$$a_{11} \cdot x_1 + a_{12} x_2 + \dots + a_{1n} x_n \leq b_1$$

$$a_{21} x_1 + \dots \quad a_{2n} x_n \leq b_2$$

$$\vdots$$

$\leq$

$\geq$

$=$

úloha ve smíšeném tvaru

$\min c \cdot x \longrightarrow \max. \ -c \cdot x$

nerovnice $\longleftrightarrow$ rovnice

$\longrightarrow$ $x + 2y \leq 30$

$x + 2y + z = 30$

$z \geq 0$

$\longleftarrow$ $x + 2y = 30$

$x + 2y \leq 30$

$x + 2y \geq 30 \longrightarrow -x - 2y \leq -30$

$$x + 2y \leq 30 \qquad y \geq 0$$

chybí $x \geq 0$ !

max. $3x + 4y$

subst $x := x' - x''$

$$x' \geq 0 \qquad x'' \geq 0$$

$$x' - x'' + 2y \leq 30$$

max. $3x' - 3x'' + 4y$

$$x', x'', y \geq 0$$

# Grafické řešení

max. $100x + 250y$

pro

$7x + 12y \leq 312$

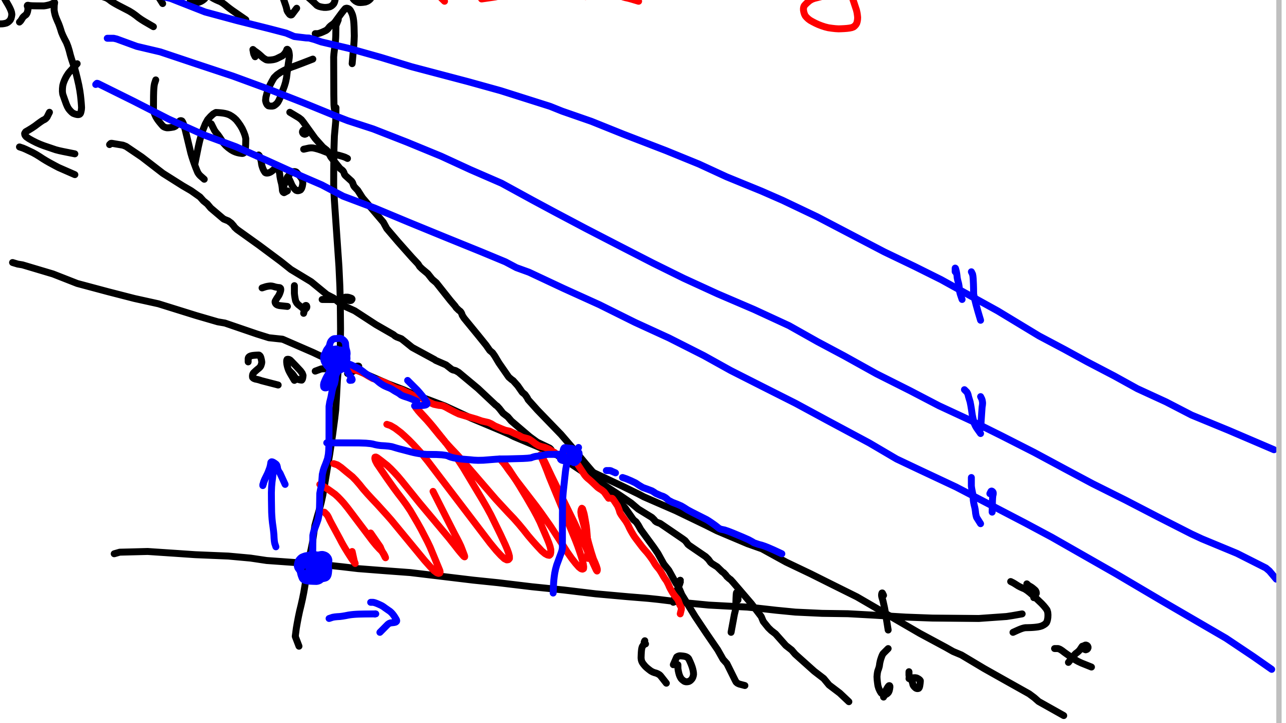
$40x + 120y \leq 2400 \iff x + 3y \leq 60$

$x + y \leq 40$

$x, y \geq 0$

$100x + 250y = c$

$y = \frac{c}{250} - \frac{2}{5}x$



$$\underline{\underline{2x}} - 3y + \underline{\underline{4z}}$$

$$\frac{3}{1}, \quad \frac{10}{1}$$

$$\cancel{M} \quad 4 \quad \rightarrow \quad 1 \quad 1 \quad 0 \quad 0 \mid 3$$

$\not{z}$

$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b c\cdot f(x)\,dx = c\cdot\int_a^b f(x)\,dx$$

$$\int_a^b : R[a,b] \to \mathbb{R}$$

$$\int_a^b f+g = \int f + \int g, \ \ldots$$

integrály závislé na parametru

$$\int_a^b f(x,y)\,dx$$

$$\int_0^1 x^2 y\,dx = \left[\frac{1}{3}x^3 y\right]_{x=0}^1 = \frac{1}{3}y =: F(y)$$

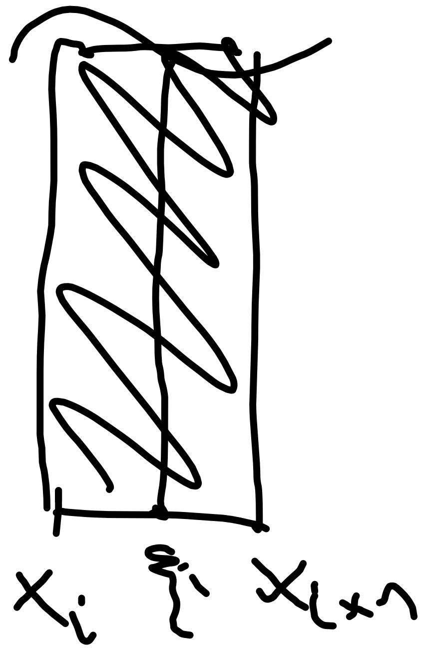
$$f(x,y) = x^2 y$$

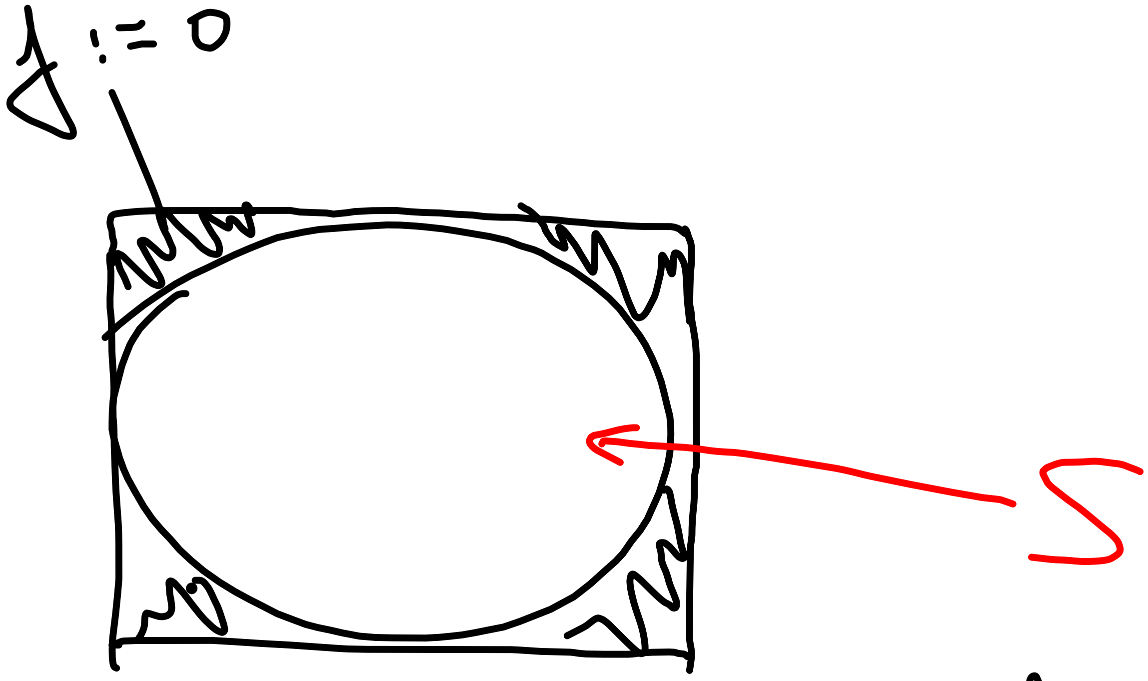
$$F'(y) = \underline{\underline{\frac{1}{3}}}$$

$$\frac{\partial f}{\partial y} = x^2$$

$$\int_0^1 x^2\,dx = \left[\frac{1}{3}x^3\right]_0^1 = \underline{\underline{\frac{1}{3}}}$$

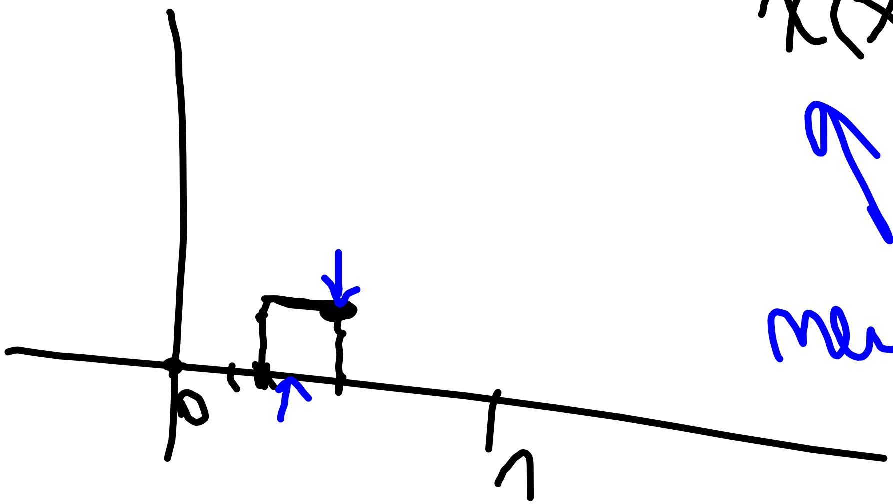

$$f(\xi)\,dx\,dy$$






$$\chi(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q} \end{cases}$$

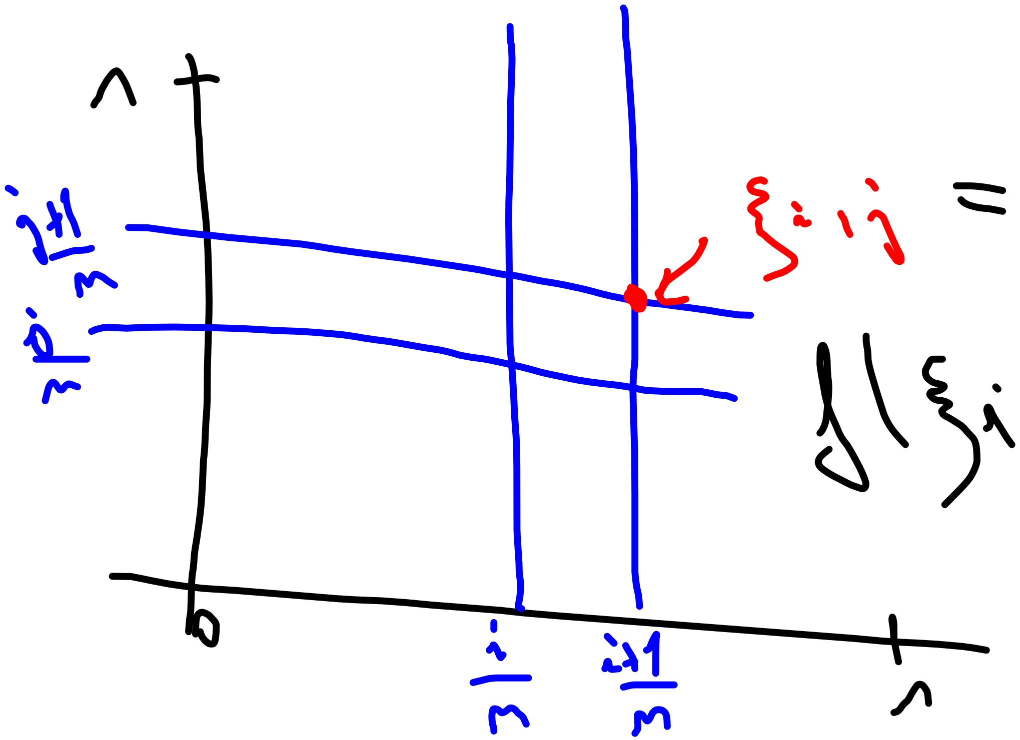
není R-integ.



$$\int_S f(x)\,dx = \int_{S_1} f(x)\,dx + \int_{S_2} f(x)\,dx + \dots$$

$$S = S_1 \,\dot\cup\, S_2 \,\dot\cup\, S_3 \,\dot\cup \dots \dot\cup\, S_n$$

$$\int_{[0,1]\times[0,1]} x y \cdot dx\,dy$$




$$\xi_{i,j} = \left[\frac{i+1}{n}, \frac{j+1}{n}\right]$$

$$f(\xi_{i,j}) = \frac{(i+1)(j+1)}{n^2}$$

$D_n$ $\quad S_{D_n} = \sum_{0 \le i,j < n} \underline{\underline{f(\xi_{i,j})}} \frac{1}{n^2}$

$$\sum_{1 \le i, j \le n} i\,j = 1\cdot 1 + 1\cdot 2 + 2\cdot 1 + 2\cdot 2 + 1\cdot 3 + 2\cdot 3 + 3\cdot 3 + \dots$$

$$= (1+2+3+\dots n)(1+2+3+\dots+n) =$$

dvojitá suma $\rightsquigarrow$ součin 2 $\sum$

$$\int_S xy\,dx\,dy \qquad \int_y \underbrace{\left(\int x\,dx\right)}\,dy$$

iterace jednodušších integrálů

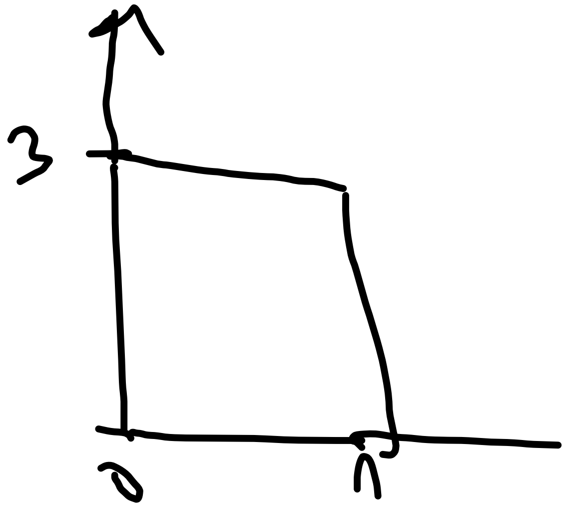
$S = 1 + 2 + \ldots + n$

$S = n + 1 + \ldots + 1$

$(n+1) \ldots (n+1) = 2S$

$$2S = n(n+1)$$

$$S = \frac{n(n+1)}{2}$$

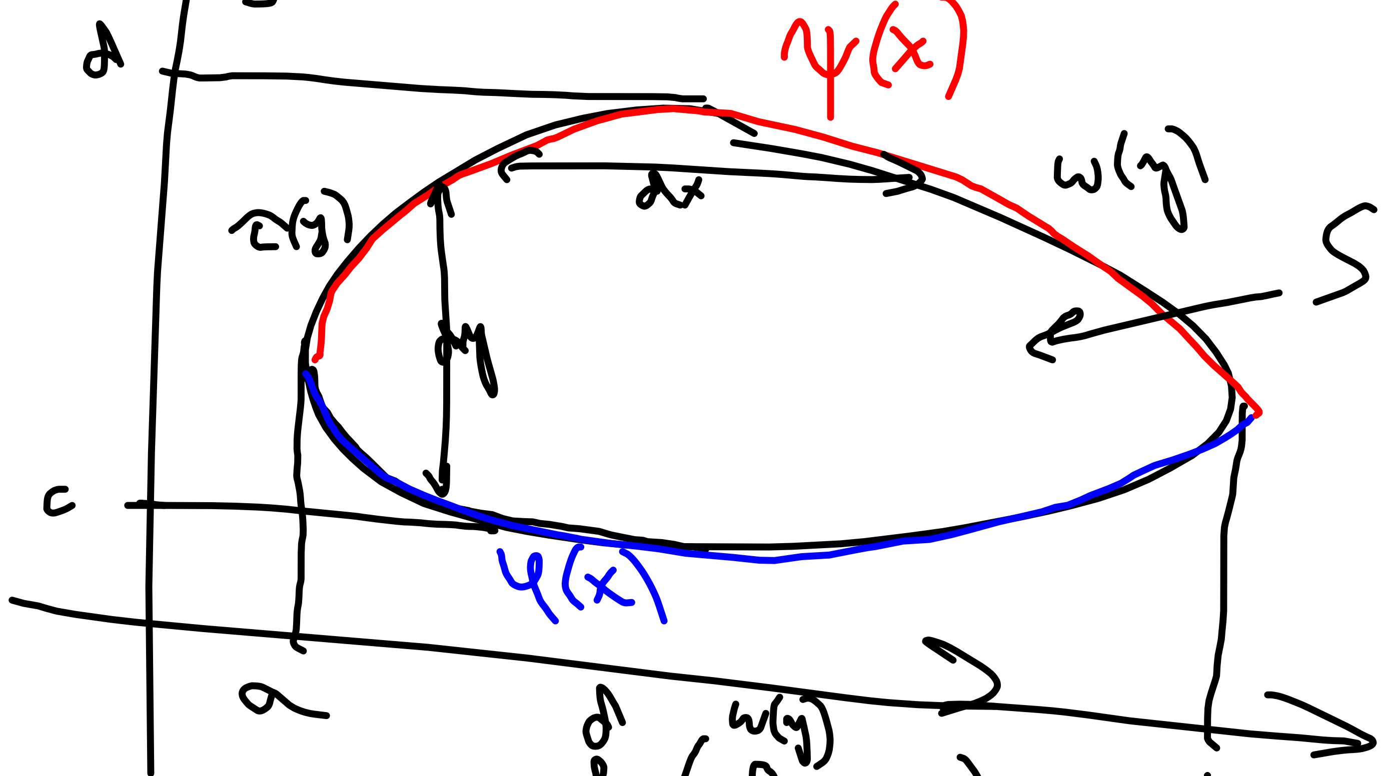



$$S = [0,1] \times [0,3]$$

$$\iint_S f(x,y)\,dx\,dy = \int_0^3 \left( \int_0^1 f(x,y)\,dx \right) dy$$

# Kdy jsou shodné násobné integrály?

$$\int_S \dots = \int_a^b \left( \int_{\varphi(x)}^{\psi(x)} \dots \, dy \right) dx$$




$$\int_S \dots \, dx\,dy = \int_c^d \left( \int_{\tau(y)}^{\omega(y)} \dots \, dx \right) dy$$

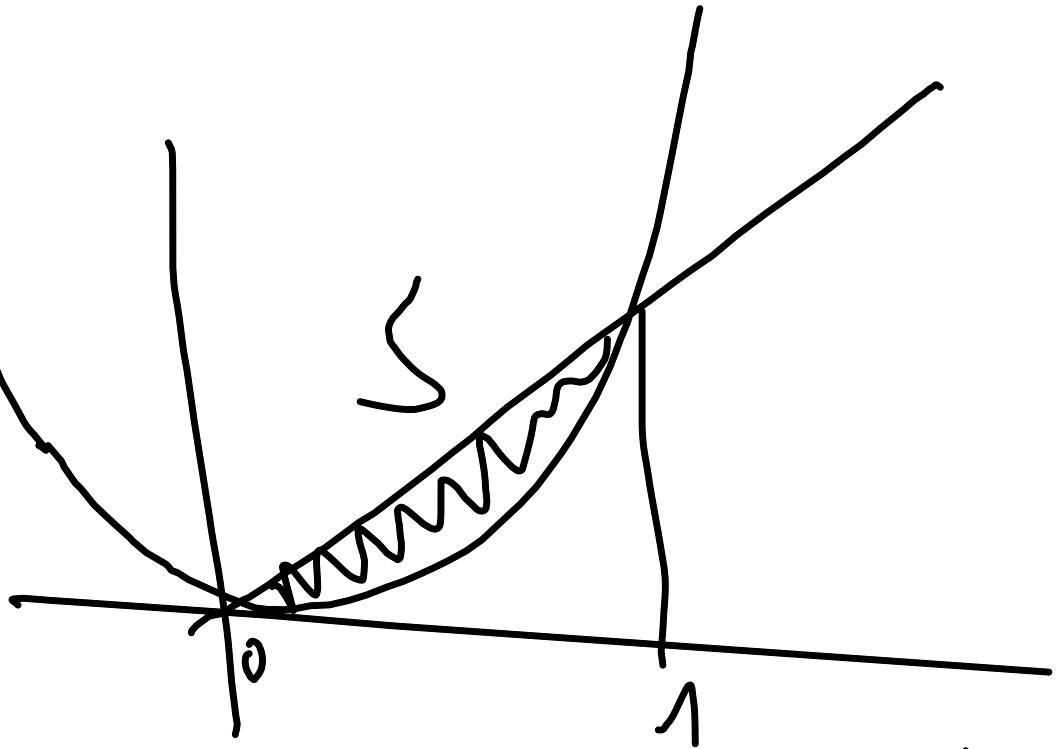
$$I = \int_0^1 \left( \int_0^3 \left[3(x-1)^2 + (y-2)^2 + 2\right] dy \right) dx =$$

$$= \int_0^1 \left[3(x-1)^2 y + \frac{1}{3}(y-2)^3 + 2y\right]_{y=0}^{3} dx =$$

$$= \int_0^1 \left[9(x-1)^2 + \frac{1}{3} + 6 + \frac{8}{3}\right] dx =$$

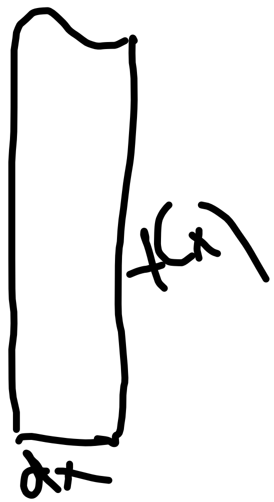
$$= \int_0^1 \left(9(x-1)^2 + 9\right) dx = \left[3(x-1)^3 + 9x\right]_0^1 =$$

$$= 9 + 3 = \underline{\underline{12}}$$




$$\int_S x y^2 \, dx \, dy$$

pro pevné $x \in [0,1]$
je $y \in [x^2, x]$



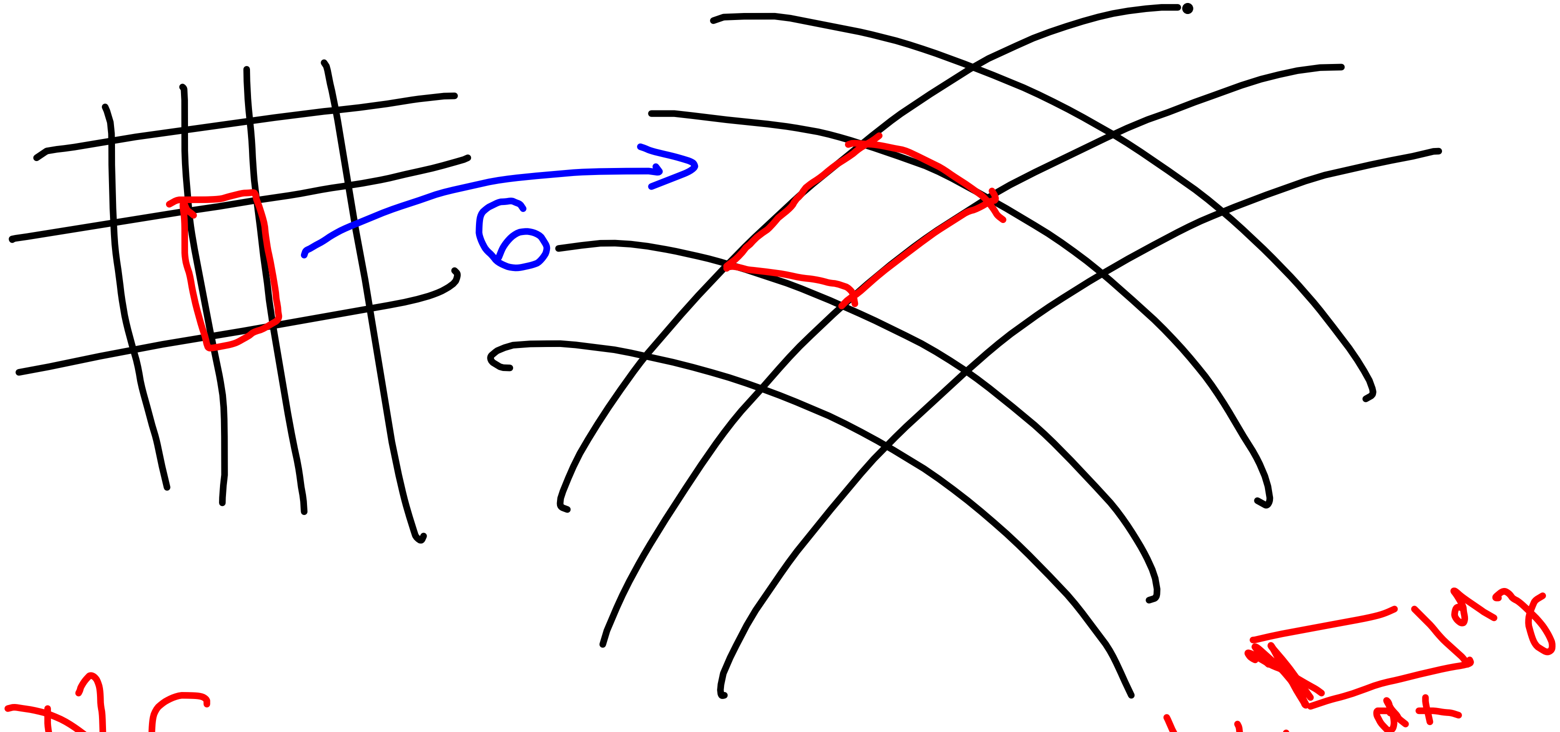

$$f(x)\,dx$$

$$x = u(t)$$

$$f(x) = f(u(t))$$

$$dx = u'(t)\,dt$$

v $E_n$




objem

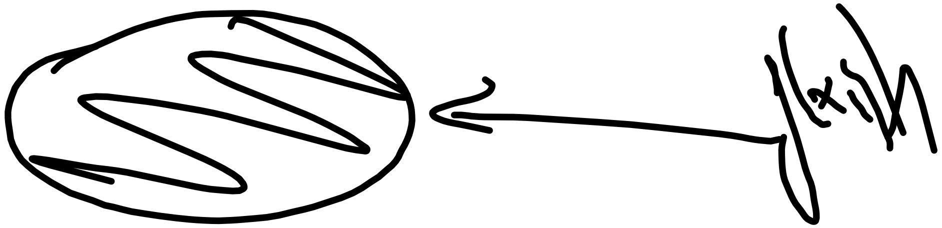
$D^1 G$

$$|\det D^1 G|$$

$$x = g(s,t)$$

$$y = h(s,t)$$

$$\int_{G(T)} f(x,y)\,dx\,dy = \int_{T} f(g,h)\,|\det DG|\,ds\,dt$$



$$\int_S dx\,dy = \int_0^{2\pi}\int_0^{R} \underline{r}\,dr\,d\varphi$$