$$[n=1] = \begin{cases} 1, & n=1 \\ 0 & \text{jinak} \end{cases}$$

$$[2|n] = \begin{cases} 1, & \text{pokud } 2|n \\ 0 & \text{jinak} \end{cases}$$

$$[x^n]F(x) = a_n$$

pro $F(x) = \sum_{n=0}^{\infty} a_n x^n$

$$\underset{F(x)}{\underset{\parallel}{\sum a_n x^n}} \longleftrightarrow (a_0, a_1, \ldots, a_n, \ldots)$$

$$\text{rshift} \quad (0, a_0, a_1, \ldots)$$

$$\text{lshift} \quad (a_1, a_2, \ldots)$$

$$\frac{F(x) - a_0}{x}$$

$$(a_0, \alpha a_1, \alpha^2 a_2, \ldots, \alpha^n a_n, \ldots)$$

$$(0, 1 \cdot a_1, 2 \cdot a_2, 3 \cdot a_3, \ldots)$$

$$\left(\sum_{n=0}^{\infty} a_n x^n\right)' = a_1 + 2a_2 x + 3a_3 x^2 + \dots$$

$$(a_1, 2a_2, 3a_3, \dots)$$

$$x \cdot \left(\quad\right)' = (0, a_1, 2a_2, 3a_3, \dots)$$

$$\binom{r}{k} = \frac{r(r-1)\dots(r-k+1)}{k!}$$

$r \in \mathbb{R}$

$k \in \mathbb{N}_0$

$$\ln\left(\frac{1}{1-x}\right) \longleftrightarrow \left(0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\right)$$

$$\ln\left(\frac{1}{1+x}\right) \longleftrightarrow \left(0, -1, \frac{1}{2}, -\frac{1}{3}, \ldots\right)$$

!

$$\|$$

$$-\ln(1+x)$$

$$\ln(1+x) \longleftrightarrow \left(0, 1, -\frac{1}{2}, \frac{1}{3}, \ldots\right)$$

$$F_{n+2} = F_{n+1} + F_n$$

$$F_0 = 0, \quad F_1 = 1$$

$$\boxed{F_{n+2} = F_{n+1} + F_n + [n=-1]}$$ lépe

$$F_2 = F_1 + F_0$$

$$\underbrace{F_1 = F_0}_{} + F_{-1} =$$

$$F_0 = F_{-1} + F_{-2} = 0$$

1

$$\boxed{F_n = F_{n-1} + F_{n-2} + [n=1]}$$!

2

$$\sum_{n=0}^{\infty} F_n x^n = \sum_{n=0}^{\infty} F_{n-1} \cdot x^n + \sum_{n=0}^{\infty} F_{n-2} x^n + \sum_{n=0}^{\infty} [n=1] x^n$$

$$\| \qquad\qquad\qquad\qquad\qquad\qquad \|$$

$$F(x) = \quad - \quad - \quad - \quad - \qquad\qquad x$$

$$F(x) = \frac{x}{1-x-x^2}$$

$$F_n = [x^n] F(x)$$

$$\frac{A}{x-\alpha}$$

$$\frac{B}{1-\beta x} = B(1+\beta x+\beta^2 x^2+\dots)$$

$$\frac{A}{x-\alpha} = \frac{-\frac{A}{\alpha}}{-\frac{x}{\alpha}+1} = \frac{-\frac{A}{\alpha}}{1-\frac{x}{\alpha}}$$

$$B := -\frac{A}{\alpha}$$

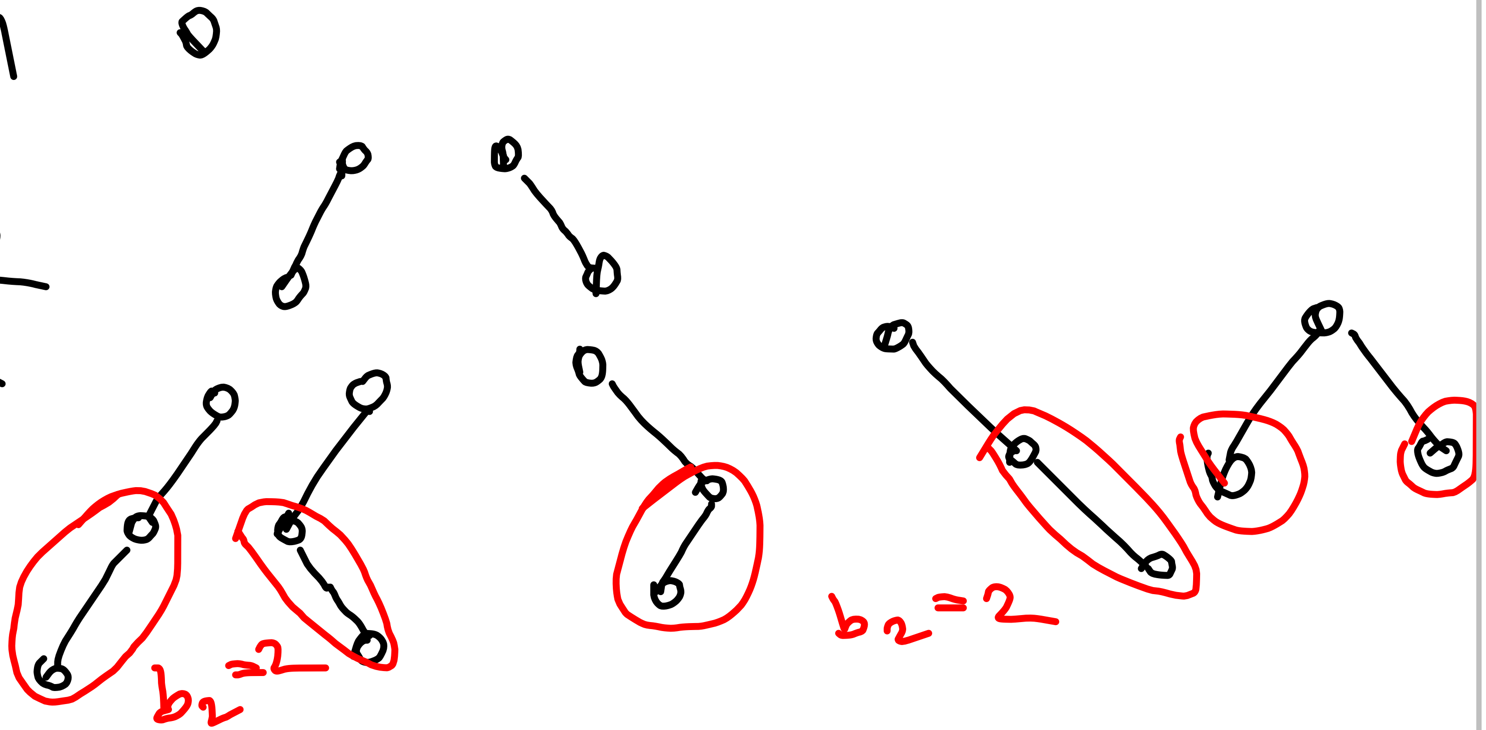
$$\beta = -\frac{1}{\alpha}$$

$b_0 = 1$

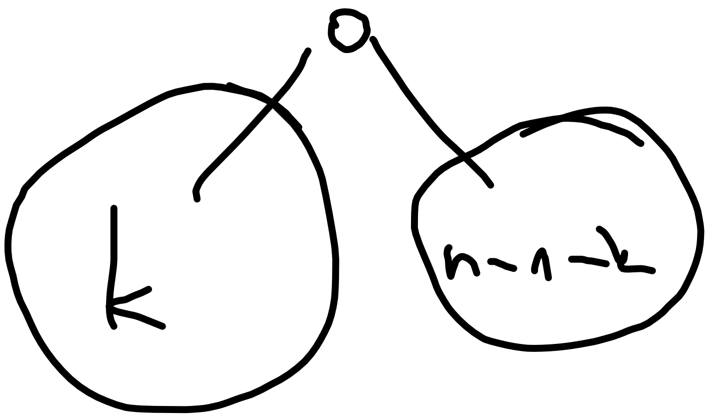
$b_1 = 1$

$b_2 = 2$

$b_3 = 5$



$b_4 = ?$



$b_k \cdot b_{n-1-k}$

$$b_n = \sum_{0 \le k < n} b_k b_{n-1-k} + [n=0]$$

žádné k

$$\sum_{n=0}^{\infty} \sum_{0 \le k < n} b_k b_{n-k-1} x^n = \boxed{\sum_{k=0}^{\infty} b_k x^k} \sum_{n=k+1}^{\infty} b_{n-k-1} x^{n-k}$$

pro $n < k+1$ je $b_{n-k-1} = 0$

$$B(x) = x\big(B(x)\big)^2 + 1$$

$$B(x) = \frac{1 \pm \sqrt{1-4x}}{2x}$$

$$\lim_{x \to 0_+} \frac{1 + \sqrt{1-4x}}{2x} = +\infty$$

$$B(0) = b_0 = 1 \implies B(x) = \frac{1 - \sqrt{1-4x}}{2x}$$

$$(1+x)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^k$$

$$\binom{r}{0} = 1 \quad \forall r \in R$$

$$\binom{1/2}{k} = \frac{1/2 \cdot \left(-1/2\right)\left(-\frac{3}{2}\right) \cdots \left(\frac{1}{2} - k + 1\right)}{k(k-1)!} =$$

$$= \frac{1}{2} \cdot \frac{1}{k} \cdot \binom{-1/2}{k-1}$$

$$-\left(\sum_{k\geq 1}\frac{1}{2k}\binom{-1/2}{k-1}(-4x)^k\right)\cdot\frac{1}{2x}=$$

$$=-\frac{1}{x}\sum_{k\geq 1}\frac{1}{4k}\binom{-1/2}{k-1}(-4x)^{k-1}$$

$$[n=k-1]$$

$$\sum_{n\geq 0}\frac{1}{n+1}\binom{-1/2}{n}(-4x)^n$$

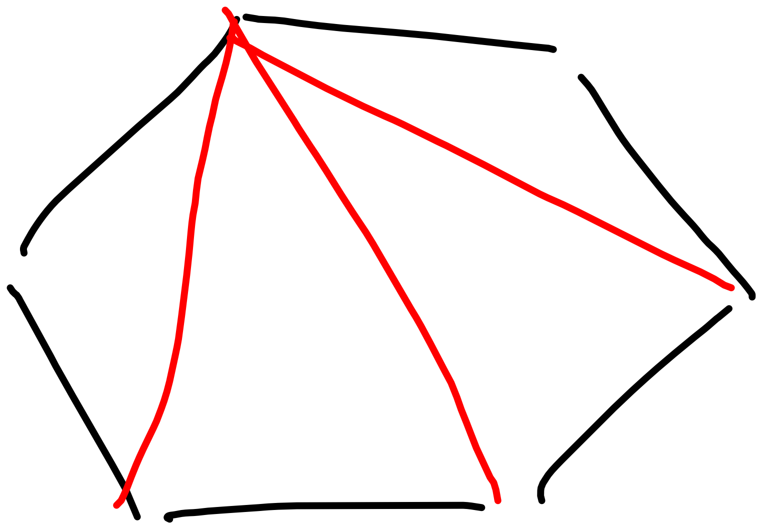
? $\binom{-1/2}{n}\cdot(-4)^n = \binom{2n}{n}$

$$\binom{-1/2}{n}\frac{(-4)^n x^n}{n+1} = \frac{\overbrace{\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\cdots\left(-\tfrac{1}{2}-n+1\right)}^{n}}{n!}\cdot(-4)^n\cdot\frac{x^n}{n+1} = \frac{\frac{1}{\cancel{2}}\cdot\frac{3}{\cancel{2}}\cdots\frac{2n-1}{\cancel{2}}}{n!}\cdot \cancel{4}^n_{2}\cdot\frac{x^n}{n+1} =$$

$$= \frac{1\cdot 2\cdot 3\cdot 4\cdot 6 \cdots 2n-1\cdot 2n}{\cancel{2^n}\cdot n!\; n!}\cdot \cancel{2^n}\cdot\frac{x^n}{n+1} = \frac{(2n)!}{n!\,n!}\cdot\frac{x^n}{n+1}$$

$$= \boxed{\binom{2n}{n}\cdot\frac{x^n}{n+1}}$$

$\binom{2n}{n}$ ... počet všech slov z n X a n Y



$$a_2 = a_1 + 2a_0 + (-1)^2 =$$

$$= 4$$

$$a_3 = a_2 + 2a_1 + (-1)^3 = 5$$

$$a_1 = \underbrace{a_0 + 2a_{-1} + (-1)}_{0} + [n = 1]$$

$$\overset{A(x)}{\overset{\shortparallel}{\sum_{n\ge 0} a_n x^n}} = \underset{\underset{x\cdot A(x)}{\shortparallel}}{\sum_{n\ge 0} a_{n-1} x^n} + \underset{\underset{2x^2 A(x)}{\shortparallel}}{2\sum a_{n-2} x^n} + \underset{\underset{\frac{1}{1+x}}{\shortparallel}}{\sum_{n\ge 0} (-1)^n x^n}$$

Krok 4:

Rozvineme $A(x)$ do mocninné řady:

$$\frac{1+x+x^2}{(1-2x)(1+x)^2} = \frac{A}{1-2x} + \frac{B}{(1+x)} + \frac{C}{(1+x)^2}$$

$$1+x+x^2 = A(1+x)^2 + B(1-2x)(1+x) + C(1-2x)$$

$x=-1$: $1 = C \cdot 3$

$x = \frac{1}{2}$: $\frac{7}{4} = \frac{9}{4} A$

$x=0$: $1 = A+B+C$

$C = \frac{1}{3}$

$A = \frac{7}{9}$

$B = -\frac{1}{9}$

$$A(x) = \frac{7}{9(1-2x)} - \frac{1}{9(1+x)} + \frac{1}{3(1+x)^2}$$

$$A_1(x) = \frac{7}{9} \cdot \frac{1}{1-2x} = \frac{7}{9} \cdot \sum_{n\geq 0} 2^n x^n$$

$$A_2(x) = -\frac{1}{9} \cdot \frac{1}{1+x} = -\frac{1}{9} \cdot \sum_{n\geq 0} (-1)^n x^n$$

$$A_3(x) = \frac{1}{3} \cdot \frac{1}{(1+x)^2} = \frac{1}{3} \cdot (1+x)^{-2} = \frac{1}{3} \cdot \sum_{n\geq 0} \binom{-2}{n} x^n =$$

$$\boxed{\binom{-2}{n} = \frac{(-2)(-3)(-4) \cdots (-2-n+1)}{n!} = \frac{(n+1)!}{n!} \cdot (-1)^n}$$

$$= \frac{1}{3} \cdot \sum_{n\geq 0} (n+1)(-1)^n \cdot x^n$$