$$P(x) = x^4 + 3x^3 - 4x$$

$x_1 = 0$ ✓

$$P(x) = (x-0)\cdot(1x^3 + 3x^2 - 4)$$

± 1 jmen.

$1, 2, 4$ čit.

PODEZŘELÉ: $\pm 1, \pm 2, \pm 4$

	1	3	0	−4
2	1	$2\cdot1+3=$ 5	$2\cdot5+0=$ 10	$16 \neq 0$ ✗
−2	1	1	−2	0 ✓

$$P_{(x)} = (x+2)\cdot(x^2+x-2)\cdot x$$

atd

$$\underline{\underline{P_{(x)} = (x+2)^2\cdot(x-1)\cdot x}}$$

Při. 1

$$l_0 = \frac{(x-x_1)\cdot(x-x_2)\cdot(x-x_3)}{(x_0-x_1)\cdot(x_0-x_2)\cdot(x_0-x_3)} =$$

$$= \frac{(x-1)\cdot(x-2)\cdot(x-5)}{(0-1)\cdot(0-2)\cdot(0-5)} = \frac{x^3-8x^2+17x-10}{-10}$$

$$l_1 = \frac{(x-0)\cdot(x-2)\cdot(x-5)}{(1-0)\cdot(1-2)\cdot(1-5)} = \frac{x^3-7x^2+10x}{4}$$

$$l_2 = \frac{x^3-6x^2+5x}{-6}, \quad l_3 = \frac{x^3-3x^2+2x}{60}$$

$$P(x) = f(x_0)\cdot l_0 + f(x_1)\cdot l_1 +$$

$$+ f(x_2)\cdot l_2 + f(x_3)\cdot l_3 =$$

$$= \overset{1}{\cancel{2}} \cdot \frac{x^3 - 8x^2 + 17x - 10}{\cancel{-10}\ -5} + 3\cdot l_1 +$$

$$+ 12\cdot l_2 + 147\cdot l_3 =$$

$$= x^3 + x^2 - x + 2$$

Př. 3

$$P_{(x)} = a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

$$P'_{(x)} = 5 \cdot a_5 \cdot x^4 + 4 \cdot a_4 \cdot x^3 + 3 \cdot a_3 x^2 +$$
$$+ 2 a_2 x + a_1$$

$P_{(x)}$

$$P_{(0)} = 2 = a_0$$

$$P_{(1)} = 5 = a_5 + a_4 + a_3 + a_2 + a_1 + a_0$$

$$P_{(4)} = 1 = 1024 a_5 + 256 a_4 + 64 a_3 +$$
$$+ 16 a_2 + 4 a_1 + a_0$$

$$P'(0) = 1 = a_1$$

$$P'(1) = -1 = 5 \cdot a_5 + 4a_4 + 3a_3 + 2a_2 + a_1$$

$$P'(4) = 2 = 1280a_5 + 256a_4 + 48a_3 + 8a_2 + a_1$$

$$
\begin{array}{cccc}
a_5 & a_4 & a_3 & a_2 \\
\end{array}
$$

$$
\begin{pmatrix}
1 & 1 & 1 & 1 \\
1024 & 256 & 64 & 16 \\
5 & 4 & 3 & 2 \\
1280 & 256 & 48 & 8
\end{pmatrix}
\cdot
\begin{pmatrix}
a_5 \\ a_4 \\ a_3 \\ a_2
\end{pmatrix}
=
\begin{pmatrix}
2 \\ -5 \\ -2 \\ 1
\end{pmatrix}
$$

$$A \cdot x = b$$

$$A^{-1} \cdot A \cdot x = A^{-1} \cdot b$$

$$x = A^{-1} \cdot b$$

$$\Rightarrow P(x) = -\frac{407}{864}x^5 + \frac{329}{72}x^4 - \frac{3953}{288}x^3 + \frac{5023}{432}x^2 + x + 2$$

Př. 4/ $f(x) = \dfrac{1}{1+x^2}$; $I = [0, 3]$

$x_0 = 0, \quad x_1 = 1, \quad x_2 = 3$

PŘIR. : $S''(x_0) = 0 = S''(x_2)$

KUB. : POLY 3. st.

---

x	0	1	3
$f(x)$	1	$\frac{1}{2}$	$\frac{1}{10}$

$$S_0(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0 \quad [0,1]$$

$$S_1(x) = b_3 x^3 + b_2 x^2 + b_1 x + b_0 \quad [1,3]$$

---

① $S(x_j) = f(x_j)$:

$$S_0(0) = 1, \quad \underbrace{S_0(1) = \frac{1}{2}, \quad S_1(1) = \frac{1}{2}}, \quad S_1(3) = \frac{1}{10}$$

② $\cancel{S_j(x_{j+1}) = S_{j+1}(x_{j+1})}$:

$$\cancel{\underbrace{S_0(1) = S_1(1)}}$$

3.
$$S_j'(x_{j+1}) = S_{j+1}'(x_{j+1})$$

$$S_0'(1) = S_1'(1)$$

$$S_j''(x_{j+1}) = S_{j+1}''(x_{j+1})$$

$$S_0''(1) = S_1''(1)$$

4.
$$S_0''(0) = 0 \quad , \quad S_1''(3) = 0$$

$$(1)\quad \begin{cases} S_0(0) = a_0 = 1 \\ S_0(1) = a_3 + a_2 + a_1 + a_0 = \dfrac{1}{2} \\ S_1(1) = b_3 + b_2 + b_1 + b_0 = \dfrac{1}{2} \\ S_1(3) = 27 b_3 + 9 b_2 + 3 b_1 + b_0 = \dfrac{1}{10} \end{cases}$$

$$S_0'(1) = S_1'(1) \Rightarrow 3a_3 + 2a_2 + a_1 =$$
$$= 3b_3 + 2b_2 + b_1$$

$$S_0''(1) = S_1''(1) \Rightarrow 6a_3 + 2a_2 = 6b_3 + 2b_2$$

$$S_0''(0) = 0 \Rightarrow 2a_2 = 0 \quad \| \quad S_1''(3) = 0 \Rightarrow 18 b_3 + 2b_2 = 0$$

6 RC o 6 nezn.

Řeš.:

$$S_0(x) = 1 - \frac{11}{20}x + \frac{1}{20}x^3$$

$$S_1(x) = \frac{43}{40} - \frac{31}{40}x + \frac{9}{40}x^2 - \frac{1}{40}x^3$$

Př. 5 (i)

$$\frac{2x^5+5x^3-x^2+2x-1}{x^6+2x^4+x^2} = R(x)$$

$$x^6+2x^4+x^2 = x^2\cdot(x^4+2x^2+1) =$$

$$= x^2\cdot(\underbrace{x^2+1}) ^2$$

$$D = 1-4 = -3 < 0$$

$$R(x) = \frac{A}{x^2} + \frac{B}{x} + \frac{Cx+D}{(x^2+1)^2} + \frac{Ex+F}{(x^2+1)}$$

$$\cdot(x^6+2x^4+x^2)$$

$$2x^5+5x^3-x^2+2x-1=$$

$$=A\cdot(x^2+1)^2+B\cdot x\cdot(x^2+1)^2+(Cx+D)\cdot x^2+$$

$$+(Ex+F)\cdot x^2\cdot(x^2+1)$$

$x^0:\quad -1=A$	$x^3:\quad 5=2B+C+E$
$x^1:\quad 2=B$	$x^4:\quad 0=A+F$
$x^2:\quad -1=2A+D+F$	$x^5:\quad 2=B+E$

4RCE O 4 NEZN.

$$\Rightarrow \quad \frac{-1}{x^2} + \frac{2}{x} + \frac{x}{(x^2+1)^2} + \frac{1}{x^2+1}$$

VOLBA x

$x = 0 \Rightarrow \quad -1 = A \cdot 1 \Rightarrow A = -1$

$x = i \Rightarrow LS = 2i^5 + 5i^3 - i^2 + 2i - 1 =$

$= \underbrace{2i - 5i + 1 + 2i - 1} = -i$

$i^5 = i^2 \cdot i^2 \cdot i = (-1) \cdot (-1) \cdot i = i$

$$0 - i = (Ci + D) \cdot (-1)$$

$$0 - i = -D - Ci$$

$\mathrm{Re}: \quad 0 = -D \quad \Rightarrow \quad D = 0$

$\mathrm{Im}: \quad -1 = -C \quad \Rightarrow \quad C = 1$

A, C, D dosadíme a ŘEŠÍME

3 RCE O 3 NEZN....

(ii)

$$(2x^5 - 5x^4 + 5x^3 - 3x^2 + 10x - 3) : (x^4 - x^3 - x + 1) = 2x - 3$$

$$-(2x^5 - 2x^4 - 2x^2 + 2x)$$

$$0 \quad -3x^4 + 5x^3 - x^2 + 8x - 3$$

$$-(-3x^4 + 3x^3 + 3x - 3)$$

$$0 \quad \underbrace{+2x^3 - x^2 + 5x}_{ZB.}$$

$$\frac{2x^5 - 5x^4 + \ldots}{x^4 - x^3 - x + 1} = \boxed{2x - 3 + \frac{2x^3 - x^2 + 5}{x^4 - x^3 - x + 1}}$$