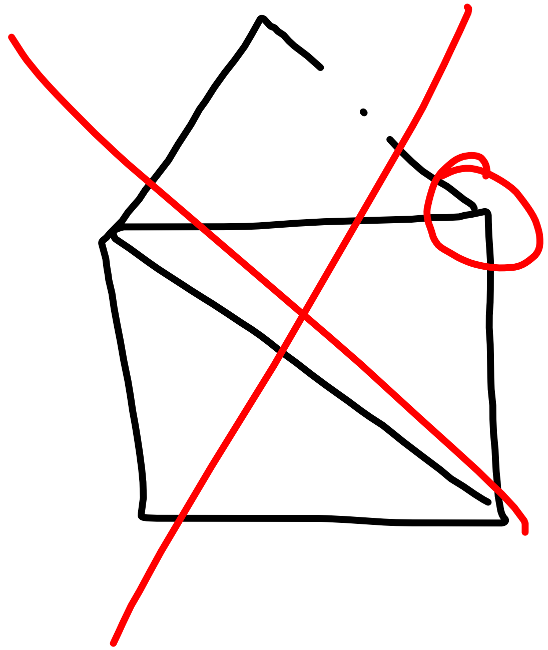
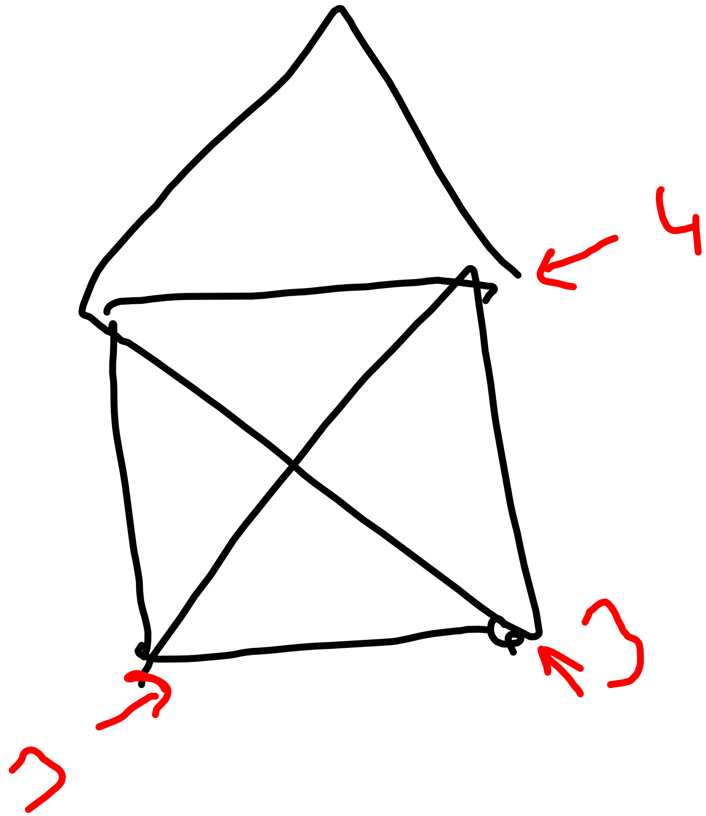
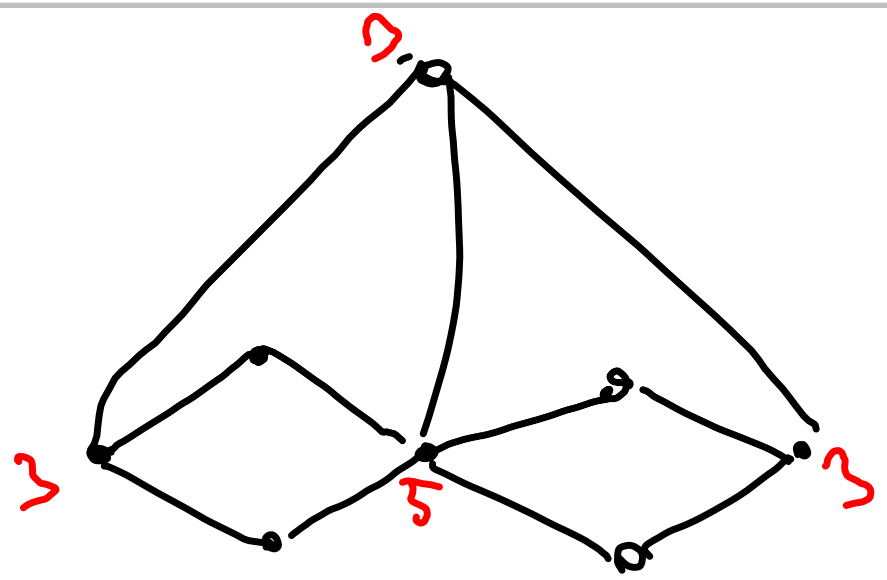
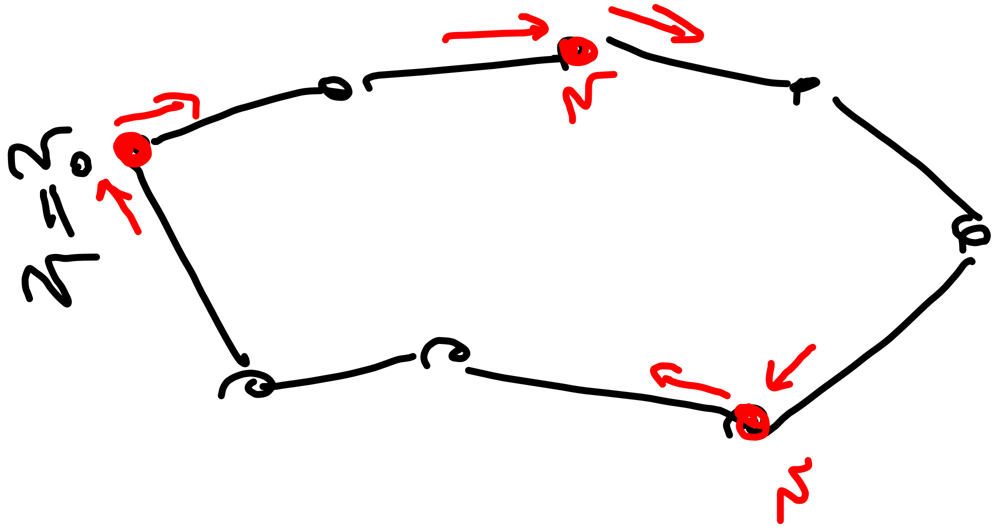



4
3
2



"⇒" souvislost zř.



⇒ deg(v) je sudý

$\deg_+$ $\deg_-$

„⟸" souvislý graf

$\forall v \in V: 2 \mid \deg(v)$

nejdelší tah v G

$P: v_0 \; v_1 \; v_2 \; \dots \; v_k$

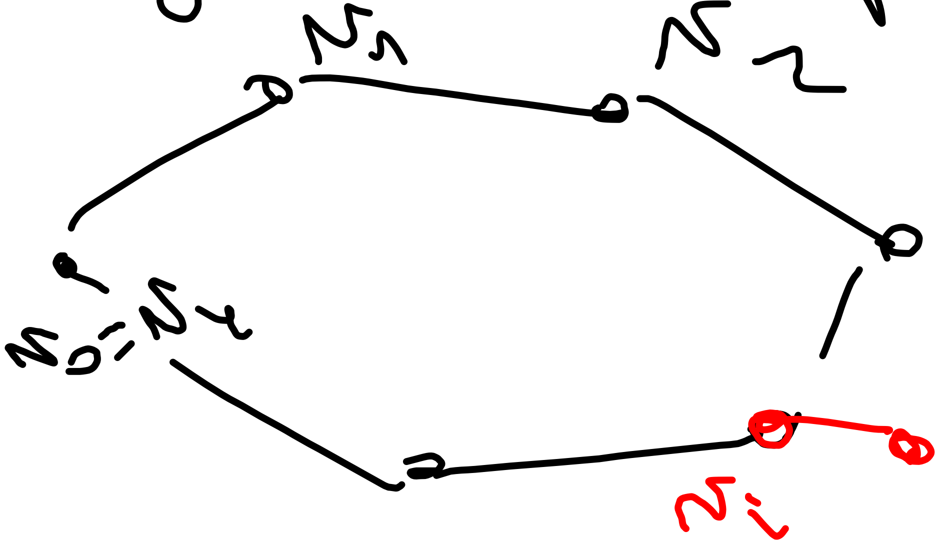
v

předpokládejme uzavřený $\Rightarrow$

ex. ~~nějaká hrana~~ mimo P

1. $v_0 = v_k$
(ne $\Rightarrow$ ex. hrana incidentní s v_0)

Proto je P uzavřený



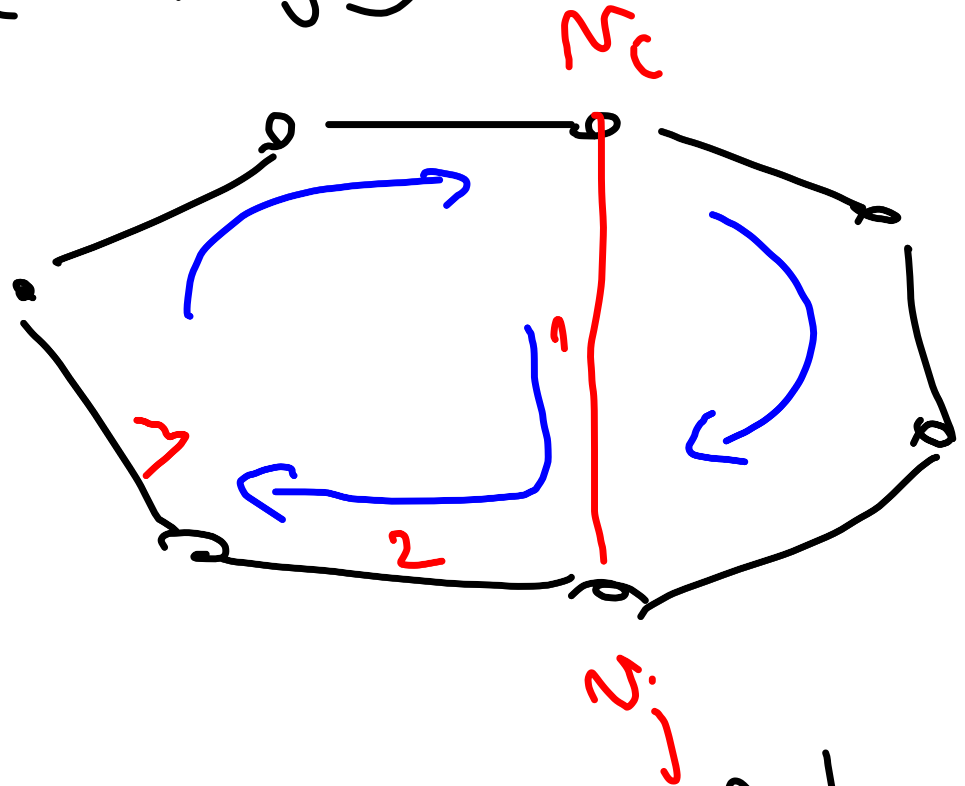
1. ukážeme, že všechny vrcholy V leží na tahu P: kdyby ne, tak najdeme hranu $\{v_i, v\}$, $v_i \in P$, $v \notin P$ $\Rightarrow$ prodloužíme

tah $v, v_i, v_{i+1}, \ldots, v_k = v_0, \ldots, v_i$

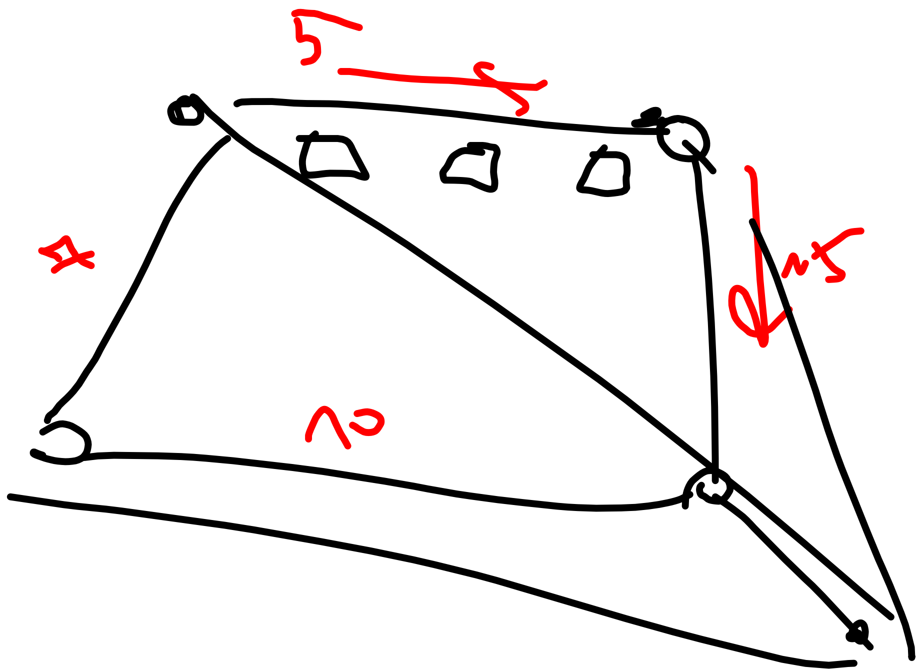
2. tah prochází všemi vrcholy.

Kdyby ex. $e \in E \setminus P$,

$e = \{v_i, v_j\}$



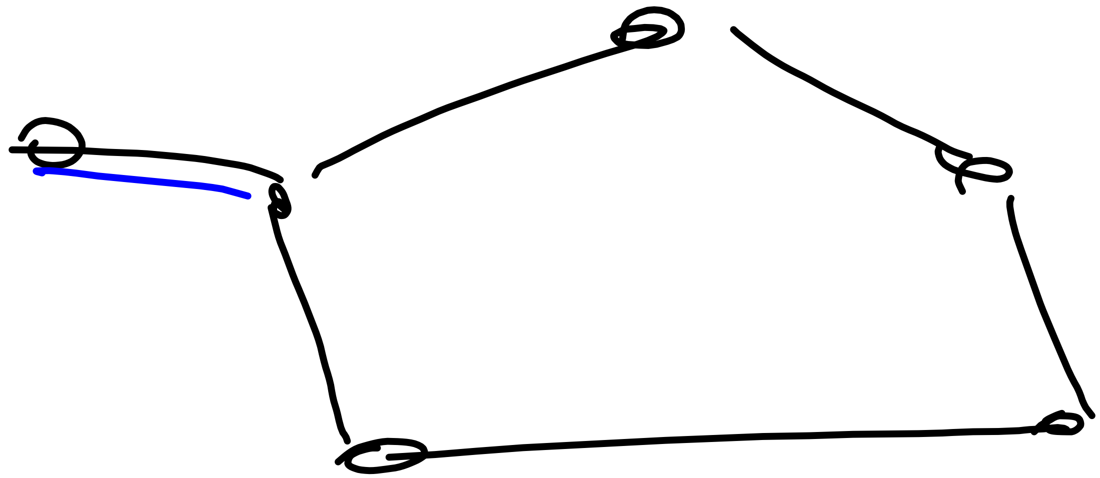
SPOR s maximalitou P

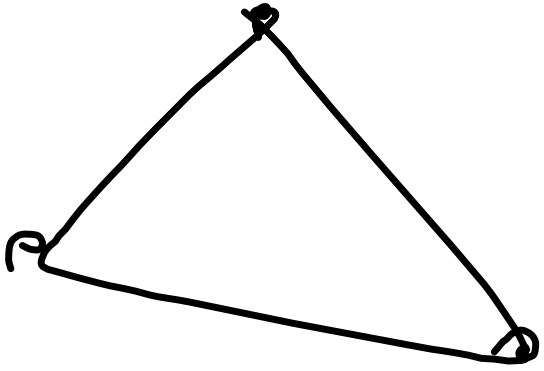


5
4
10
1,5

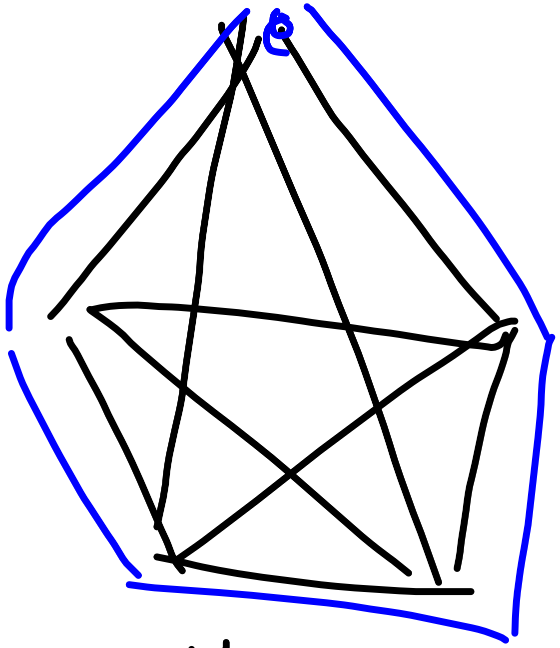
Počet vrcholů lichého stupně je sudý

$$\sum_{v \in V} \deg(v) = 2|E|$$



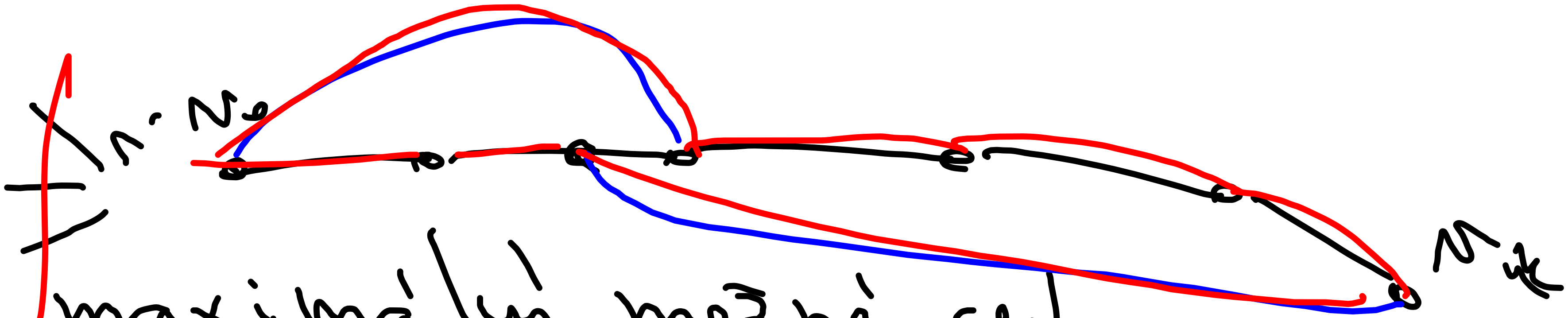


K_3



K_5

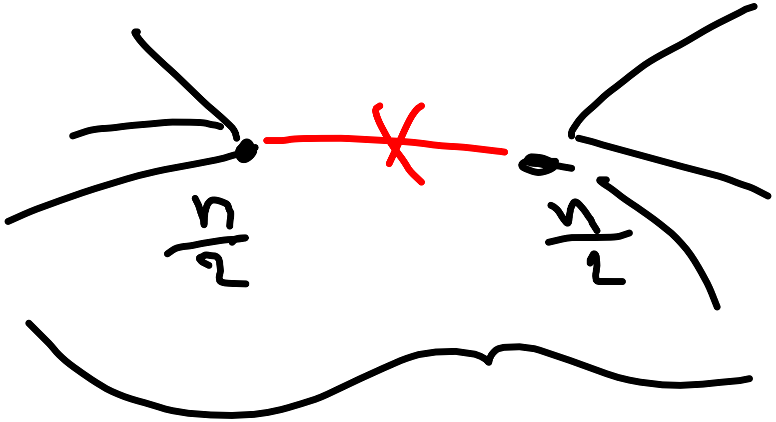
? Petersenův



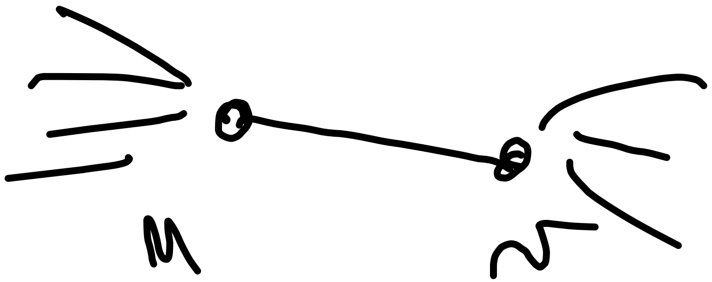
maximální možná cesta v G

lze ukázat ex. $\boxed{\{v_0, v_{i+1}\}, \{v_i, v_k\}}$

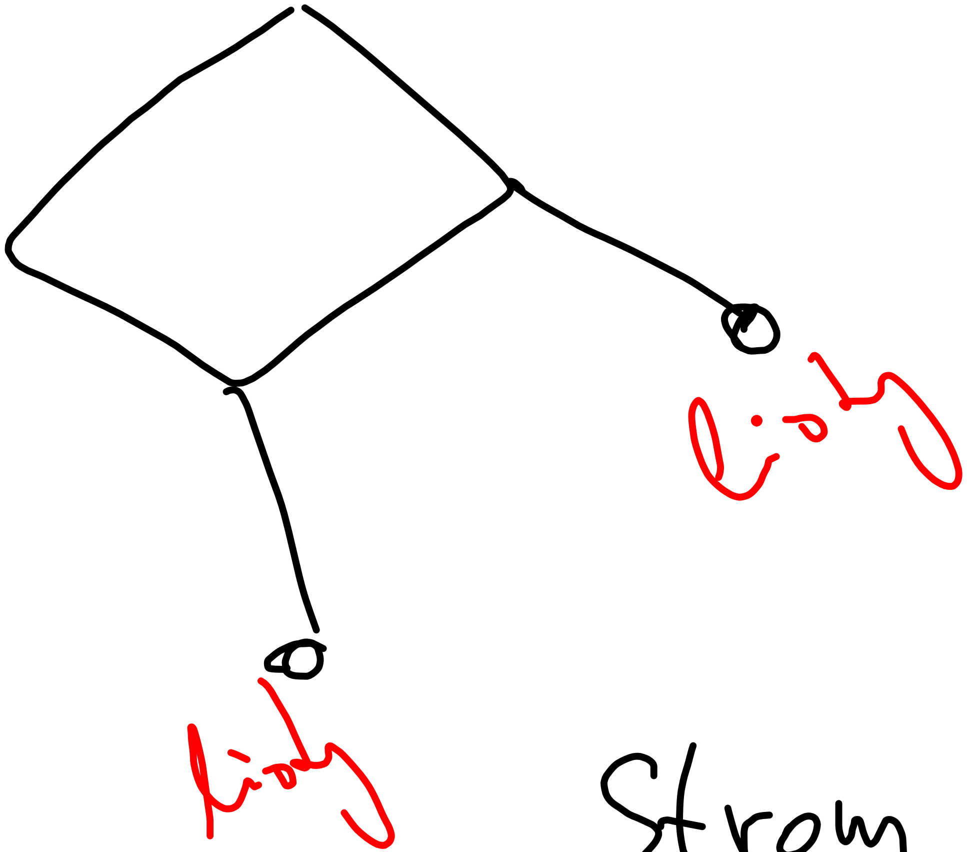
Ukáže se, že $v_0, v_{i+1}, \ldots, v_k, v_i, \ldots, v_0$ je ham. kružnice



$\sum \geq 4$



$E \cup \{\{u, v\}\}$

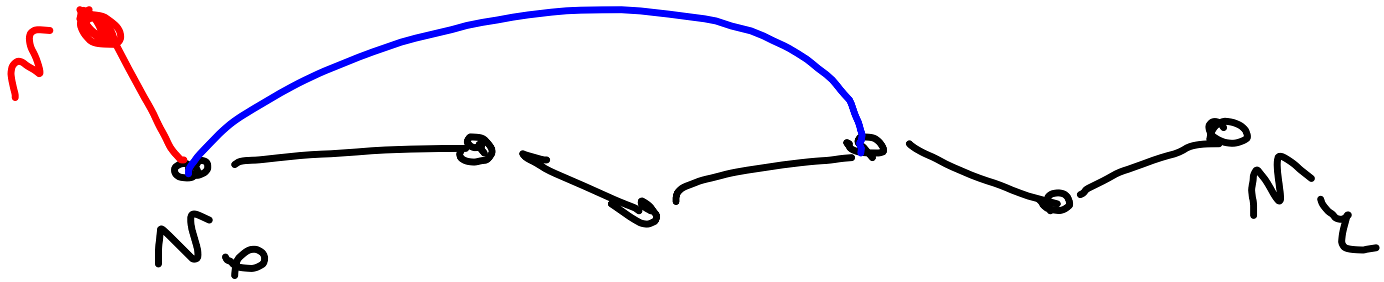


Strom je souvislý

lu

Každý strom s $n \geq 2$ vrcholy obs. ≥ 2 listy

Dk: buď P maximální cesta ve stromu T



v_0, v_k jsou listy

1. $v_0 \neq v_k$ (když $v_0 = v_k \Rightarrow P$ je kružnice)
2. $\deg(v_0) > 1 \Rightarrow$ hrana
   - mimo $P \Rightarrow$ SPOR s MAX
   - $\{v_0, v_i\} \Rightarrow$ kružnice

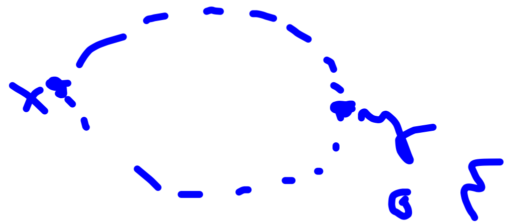
G je strom $\iff$ $G \setminus v$ je strom
(v list)

Dk: G neobs. kružnice $\implies$ $G \setminus v$ neobs. kruž.

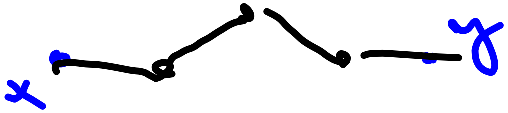
$G \setminus v$ je souvislý $\implies$ G je souvislý
cesta mezi v a x

$G \setminus v$ neobs. kružnici $\implies$ G neobs. kružnice

kružnice v G $\Longleftarrow$



G souvislý $\Rightarrow$ $G \setminus v$ souvislý

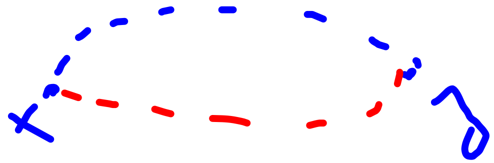


$\Rightarrow$ cesta i v $G \setminus v$

$\cdot v$

Dk charakterizační věty:

(i) $\Rightarrow$ (ii) souvislost $\Rightarrow$ aspoň 1 cesta

kdyby ≥ 2 cesty 

$\Rightarrow$ kružnice

(i) $\Rightarrow$ (iii) indukcí: v list $\Rightarrow G - v$ je strom

$\overset{IP}{\Rightarrow}$ nejmenším hran z $G - v$ zdratlme

souvislost, nejmenším hran

z listu v tahu!

(i) $\Rightarrow$ (iv) přidáme hranu $\{x, y\}$, mezi

x a y ex. cesta 

(i) ⇒ (v) indukcí

$G' = (V', E') = G \setminus v$ [v list]

Podle IP: $|V'| = |E'| + 1$

$|V| = |V'| + 1 \quad |E| = |E'| + 1$ ✓

(ii) ⇒ (i) souv. gr. ; kdyby kružnice ⇒ 2 cesty

(iii) ⇒ (i) zřejmé

(iv) ⇒ (i) přidáním hrany vznikne strom

⇒ G je souvislý gr.

(v) ⇒ (i) indukcí přes $|V| = n$. $n = 1$ •

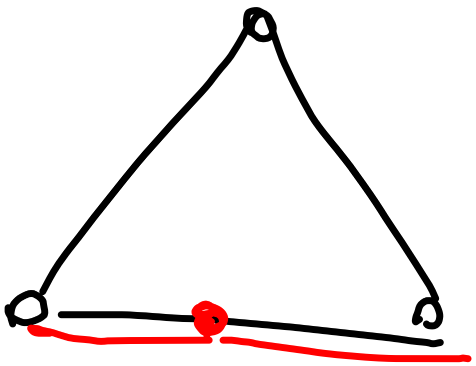
$1 = 0 + 1$ ✓

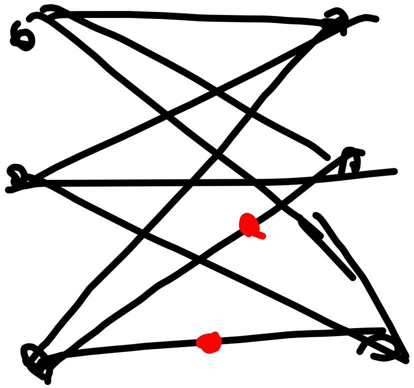
$G' = G \setminus v$

Předp. , že $|V| = |E| + 1$

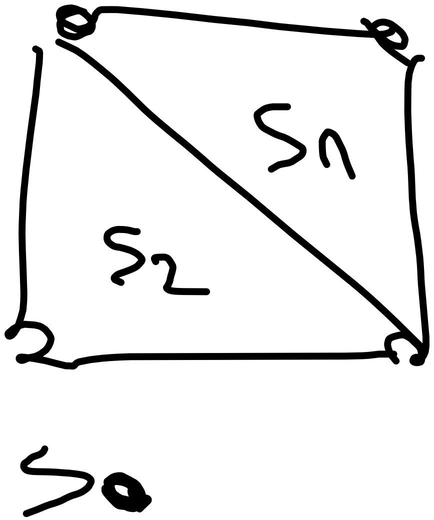
zřejmě i $|V'| = |E'| + 1 \overset{IP}{\Longrightarrow}$

$\Longrightarrow G'$ je strom $\overset{\text{Lemma}}{\Longrightarrow} G = G' \cup \{v\}$ je strom. $\square$

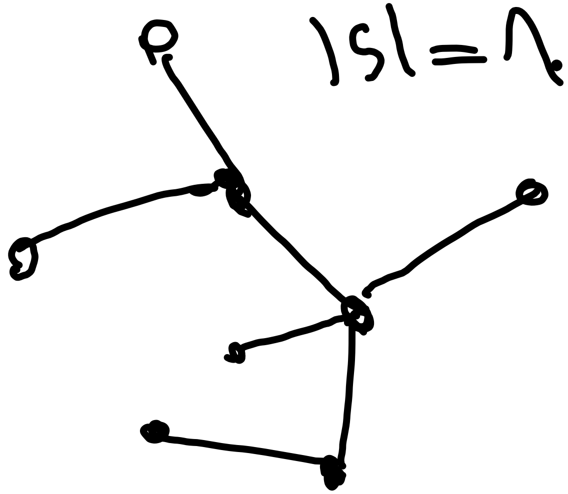




$|V| = |E| + 1$



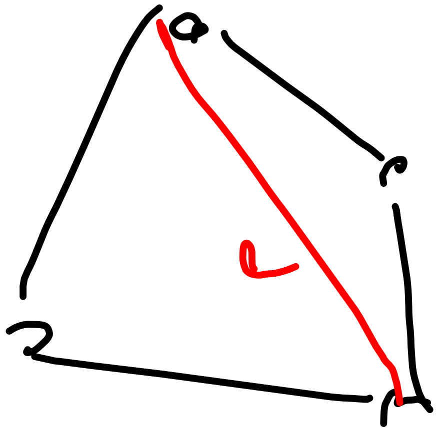
$|S| = 1.$



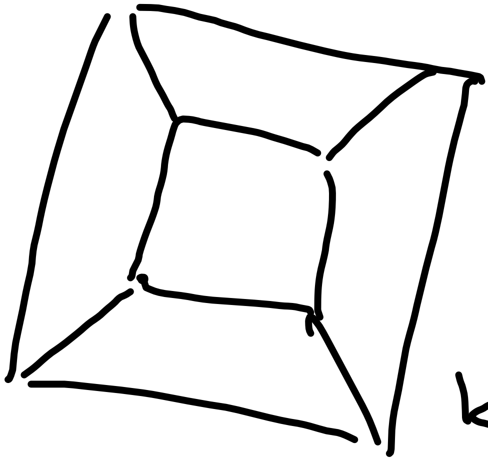
Dk: (Eulerova vztahu) : indukcí přes $|E|$

I. $|E| = 0$ $\quad |V| = 1$, $|S| = 1$

II. $G = (V, E, S)$ souvislý rovinný

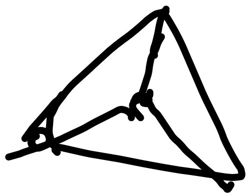


odebráním hrany se počet stěn sníží o 1! (topologie)

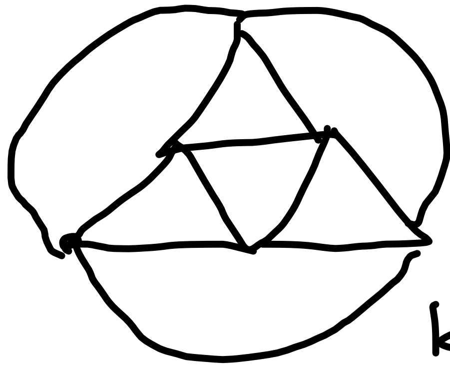


je dokonce 3-souvislý

$k=4$
$d=3$



$k=3$
$d=3$



$k=3$
$d=4$

k ... počet vrcholů u 1 stěny

d ... počet stěn u vrcholu

$$dn = 2e$$

$$\boxed{\begin{array}{l} k \geq 3 \\ d \geq 3 \end{array}}$$

$$2e = k \cdot s$$

$$2 = n - e + s = \frac{2e}{d} - e + \frac{2e}{k}$$

$$\Rightarrow \frac{2e}{d} + \frac{2e}{k} = 2 + e \quad \Big| : 2e$$

$$\frac{1}{d} + \frac{1}{k} = \frac{1}{e} + \frac{1}{2}$$

kdyby $d \geq 6 \Rightarrow \frac{1}{d} + \frac{1}{k} \leq \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$

SPOR

Proto $3 \leq k, d < 6$

$\frac{1}{3} + \frac{1}{3}$

$\frac{1}{3} + \frac{1}{4}$

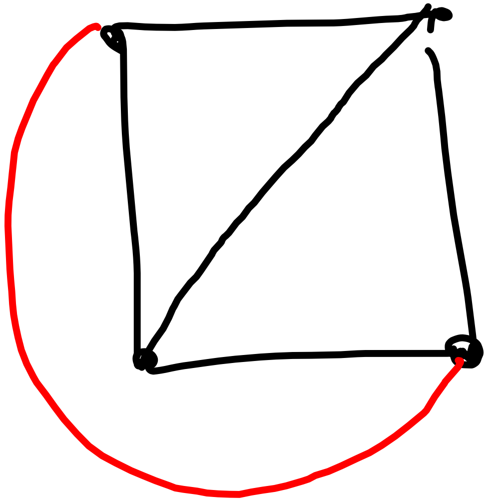
$\frac{1}{4} + \frac{1}{3}$

$\frac{1}{5} + \frac{1}{3}$

$\frac{1}{3} + \frac{1}{5}$

$\cancel{\frac{1}{4} + \frac{1}{4}} = \frac{1}{2} < \frac{1}{2} + \frac{1}{e}$

$\cancel{\frac{1}{4} + \frac{1}{5}}$



není max.

je max.

$$3|S| = 2|E|$$

$$2 = |V| - |E| + |S| = |V| - |E| + \frac{2}{3}|E|$$

$$= |V| - \frac{1}{3}|E|$$

$$6 = 3|V| - |E| \Rightarrow \boxed{|E| = 3|V| - 6}$$

K_5 $\quad |V| = 5$

$|E| = \binom{5}{2} = 10$

rovinný $\rightarrow |E| \leq 3 \cdot 5 - 6 = 9 < 10$

|