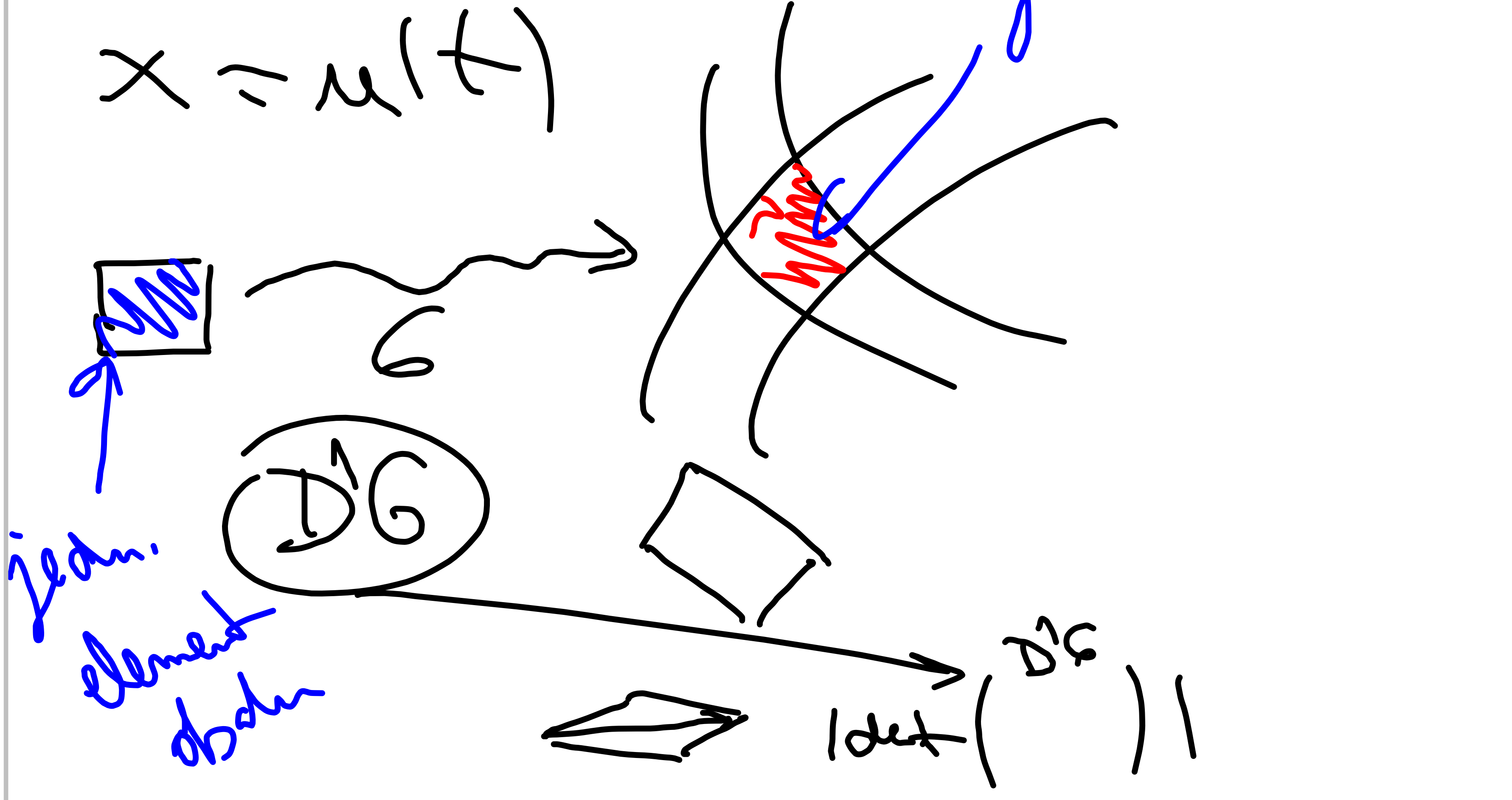
$$dx = u'(t)\,dt$$

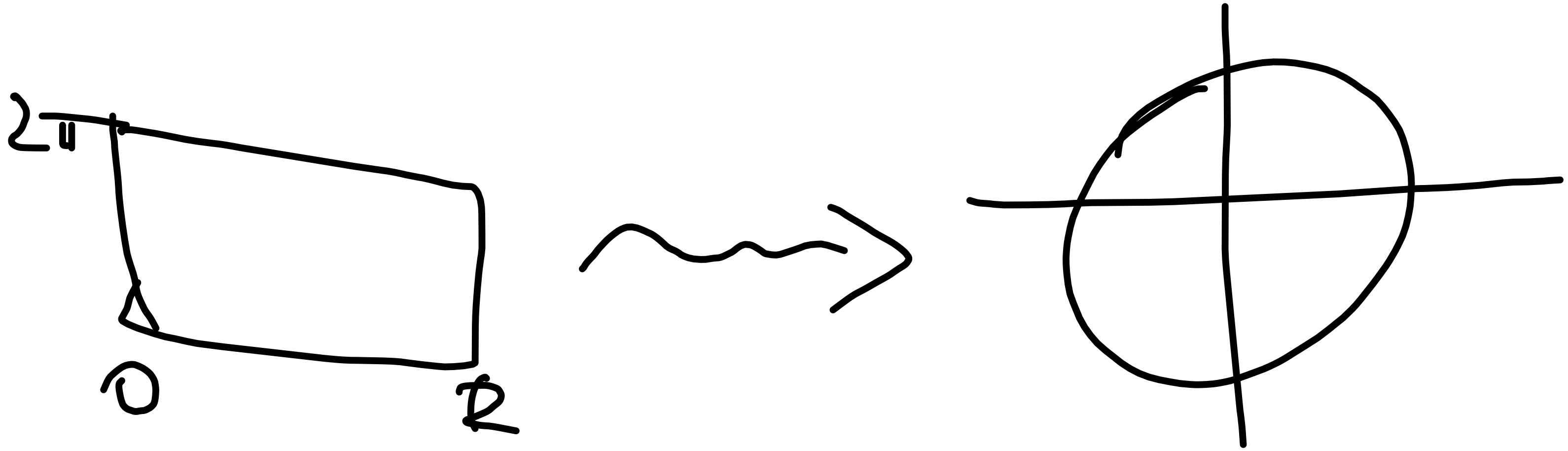
$$x = u(t)$$



$$[s,t] \longrightarrow [x,y]$$

$$G(s,t) = [g, h]$$

$$\begin{pmatrix} \frac{\partial g}{\partial s} & \frac{\partial g}{\partial t} \\ \frac{\partial h}{\partial s} & \frac{\partial h}{\partial t} \end{pmatrix}$$



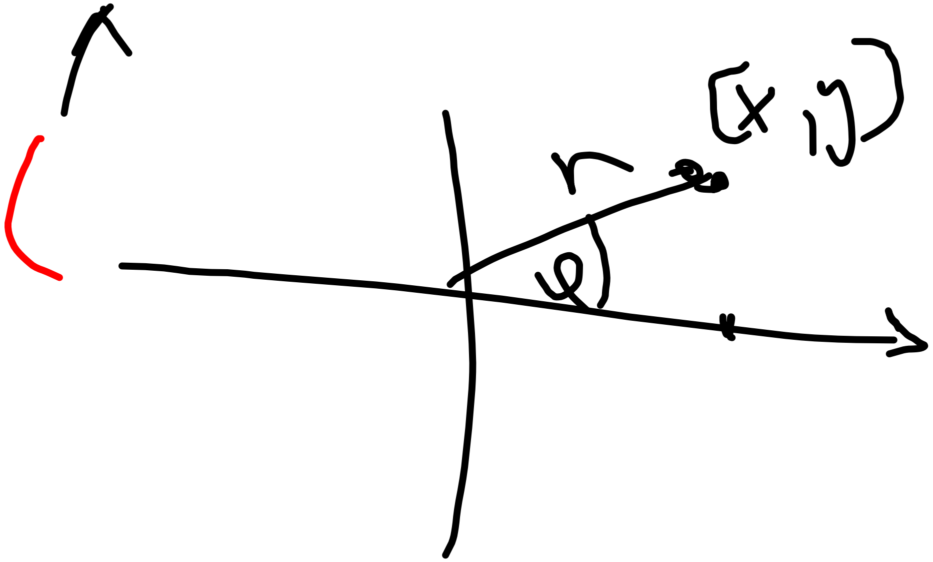
$$\int_S 1\,dx\,dy = \int_S 1\cdot r\,dr\,d\varphi =$$

$$= \int_0^{2\pi} d\varphi \int_0^R r\,dr = \int_0^{2\pi} \frac{R^2}{2}\,d\varphi = \frac{R^2}{2}\int_0^{2\pi} d\varphi =$$

$$= \pi R^2$$

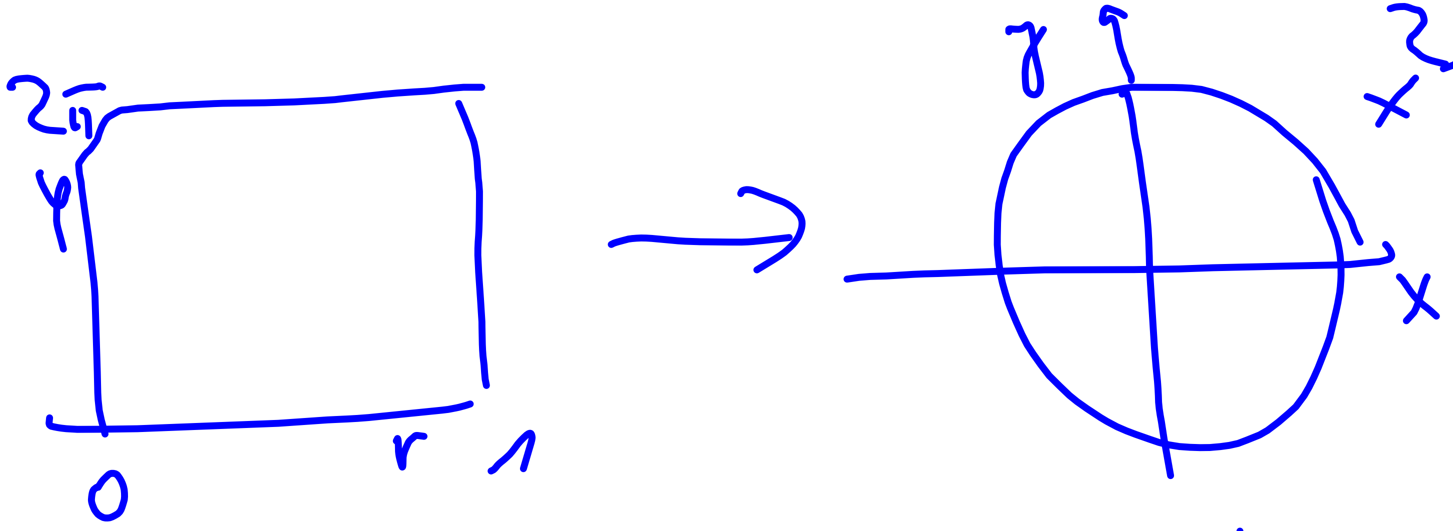
pol. souř.

$$[0,+\infty) \times [0,2\pi) \longrightarrow E_2$$



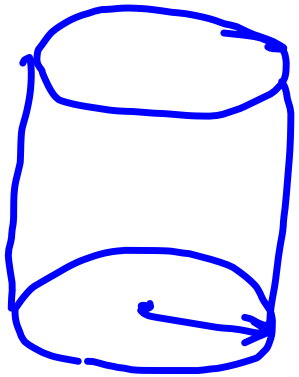
$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$x^2 + y^2 \le 1$

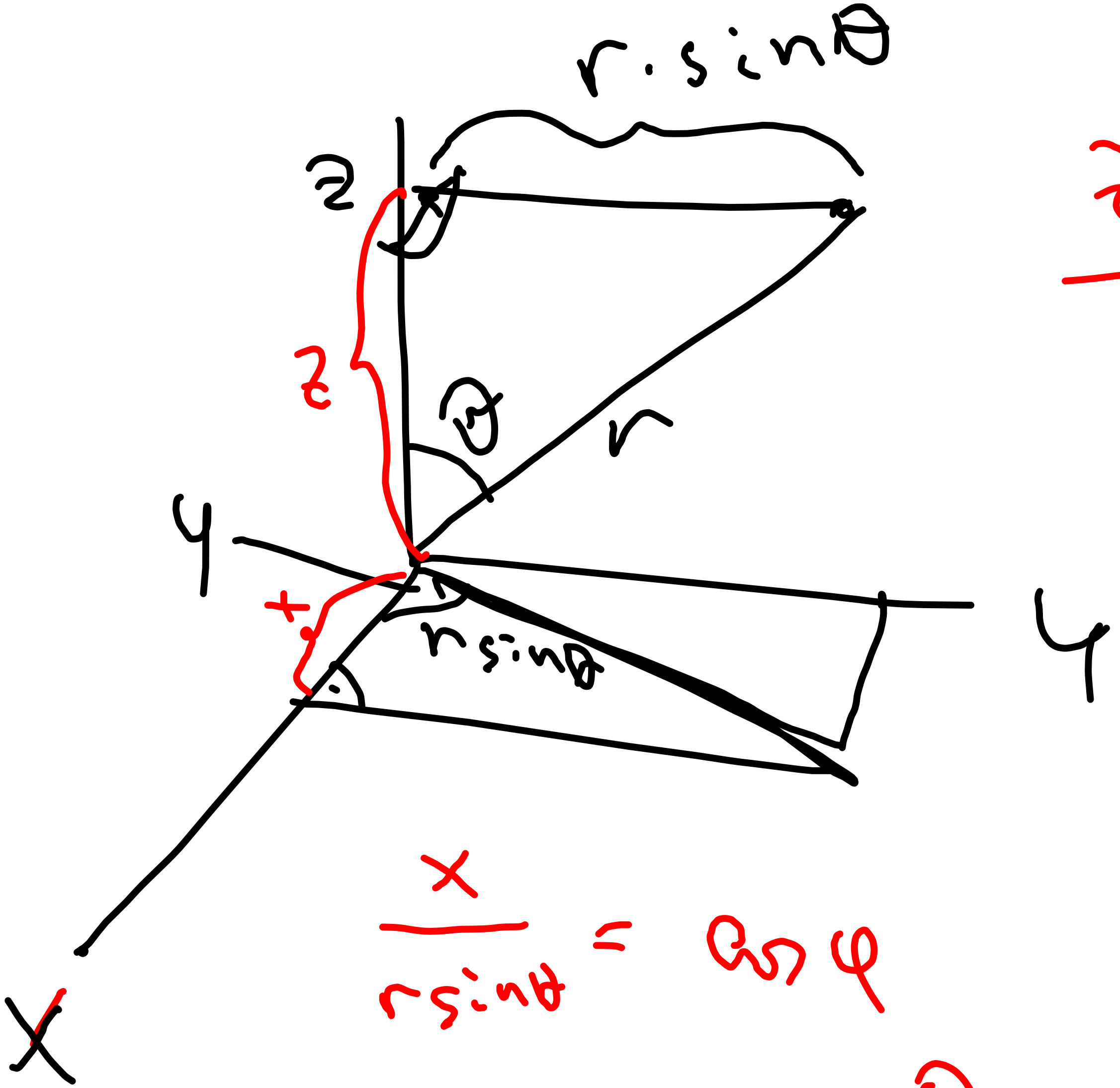


pol. souř.

kart. souř.



$$[r\cos\varphi, r\sin\varphi, z] \rightsquigarrow \underbrace{[x, y, z]}$$



$$\frac{z}{r} = \cos\theta$$

$$\frac{x}{r\sin\theta} = \cos\varphi$$

$$x = r \cdot \sin\theta \cos\varphi$$

$$r^2 \cdot \underbrace{\sin\theta}_{\geq 0} \geq 0 \Rightarrow |r^2 \sin\theta| = r^2 \sin\theta$$

$$\theta \in [0,\pi] \Rightarrow \sin\theta \in [0,1]$$

$$\int_{B(R)} 1 \, dx\, dy\, dz = \int_{\substack{B(R) \\ [0,R]\times[0,\pi]\times[0,2\pi]}} 1 \cdot r^2 \sin\theta \, dr\, d\theta\, d\varphi =$$

$$= \int_0^R dr \int_0^\pi r^2 \sin\theta \cdot 2\pi \, d\theta = \int_0^R 2\pi r^2 dr \int_0^\pi \sin\theta \, d\theta =$$

$$= \int_0^R 2\pi r^2 \left[-\cos\theta\right]_{\theta=0}^{\pi} dr = \int_0^R 4\pi r^2 dr = \underline{\underline{4\pi \cdot \frac{R^3}{3}}}$$

$$x^2+y^2+z^2-z=0$$

$$x^2+y^2+\left(z-\frac{1}{2}\right)^2=\frac{1}{4}=R^2$$

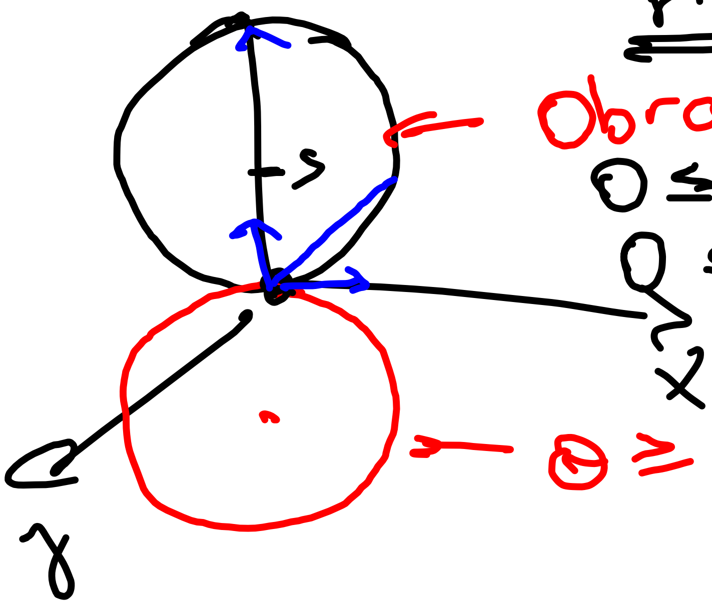
sféra o poloměru $R=\frac{1}{2}$, se středem v $[0,0,\frac{1}{2}]$.

$$r=\sqrt{x^2+y^2+z^2}$$

$$x^2+y^2+z^2\le z$$

$$r^2\le r\cdot\cos\theta$$

$$r\le\cos\theta$$



Obraz množiny:

$$0\le\varphi<2\pi$$

$$0\le\theta\le\frac{\pi}{2}$$

$$I = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} d\theta \int_0^{\cos\theta} r \cdot r^2 \sin\theta \, dr =$$

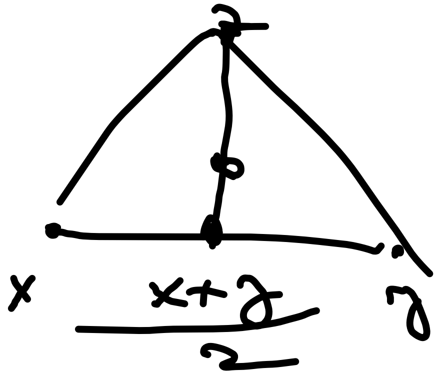
$$= \frac{1}{4} \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} \sin\theta \cos^4\theta \, d\theta =$$

$$\left(\cos^5\theta\right)' = 5 \cdot \cos^4\theta \cdot (-\sin\theta)$$

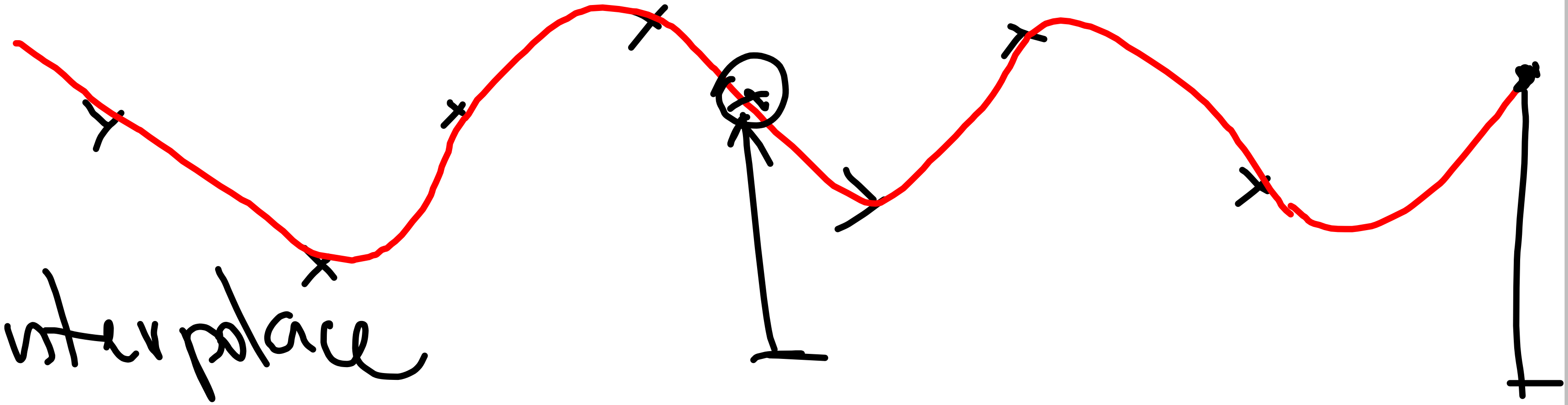
$$= \frac{1}{4} \int_0^{2\pi} d\varphi \left[-\frac{1}{5} \cos^5\theta \right]_{\theta=0}^{\frac{\pi}{2}} =$$

$$\left(\cos^5\frac{\pi}{2} - \cos^5 0\right)$$

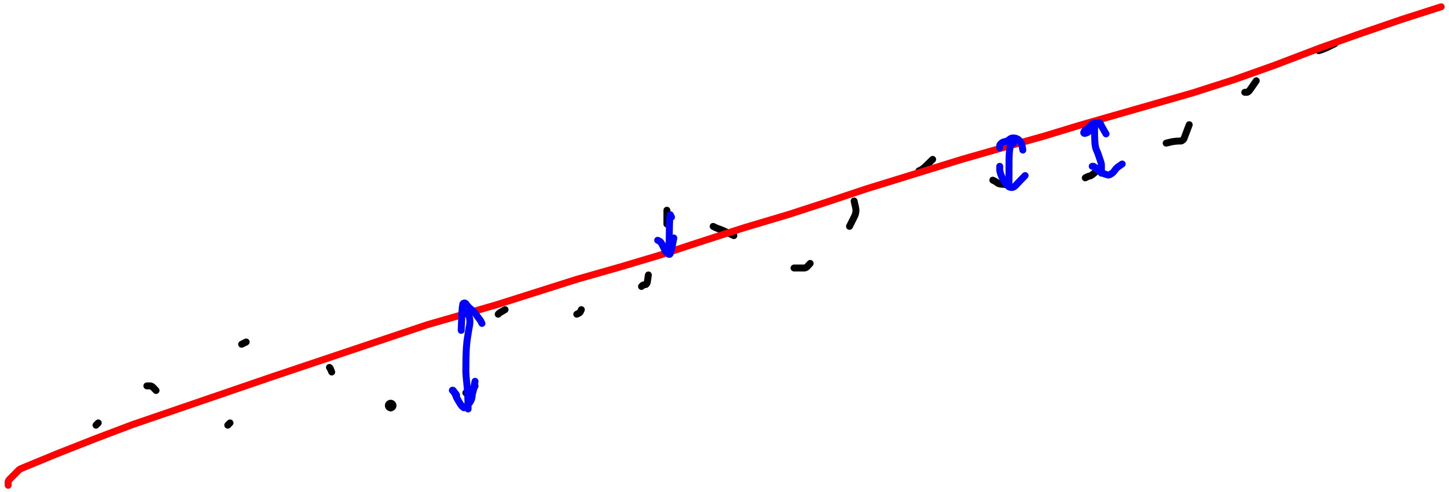
$$= \frac{1}{20} \int_0^{2\pi} -1 \, d\varphi = \frac{2\pi}{20} = \frac{\pi}{10}.$$



$$\frac{2}{3}\,\frac{x+y}{2} + \frac{1}{3} = \frac{x+y+z}{3}$$



interpolace



aproximace

$[x_0, y_0] \quad | \quad \cdots$

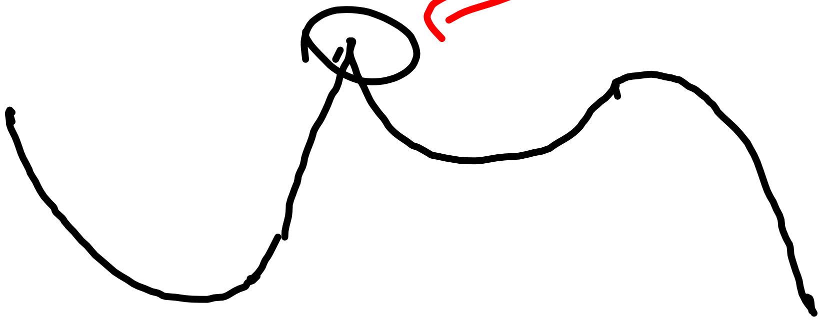
$y_0 = f(x_0)$

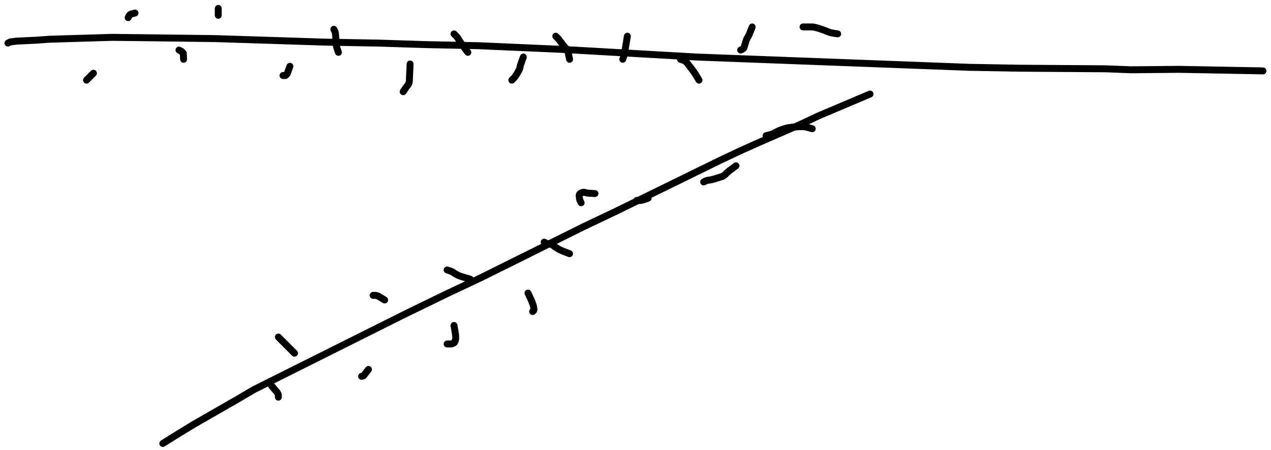
$$l_0(x_0) = \frac{\overline{\prod_{j \neq 0} (x - x_j)}}{\prod_{j \neq 0} (x_0 - x_j)} = 1$$

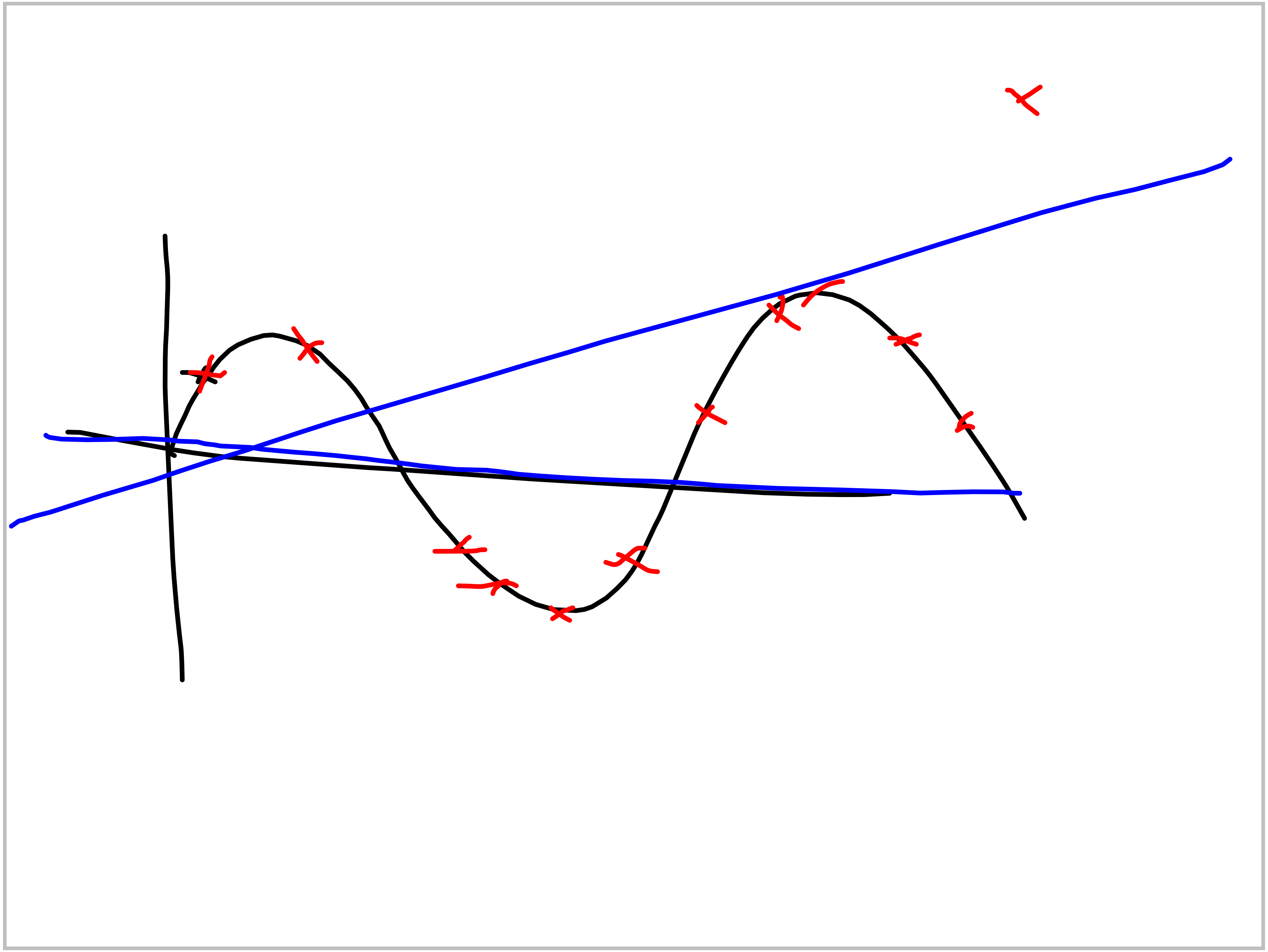
$l_0(x_0) = 1 \qquad l_0(x_i) = 0 \quad \text{pro } x_i \neq x_0$

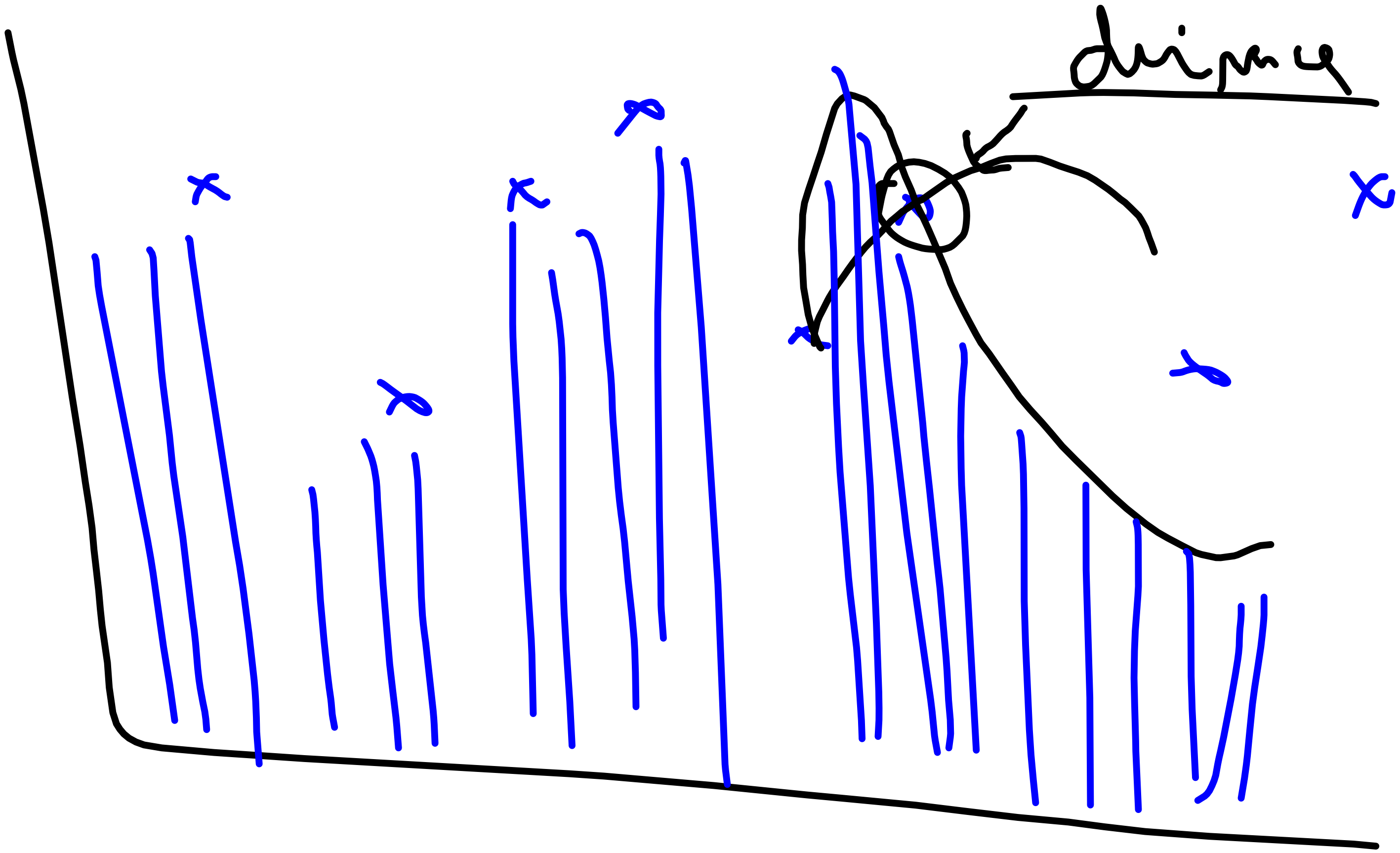
$$\boxed{f(x_i) = y_i}$$

není hladká

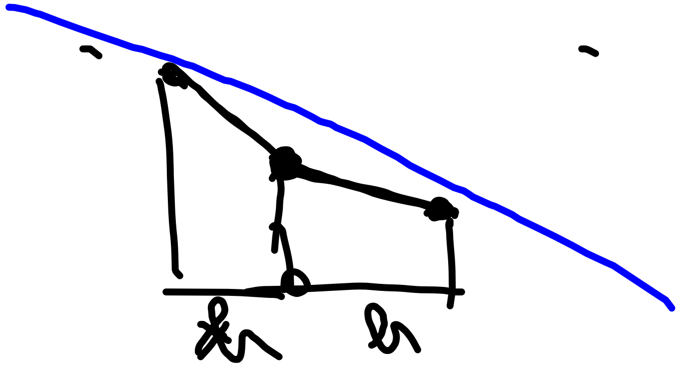






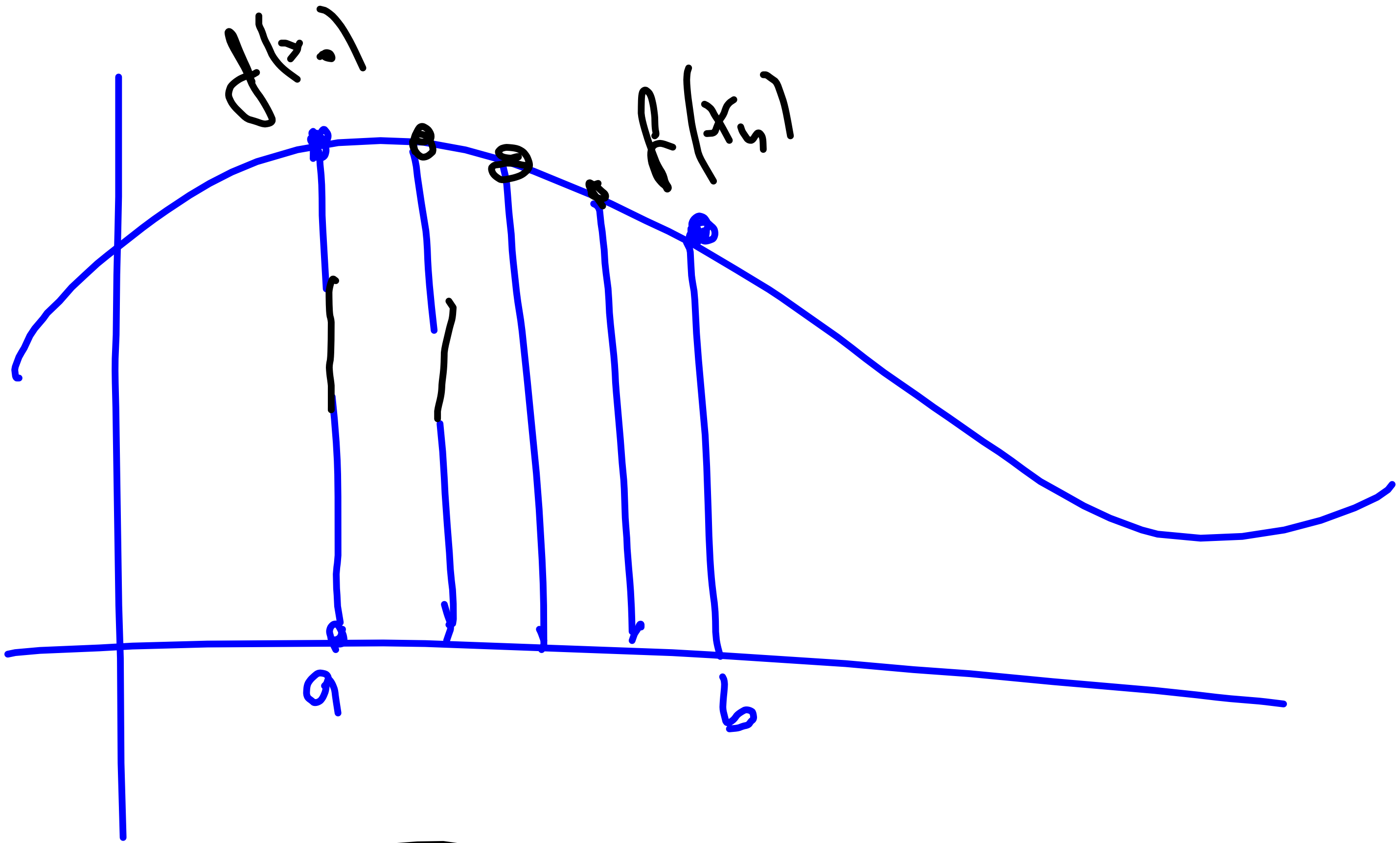


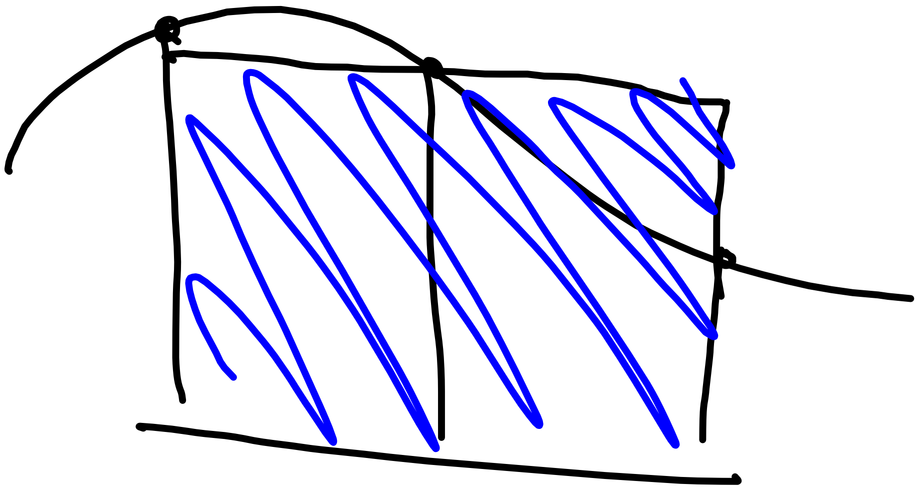
$$f'(x_0) = \lim_{\substack{x \to x_0 \\ h \to 0}} \frac{f(x+h) - f(x)}{h}$$



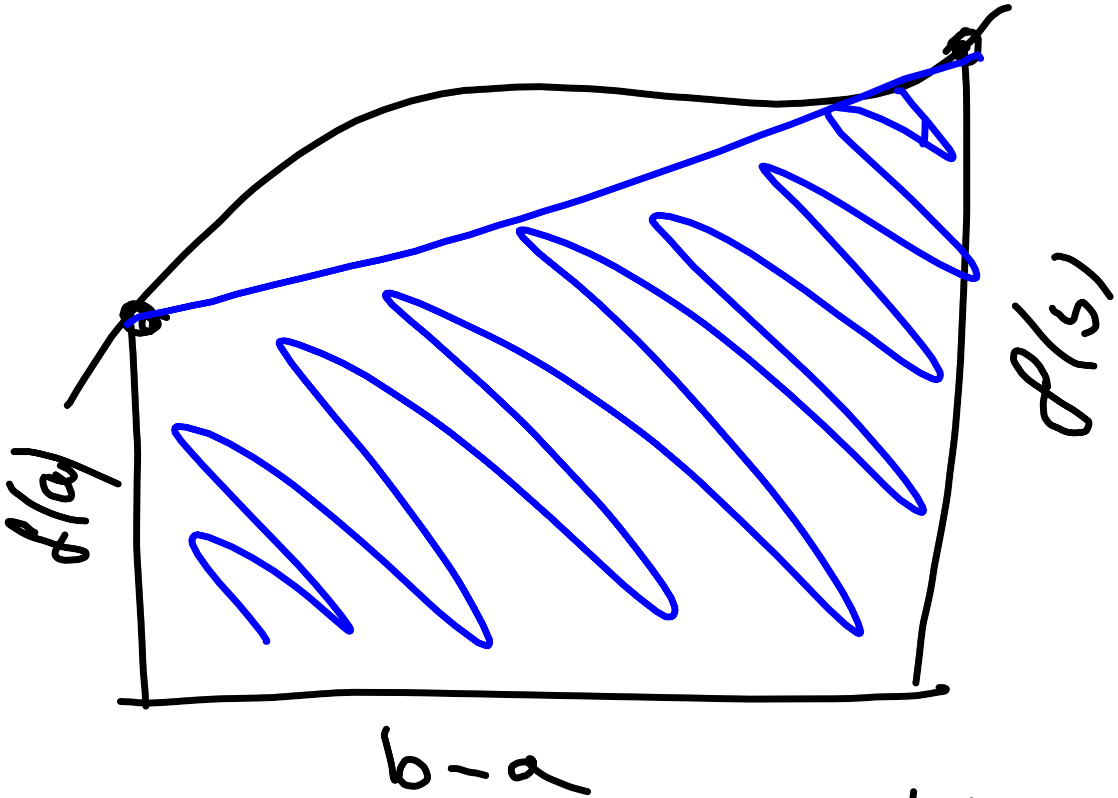
$$\int_0^3 e^{-\frac{x^2}{2}}$$

$$\int_0^\alpha e^{-x^2} = \mathrm{erf}(\alpha)$$

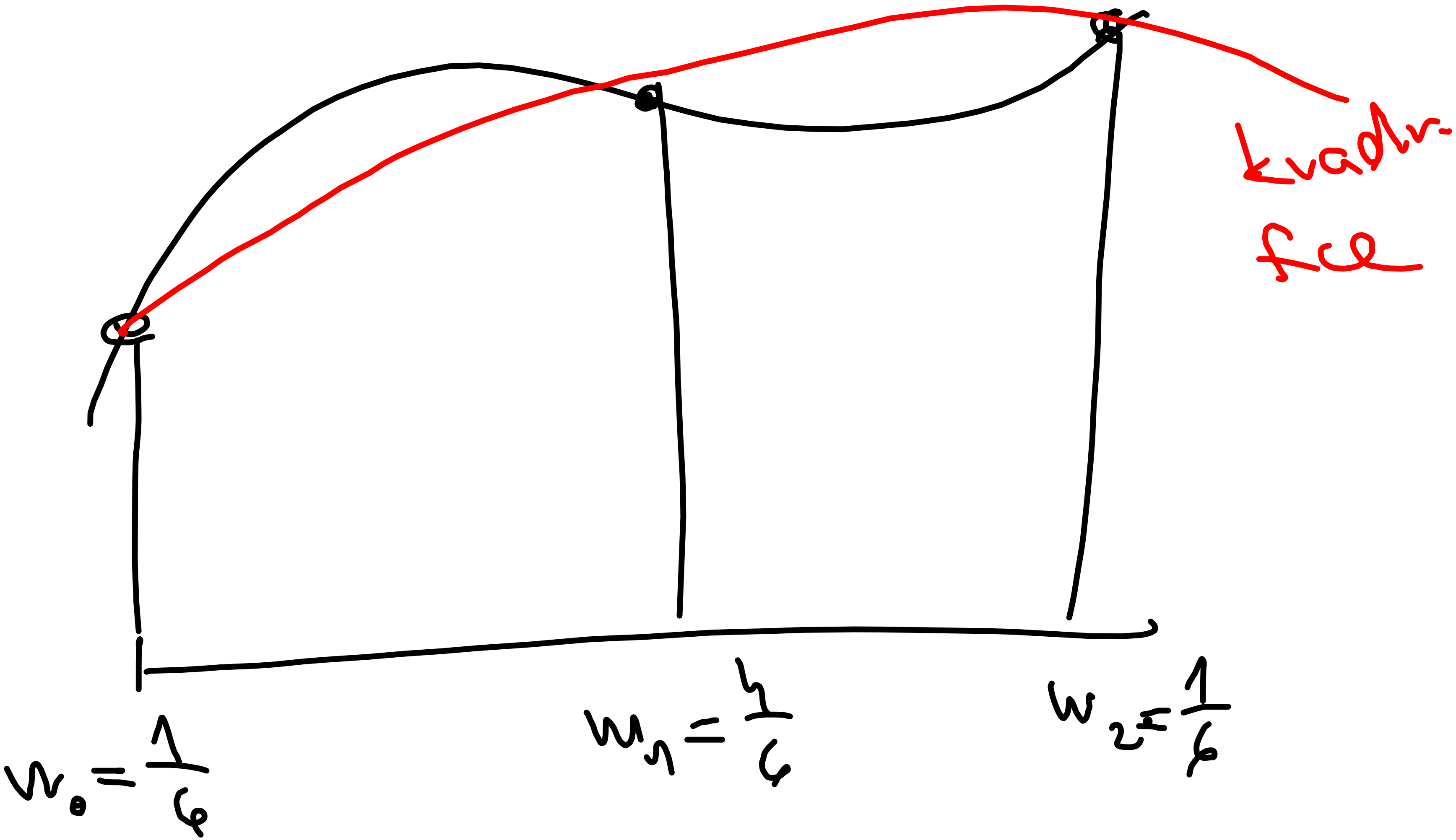




$(b-a)\, f\left(\frac{a+b}{2}\right)$



$$(b-a)\frac{f(b)+f(a)}{2}$$


kvadr. fce
$w_0 = \frac{1}{6}$
$w_1 = \frac{1}{6}$
$w_2 = \frac{1}{6}$

odvození Simpsonova pravidla:

$$\alpha \cdot f(a) + \beta \cdot f\left(\frac{a+b}{2}\right) + \gamma \cdot f(b)$$

pro jednoduchost předp. $\underline{a=0}$ $\underline{b=1}$

tvaru $\underline{\alpha} \cdot f(0) + \underline{\beta} \cdot f\left(\frac{1}{2}\right) + \underline{\gamma} \cdot f(1)$

neurčité koeficienty

$$\int_0^1 f(x)\,dx \approx \alpha \cdot f(0) + \beta \cdot f\left(\frac{1}{2}\right) + \gamma \cdot f(1)$$

musí platit $=$ pro $f(x)$ polynom st. ≤ 2

stačí najít α, β, γ tak aby = platila

pro $1, x, x^2$:

$$\int_0^1 1\,dx = 1 = \alpha \cdot f(0) + \beta \cdot f\left(\tfrac{1}{2}\right) + \gamma \cdot f(1) = \alpha + \beta + \gamma$$

$f(x) = 1$:

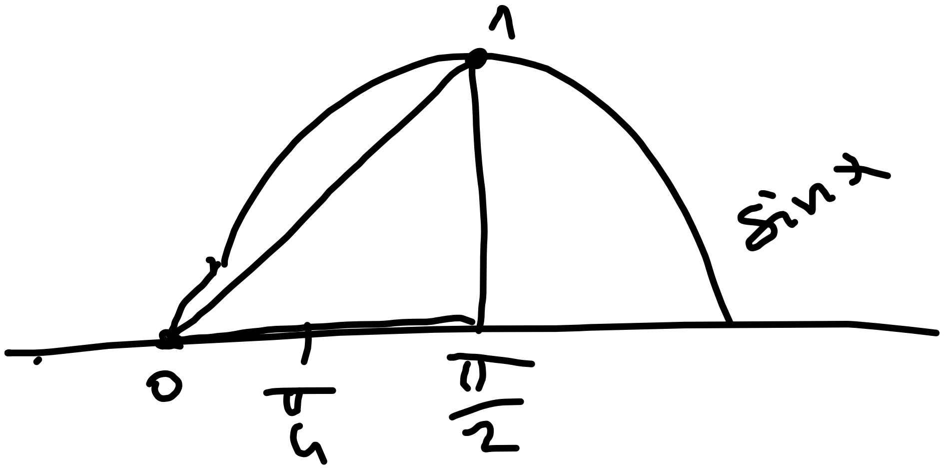
$$f(x) = x : \int_0^1 x\,dx = \frac{1}{2} = \frac{\beta}{2} + \gamma$$

$$f(x) = x^2 : \int_0^1 x^2\,dx = \frac{1}{3} = \frac{\beta}{4} + \gamma$$

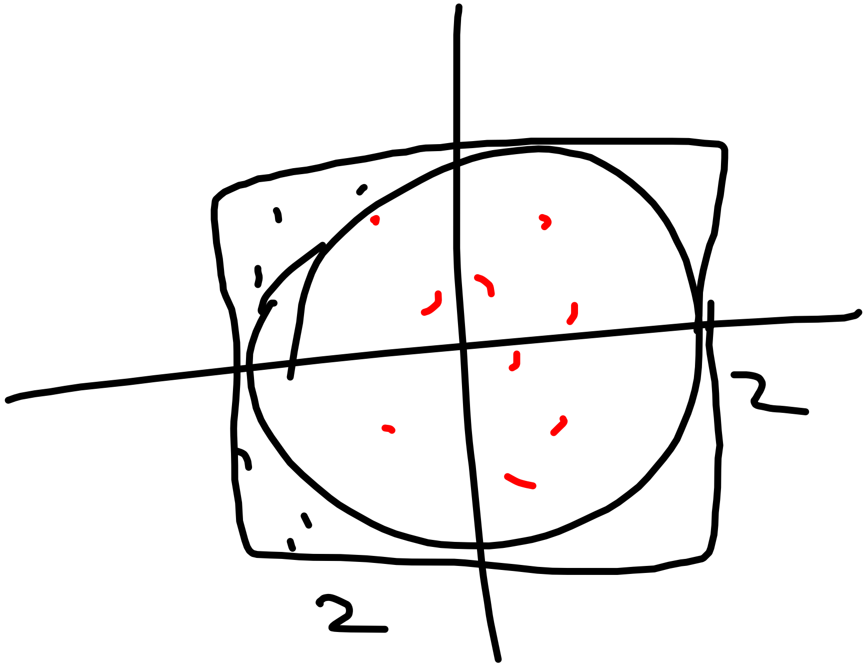
$\beta = \frac{2}{3}$

$\alpha = \gamma = \frac{1}{6}$

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = \left[-\cos x\right]_0^{\frac{\pi}{2}} = 1$$



$$\frac{\pi}{2}\left(\frac{1}{6}\sin 0 + \frac{2}{3}\sin\left(\frac{\pi}{4}\right) + \frac{1}{6}\sin\left(\frac{\pi}{2}\right)\right)$$



očekávaná
čet. hod. $\frac{\pi^2}{4}$