2n} = \underline{\underline{\frac{1}{4}}}$$



$$\sum S_n =$$

$$= a^2 + \left(\frac{a}{\sqrt{2}}\right)^2 + \dots +$$

$$= a^2 + \frac{1}{2}a^2 + \frac{1}{2}\cdot\frac{1}{2}a^2 + \dots =$$

$$= a^2 + \frac{1}{2}a^2 + \frac{1}{4}a^2 + \dots = a^2\cdot\left(\underbrace{\frac{1}{2^0} + \frac{1}{2} + \frac{1}{2^2} + \dots}_{\text{GEOM. Ř.}}\right) =$$

$$= \underline{\underline{2a^2}}$$

$$\boxed{\begin{array}{l} a = \frac{1}{4} \\ \text{POUŽIJEME } \frac{2}{16} = \underline{\underline{\frac{1}{8}\,m^2}} \end{array}}$$

$q = \frac{1}{2}$

$$\frac{1}{1-\frac{1}{2}} = 2$$

$$\sum o_n = 4a + 4\cdot\left(\frac{\sqrt{2}}{2}a\right) + 4\cdot\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}a\right) + \dots =$$

$$= 4a\cdot\underbrace{\left(1 + \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^3 + \dots\right)}_{} =$$

$$\left[\begin{array}{l} \text{G. ř.}, \quad q = \frac{\sqrt{2}}{2} < 1 \\ \frac{1}{1-\frac{\sqrt{2}}{2}} = \frac{1}{\frac{2-\sqrt{2}}{2}} = \frac{2}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}} = \\ = \frac{\cancel{2}\cdot(2+\sqrt{2})}{\cancel{4-2}} = 2+\sqrt{2} \end{array}\right]$$

$$= 4a\,(2+\sqrt{2}) = \left|a = \frac{1}{4}\right| = 2+\sqrt{2} \doteq \underline{\underline{3{,}41\,m}}$$

$$\sum_{0}^{\infty} \frac{1}{(n+1)\cdot 3^n}$$

$$\frac{1}{(n+1)\cdot 3^n} \le \frac{1}{3^n} \quad \ldots \quad \text{GEOM. ŘI}, \; q = \frac{1}{3} < 1$$

KONV.

$$\left(\frac{1}{1-\frac{1}{3}} = \frac{3}{2} = \sum_{0}^{\infty} \left(\frac{1}{3}\right)^n \right)$$

$$\Downarrow$$

$$\sum \frac{1}{(n+1)\, 3^n} \quad \text{KONV.}$$

$$\sum_{1}^{\infty} \frac{n^2+1}{n^3} = \sum_{1}^{\infty} \left(\frac{1}{n} + \frac{1}{n^3} \right) \geq \underbrace{\sum_{1}^{\infty} \frac{1}{n}}_{\text{DIV.}}$$

$$\Rightarrow \quad \sum_{1}^{\infty} \frac{n^2+1}{n^3} \quad \underline{\underline{\text{DIV.}}}$$

NUTNÁ PODM. KONVERGENCE : $\lim\limits_{n\to\infty} a_n = 0$

$\lim\limits_{n\to\infty} a_n \neq 0 \Rightarrow$ <u>NEKONV.</u>

NENÍ TO DOST. PODM. : $\lim\limits_{n\to\infty} a_n = 0 \not\Rightarrow$ KONV.

$\Rightarrow$ MŮŽE KONV.

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)\cdot \ln^2(n+1)}$$

$$f(x) = \frac{1}{(x+1)\cdot \ln^2(x+1)}$$ JE NEROSTOUCÍ ?

$$f'(x) = \frac{0 - 1\cdot[(x+1)\cdot \ln^2(x+1)]'}{[(x+1)\cdot \ln^2(x+1)]^2} =$$

$$= \frac{1\cdot \ln^2(x+1) + (x+1)\cdot 2\cdot \ln(x+1)\cdot \frac{1}{x+1}}{[(x+1)\cdot \ln^2(x+1)]^2} =$$

$$= - \frac{\ln^2(x+1) + 2\cdot\ln(x+1)}{[\qquad]^2} \leq 0 \quad \text{VŽDY} \checkmark$$

$\Rightarrow$ NEROST.

$$\int_1^\infty \frac{1}{(x+1)\cdot\ln^2(x+1)}\,dx = \left| \begin{array}{l} t = \ln(x+1) \\ dt = \frac{1}{x+1}\,dx \end{array} \right| \begin{array}{l} x=\infty \Rightarrow t=\infty \\ x=1 \Rightarrow t= \\ \quad = \ln 2 \end{array} \Bigg|$$

$$= \int_{\ln 2}^{\infty} \frac{1}{t^2}\,dt = \left[\frac{t^{-1}}{-1}\right]_{\ln 2}^{\infty} = \lim_{t\to\infty}\left(-\frac{1}{t}\right) -$$

$$- \left[-(\ln 2)^{-1}\right] =$$

$$= 0 + \frac{1}{\ln 2} < \infty \Rightarrow \underline{\underline{\text{KONV.}}}$$

$\Rightarrow$ <u>ŘADA KONV.</u>

$$\sum_{n=0}^{\infty} \frac{(n+1)!}{2^n \cdot n!}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{(n+2)!}{2^{n+1}\cdot(n+1)!}}{\dfrac{(n+1)!}{2^n \cdot n!}} = \frac{\dfrac{(n+2)\cdot\cancel{(n+1)!}}{\cancel{2}^n\cdot 2\cdot\cancel{(n+1)!}}}{\dfrac{(n+1)\cdot\cancel{n!}}{\cancel{2}^n\cdot\cancel{n!}}} =$$

$$= \frac{n+2}{2\cdot(n+1)}$$

KONV.

$\Uparrow$

$$\lim_{n\to\infty} \frac{n+2}{2\cdot(n+1)} = \lim_{n\to\infty} \frac{1+\frac{2}{n}}{2+\frac{2}{n}} = \frac{1}{2} < 1$$

$$\sum_{n=25}^{\infty} \frac{n}{2n-1}$$

$$\lim_{n\to\infty} \frac{n}{2n-1} \overset{?}{=} 0$$

$$\| $$

$$\frac{1}{2} \neq 0 \Rightarrow \text{NEKONV.}$$

$$(2\cdot(n+1)-1 = 2n+2-1)$$

$$\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{2n+1}}{\frac{n}{2n-1}} = \frac{(n+1)\cdot(2n-1)}{n\cdot(2n+1)} =$$

$$= \frac{2n^2+n-1}{2n^2+n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2n^2+n-1}{2n^2+n} =$$

$$= 1 \quad \ldots \text{NELZE ROZH.}$$

$$\sum_{n=0}^{\infty} \frac{3^n}{2^n(2n+1)}$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{3^{n+1}}{2^{n+1}\cdot(2n+3)}}{\dfrac{3^n}{2^n\cdot(2n+1)}} = \frac{\dfrac{\cancel{3^n}\cdot 3}{\cancel{2^n}\cdot 2\cdot(2n+3)}}{\dfrac{\cancel{3^n}}{\cancel{2^n}(2n+1)}} =$$

$$= \frac{3\cdot(2n+1)}{2\cdot(2n+3)} \xrightarrow{n\to\infty} \frac{3}{2} > 1$$

$$\Downarrow$$

$$\underline{\underline{\text{DIV.}}}$$