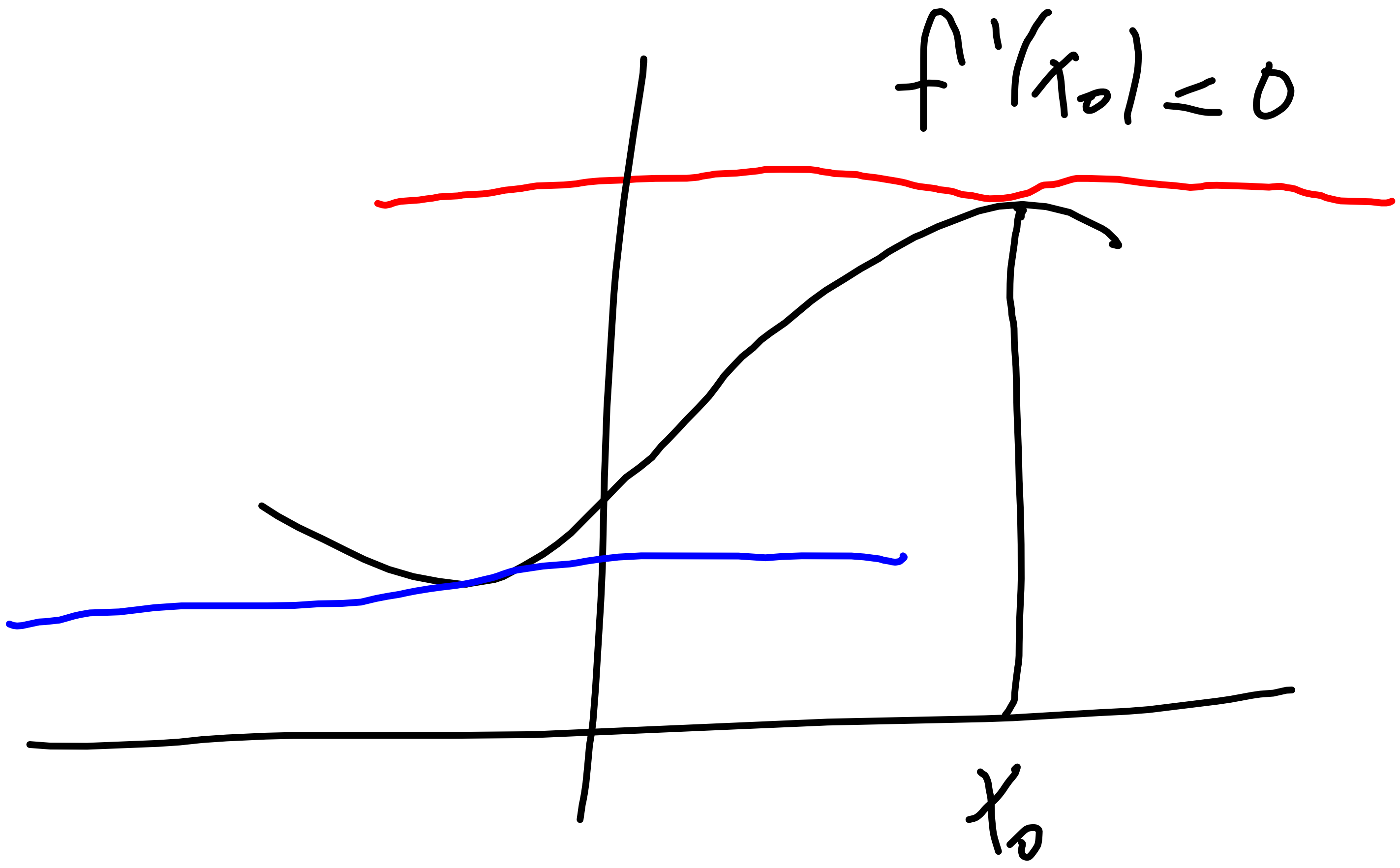
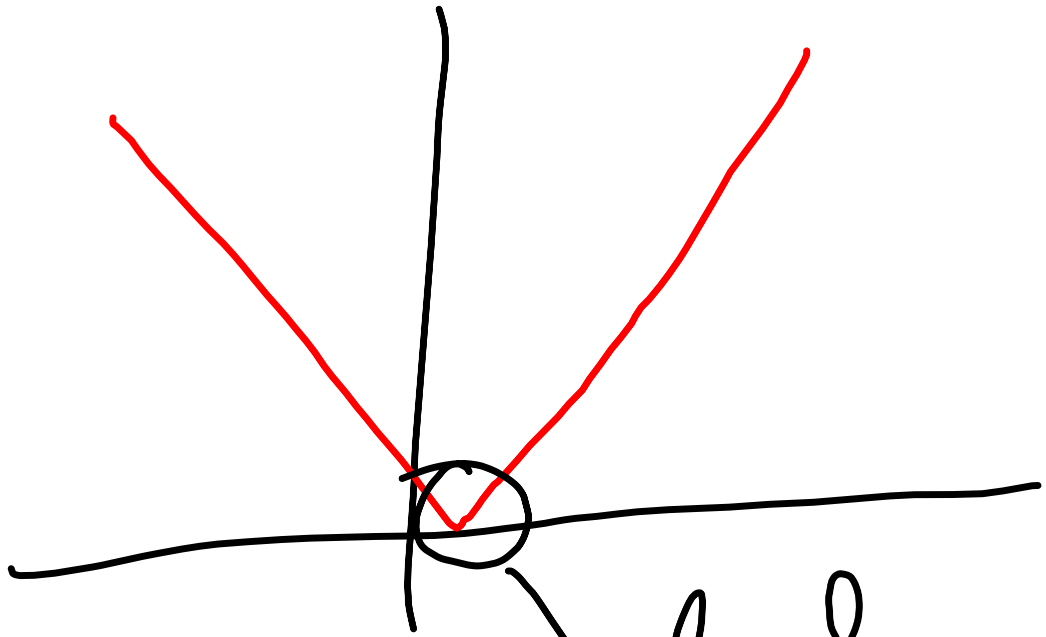

$f'(x_0) = 0$
x_0



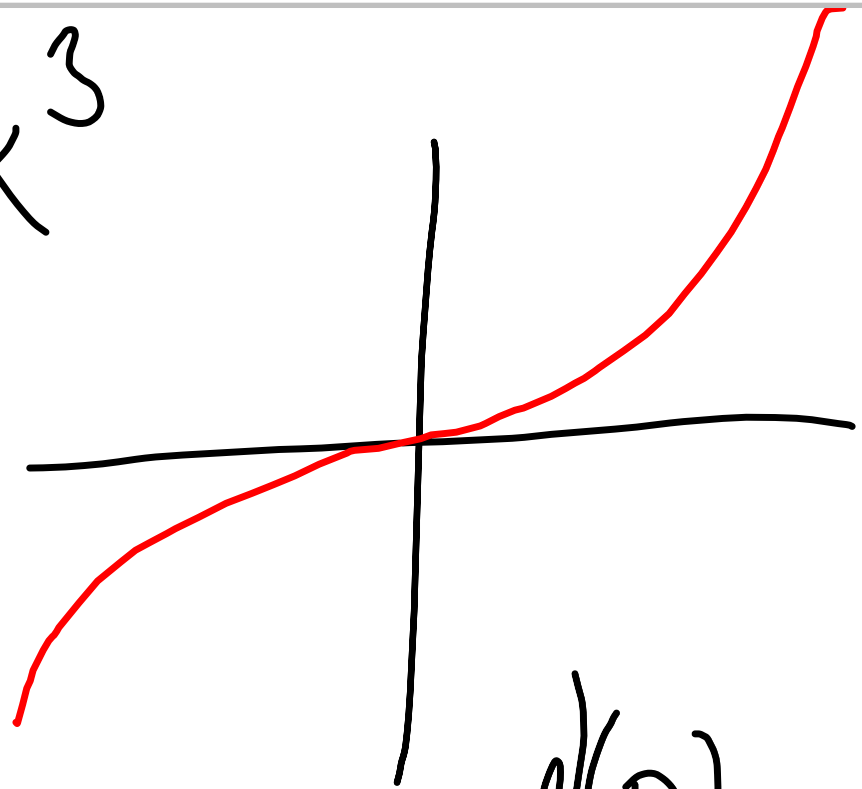
lok. extrém

není derivace

$$df(x^*)(v) =$$

$$\left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_m}\right) \cdot (v_1, \dots, v_m)$$

$f: y = x^3$



← tečna v $[0,0]$

$f'(0) = 0$

není extrém

$y = x^3$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x =$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{y}{\sqrt{x^2+y^2}}$$

$$\frac{\partial f}{\partial x}(0,0) = \ ?$$

neexistuje

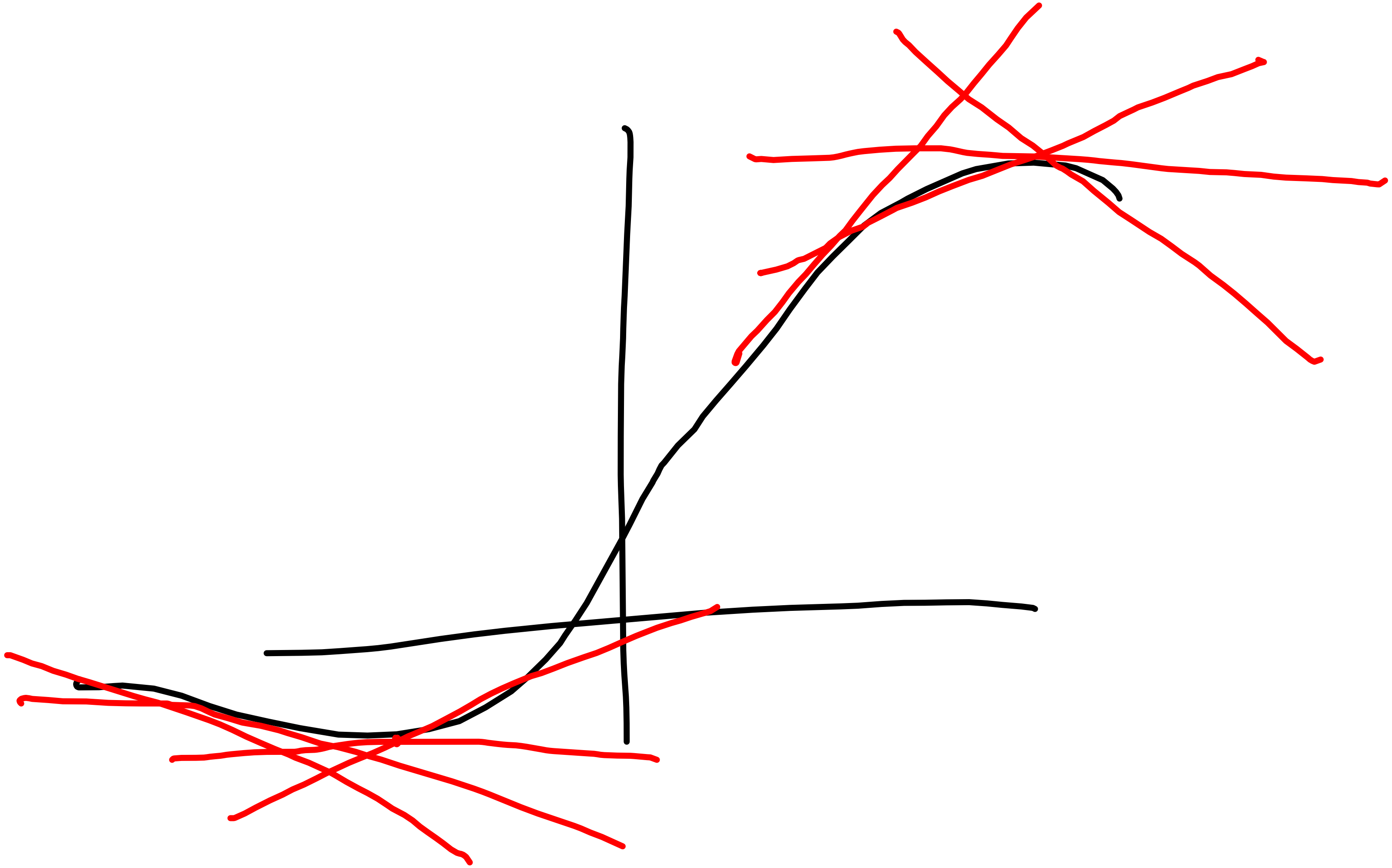
$$\lim_{(x,y)\to 0} \frac{\partial f}{\partial x}(x,y) = \lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \left| y = k\cdot x \right| = \lim_{x\to 0} \frac{x}{\sqrt{x^2+k^2x^2}} =$$

$$= \lim_{x\to 0} \frac{x}{|x|\sqrt{1+k^2}} = \lim_{x\to 0} \frac{\operatorname{sgn} x}{\sqrt{1+k^2}}$$

$$\frac{\partial f}{\partial x}(x,y) = (y - y_0)$$

$$f(x,y) = (x - x_0)(y - y_0)$$



$$f''(\xi) < 0$$

$$(x - x_0)^2 \geq 0 \quad > 0 \text{ pro } x \neq x_0$$

$h: E_n \to \mathbb{R}$

$h(u) = r$

$h_b(u) = b(u,u)$

b ... bilineární forma

$b(x,y)$

$b(x_1+x_2,y) = b(x_1,y) + b(x_2,y)$

$b(r\cdot x,y) = r\cdot b(x,y)$

např.: skal. součin

kvadr. forma $\approx$ sym. matice

$$h(u) \qquad u^T \cdot H \cdot u$$

$$(x, y)\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= (2x+y,\; x-y)\begin{pmatrix} x \\ y \end{pmatrix} =$$

$$= 2x^2 + xy + xy - y^2 = 2x^2 + 2xy - y^2$$

$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix}$$

$$|2| = 2$$

$$\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -3$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = \lambda^2 - \lambda - 3$$

$$\lambda_{1,2} = \frac{1 \pm \sqrt{13}}{2}$$

$$f(x,y) = \sin(x)\cos(y)$$

$$f_x(x,y) = \cos x \cos y = 0$$

$$f_y(x,y) = -\sin x \sin y = 0$$

$\cos x = 0$ $\qquad$ $\sin x = 0$

$\cos y = 0$ $\qquad$ $\sin y = 0$

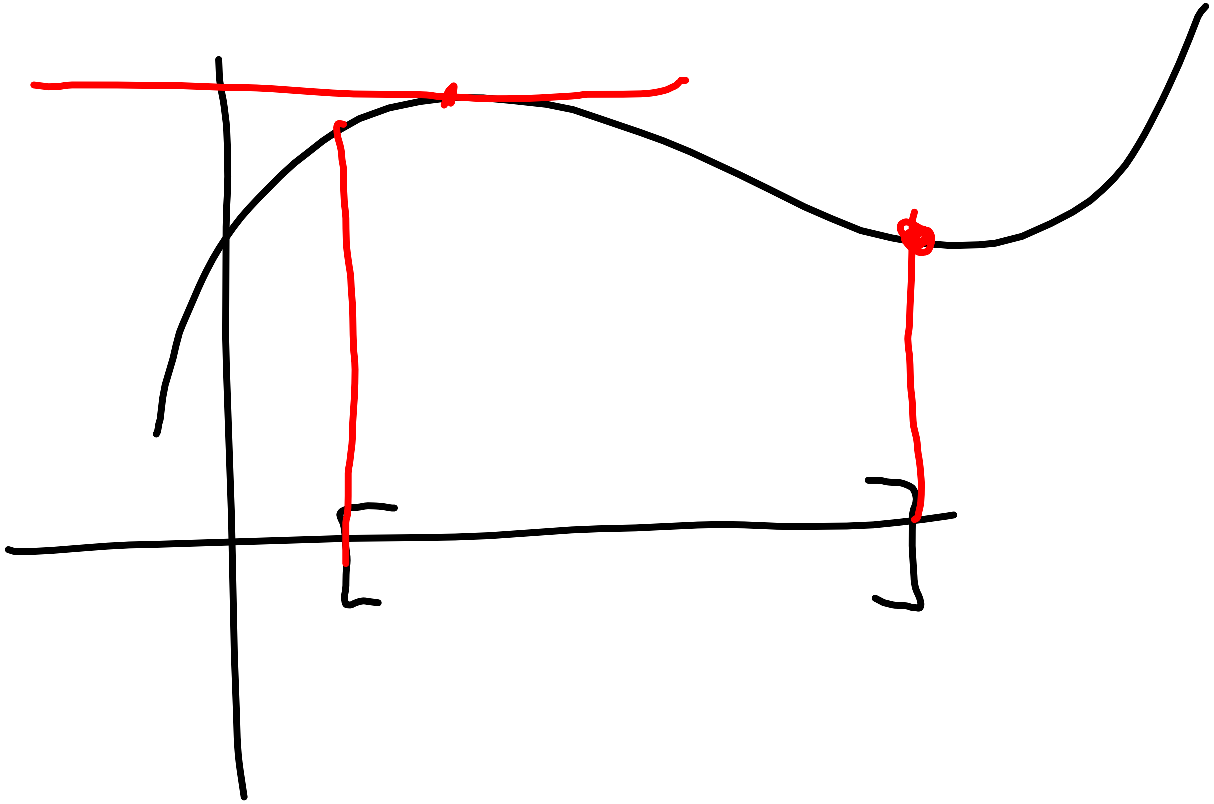
$$\left[(2k+1)\frac{\pi}{2},\ l\pi\right]$$

$$\left[k\cdot\pi,\ (l+1)\frac{\pi}{2}\right]$$

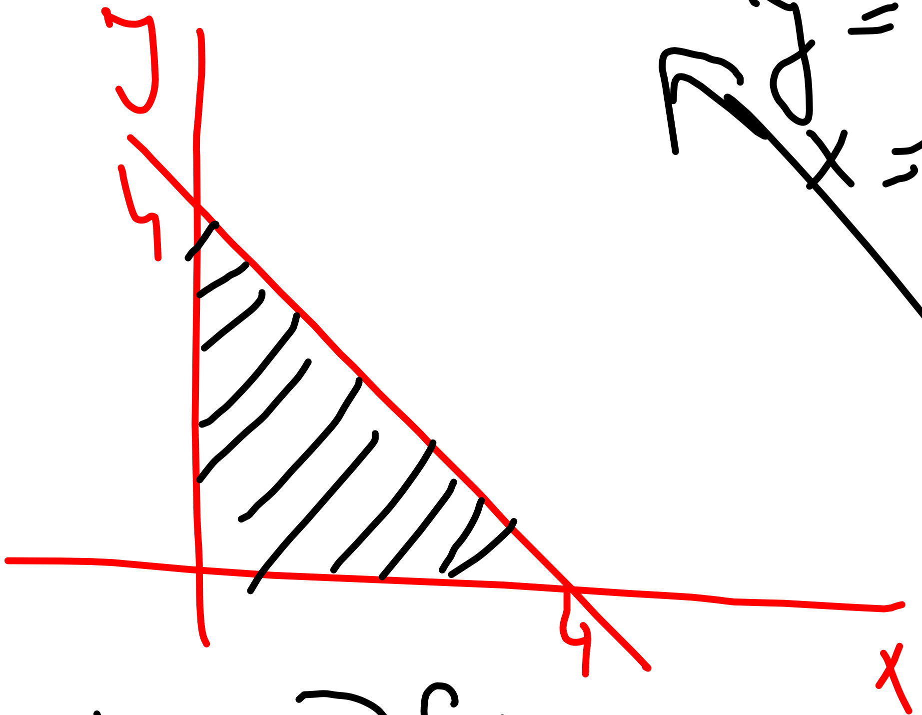
$$Hf = \begin{pmatrix} -\sin x\cos y & -\cos x\sin y \\ -\cos x \sin y & -\sin x\cos y \end{pmatrix}$$

$$(x, y)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = x^2 + y^2$$

$$(x, y)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 2xy$$




abs. max.
abs. min.



$$y = 2x - 1$$
$$x = 2y - 1$$
$$\Downarrow$$
$$y = 4y - 3$$

$$\boxed{\begin{array}{c} y = 1 \\ \hline x = 1 \end{array}}$$

stac. body: $\dfrac{\partial f}{\partial x}(x,y) = y - 2x + 1$

$$\frac{\partial f}{\partial y}(x,y) = x - 2y + 1$$

$\boxed{f(1,1) = 1}$

na hranici $\quad x \in \langle 0, 4 \rangle$

I: $f(x,0) = -x^2 + x \qquad f\left(\frac{1}{2}, 0\right) = \frac{1}{4}$

$u(x) =$

$$u'(x) = -2x + 1 = 0 \iff \underline{x = \frac{1}{2}}$$

$$f(0,0) = 0 \qquad f(4,0) = -12$$

II, $v(x) = f(0,y) = -y^2 + y$

$$\underline{y = \frac{1}{2}} \qquad f(0,4) = -12$$

$\underline{\text{III.}}$ $x+y=4 \Rightarrow y=4-x$

$m(x)=f(x,y)=x(4-x)-x^2-(4-x)^2+x+4-x$

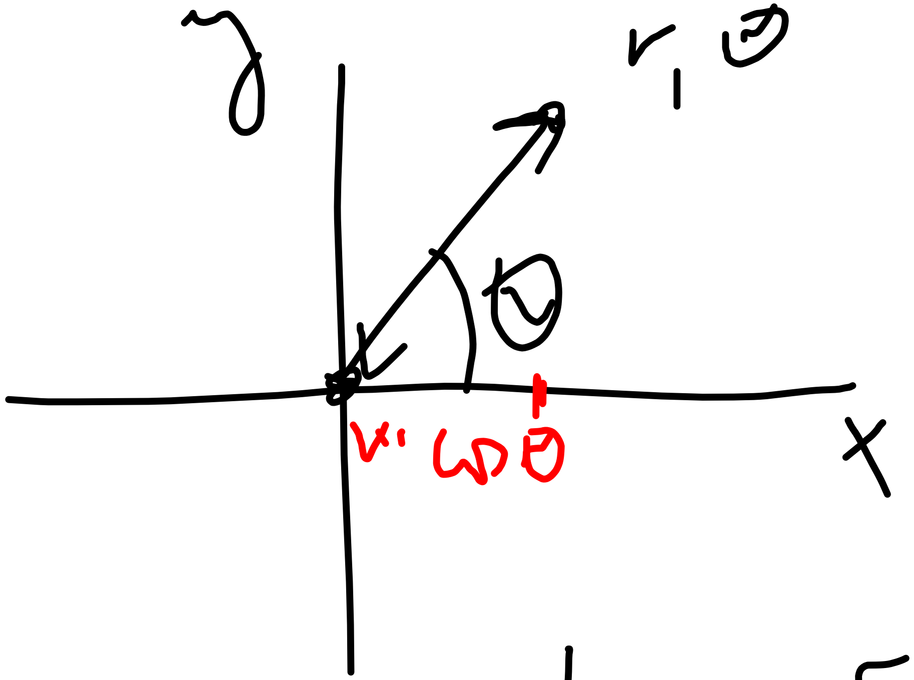
$=-3x^2+12x-12$

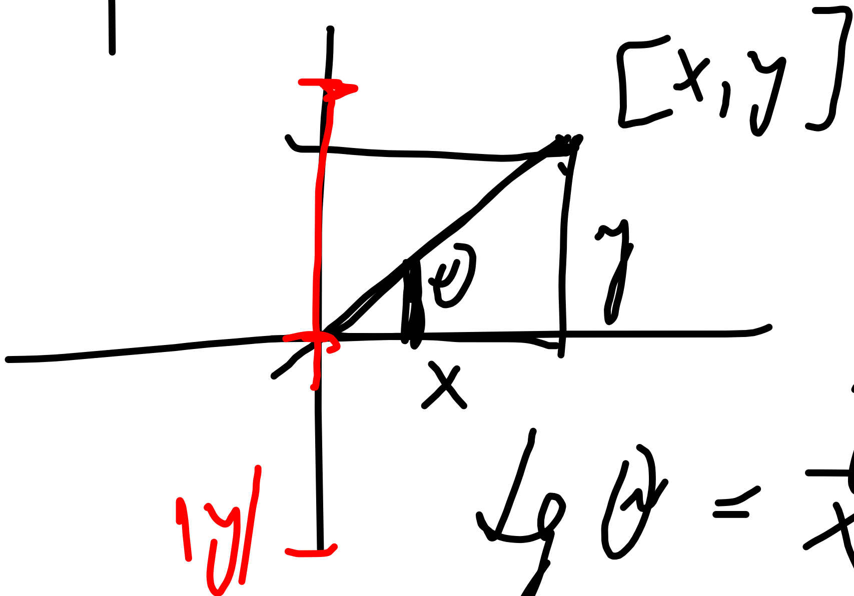
$m'(x)=-6x+12=0 \quad \underline{x=2}$

$[2,2]$ ... lok. max.

$f(2,2)=0$

$f(4,0)=f(0,4)=-12$





$$\operatorname{tg} \theta = \frac{y}{x}$$

$$\frac{\partial g}{\partial x} = \frac{\partial g}{\partial r} \cdot \frac{\partial r}{\partial x} + \frac{\partial g}{\partial \varphi} \cdot \frac{\partial \varphi}{\partial x}$$

$$\frac{\partial g}{\partial y} = \dots$$

$$g_1(r, \varphi) = \sin(r - t)$$

$$r = \sqrt{x^2 + y^2} \qquad \frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2 + y^2}}$$

$y = f(x)$

$x = g(y)$

$$f'(x) = \frac{1}{g'(y)}$$

$(\arcsin x)'$

$x = \sin y$

$(\sin y)' = \cos y$

$$(\arcsin x)' = \frac{1}{\cos y} = \frac{1}{\cos(\arcsin x)}$$

$= \%$

$$\frac{1}{\cos(\arcsin x)} = \frac{1}{\sqrt{1-x^2}}$$

$$\cos^2(\arcsin x) + \underbrace{\sin^2(\arcsin x)}_{x^2} = 1$$

$$\cos(\arcsin x) = \sqrt{1-x^2}$$

$$\mathrm{id} = F \circ F^{-1}$$

$$\mathbb{I}_n = D^1 F \circ D^1 F^{-1}$$

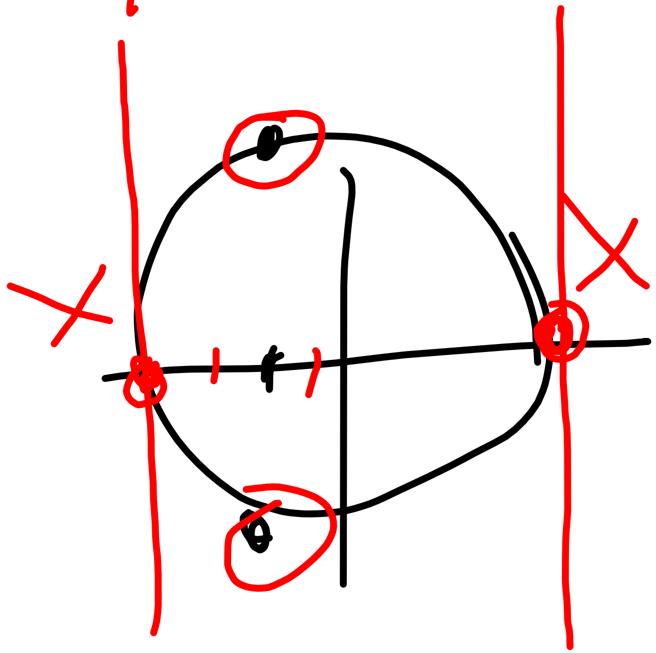
$y = f(x)$ explicitní

$F(x,y) = 0$ implicitní

$x^2 + y^2 = 1$

$f_1: \; y = \sqrt{1-x^2}$

$f_2: \; y = -\sqrt{1-x^2}$



$$f(x) = t + \sqrt{(x-s)^2 - r}$$

$$F(x,y) = (x-s)^2 + (y-t)^2 - r^2 = 0$$

$$0 = dF = F_x\,dx + F_y \cdot dy =$$

$$= F_x \cdot dx + F_y \cdot f'(x)\,dx$$

$$\boxed{f'(x) = -\frac{F_x}{F_y}}$$