1.a)

$$\left(\sin\left[\ln\left(x^3+2x\right)\right]\right)' =$$

$$= \cos\left[\ln\left(x^3+2x\right)\right] \cdot \frac{1}{x^3+2x} \cdot \left(3x^2+2\right) =$$

$$= \frac{3x^2+2}{x^3+2x} \cdot \cos\left[\ln\left(x^3+2x\right)\right]$$

1.b) $\left(\operatorname{cotg}\left[e^{(x^2+1)\cdot\sin x}\right]\right)' =$

$$= \underbrace{\frac{-1}{\sin^2\left[e^{(x^2+1)\cdot\sin x}\right]}}_{\text{DERIV. cotg.}} \cdot \underbrace{e^{(x^2+1)\cdot\sin x}}_{\text{DERIV. } e^x} \cdot$$

$$\cdot \underbrace{\left[2x\cdot\sin x + (x^2+1)\cdot\cos x\right]}_{\text{DERIV. SOUČINU}}$$

2.1 $(\sinh x)' = \lim\limits_{h \to 0} \dfrac{\sinh(x+h) - \sinh x}{h} =$

$$= \lim_{h \to 0} \frac{\dfrac{e^{x+h} - e^{-(x+h)}}{2} - \dfrac{e^{x} - e^{-x}}{2}}{h} =$$

$$= \lim_{h \to 0} \frac{\dfrac{e^{x+h} - e^{-x-h} - e^{x} + e^{-x}}{2}}{h} =$$

$$= \lim_{h \to 0} \frac{e^{x+h} - e^{x} - e^{-x-h} + e^{-x}}{2h} =$$

$$= \frac{1}{2} \cdot \lim_{h \to 0} \left[\frac{e^x \cdot e^h - e^x}{h} + \frac{e^{-x} \cdot e^{-h} - e^{-x}}{-h} \right] =$$

$$= \frac{1}{2} \cdot \lim_{h \to 0} \left[e^x \cdot \frac{e^h - 1}{h} + e^{-x} \cdot \frac{e^{-h} - 1}{-h} \right] =$$

$$= \frac{1}{2} \cdot \left[e^x \cdot \underbrace{\lim_{h \to 0} \frac{e^h - 1}{h}}_{\to 1} + e^{-x} \cdot \underbrace{\lim_{h \to 0} \frac{e^{-h} - 1}{-h}}_{\to 1} \right]$$

$$= \frac{1}{2} \cdot \left(e^x + e^{-x} \right) = \frac{e^x + e^{-x}}{2} = \cosh x$$

$$(\operatorname{tgh} x)' = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)' =$$

$$= \frac{(e^x - e^{-x})' \cdot (e^x + e^{-x}) - (e^x - e^{-x}) \cdot (e^x + e^{-x})'}{(e^x + e^{-x})^2} =$$

$$= \frac{[e^x - e^{-x} \cdot (-1)] \cdot (e^x + e^{-x}) - (e^x - e^{-x}) \cdot}{(e^x + e^{-x})^2}$$

$$\frac{\cdot [e^x + e^{-x} \cdot (-1)]}{} =$$

$$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} =$$

$$= \frac{\cancel{e^{2x}} + 2e^x e^{-x} + \cancel{e^{-2x}} - \cancel{e^{2x}} + 2e^x e^{-x} - \cancel{e^{-2x}}}{(e^x + e^{-x})^2} =$$

$$= \frac{4 e^x \cdot e^{-x}}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2} =$$

$$= \left(\frac{2}{e^x + e^{-x}}\right)^2 = \frac{1}{\left(\frac{e^x + e^{-x}}{2}\right)^2} = \frac{1}{\cosh^2 x}$$

3.1 $\left(f^{-1}\right)'(x_0) = \dfrac{1}{f'(y_0)} = \dfrac{1}{f'\left(f^{-1}(x_0)\right)}$

$$(\operatorname{argsinh} x)' = \frac{1}{\left[\sinh(\operatorname{argsinh} x)\right]'} =$$

$$= \frac{1}{\cosh(\operatorname{argsinh} x)} = \left| \cosh^2 x - \sinh^2 x = 1 \right| =$$

$$= \frac{1}{\sqrt{1 + \underbrace{\sinh^2(\operatorname{argsinh} x)}_{x^2}}} = \frac{1}{\sqrt{1+x^2}}.$$

4.1

$$f(x) = x^x = e^{\ln x^x} = e^{x \cdot \ln x}$$

$$f'(x) = \left(e^{x \cdot \ln x}\right)' =$$

$$= e^{x \cdot \ln x} \cdot (x \cdot \ln x)' =$$

$$= x^x \cdot \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) =$$

$$= x^x \cdot (\ln x + 1)$$

$$f(x) = x^{\sin x} = e^{\sin x \cdot \ln x}$$

$$f'(x) = x^{\sin x} \cdot \left(\sin x \cdot \ln x\right)' =$$

$$= x^{\sin x} \cdot \left(\cos x \cdot \ln x + \sin x \cdot \frac{1}{x}\right) =$$

$$= x^{\sin x} \cdot \left(\cos x \cdot \ln x + \frac{\sin x}{x}\right)$$

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(i) $\lim_{x\to 1} \frac{\ln x}{\cos(\frac{\pi}{2}x)} = \left[\frac{0}{0}\right] \overset{\text{L'H.}}{=}$

$= \lim_{x\to 1} \frac{\frac{1}{x}}{-\sin(\frac{\pi}{2}x)\cdot\frac{\pi}{2}} \overset{\text{DOS.}}{=} \left[\frac{1}{-1\cdot\frac{\pi}{2}}\right] = -\frac{2}{\pi}$

$\frac{0}{0} = \frac{\frac{1}{0}}{\frac{1}{0}} = \frac{\infty}{\infty}$

(ii) $\lim\limits_{x\to 0^+} \dfrac{\ln x}{\cot x} = \left[\dfrac{\infty}{\infty}\right] \overset{\text{l'H.}}{=}$

$= \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{\frac{-1}{\sin^2 x}} = \lim\limits_{x\to 0^+} \dfrac{-\sin^2 x}{x} =$

$= \left[\dfrac{0}{0}\right] \overset{\text{l'H.}}{=} \lim\limits_{x\to 0^+} \dfrac{-2\cdot \sin x \cdot \cos x}{1} =$

$= \left[\dfrac{-2\cdot 0\cdot 1}{1}\right] = \dfrac{0}{1} = \underline{\underline{0}}$

(iii)
$$\lim_{x \to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right) = \left[\frac{1}{0} - \frac{1}{0} = \infty - \infty \right]$$

$$= \lim_{x \to 1} \frac{x \cdot \ln x - x + 1}{(x-1) \cdot \ln x} = \left[\frac{1 \cdot 0 - 1 + 1}{(1-1) \cdot 0} = \frac{0}{0} \right]$$

$$\overset{\text{L'H.}}{=} \lim_{x \to 1} \frac{1 \cdot \ln x + x \cdot \frac{1}{x} - 1}{1 \cdot \ln x + (x-1) \cdot \frac{1}{x}} =$$

$$= \lim_{x \to 1} \frac{\ln x}{\ln x + 1 - \frac{1}{x}} = \left[\frac{0}{0 + 1 - 1} = \frac{0}{0} \right] \overset{\text{L'H.}}{=}$$

$$= \lim_{x \to 1} \frac{\frac{1}{x}}{\frac{1}{x} + 0 + \frac{1}{x^2}} = \left[\frac{1}{1+1} \right] = \frac{1}{2}$$

$$\left(- \frac{1}{x} \right)' = -\left(x^{-1} \right)' = -(-1) \cdot (x^{-2}) = \frac{1}{x^2}$$

(iv)

$$\lim_{x\to 1^+} \ln x \cdot \ln(x-1) = \left[0\cdot\infty\right] =$$

$$= \lim_{x\to 1^+} \frac{\ln(x-1)}{\frac{1}{\ln x}} = \left[\frac{\infty}{\infty}\right] \overset{\text{L'H.}}{=}$$

$$= \lim_{x\to 1^+} \frac{\frac{1}{x-1}}{-\frac{1}{\ln^2 x}\cdot\frac{1}{x}} = \lim_{x\to 1^+} \frac{x\cdot\ln^2 x}{1-x} =$$

$$= \left[\frac{1\cdot 0}{0} = \frac{0}{0}\right] \overset{\text{L'H.}}{=} \lim_{x\to 1^+} \frac{1\cdot\ln^2 x + x\cdot 2\cdot\ln x\cdot\frac{1}{x}}{-1} =$$

$$= \lim_{x\to 1^+} \left(-\ln^2 x - 2\ln x\right) = \left[-0^2 - 2\cdot 0\right] = 0$$

(v) $$\lim_{x\to 0^+} (\cot x)^{\frac{1}{\ln x}} = \left[\infty^0\right] =$$

$$= \lim_{x\to 0^+} e^{\ln (\cot x)^{\frac{1}{\ln x}}} = \lim_{x\to 0^+} e^{\frac{1}{\ln x}\cdot \ln \cot x} =$$

$$= \lim_{x\to 0^+} e^{\frac{\ln \cot x}{\ln x}} = *$$

$$\lim_{x\to 0^+} \frac{\ln(\cot x)}{\ln x} = \left[\frac{\infty}{\infty}\right] \overset{\text{L'H.}}{=}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\cot x}\cdot\frac{-1}{\sin^2 x}}{\frac{1}{x}} =$$

$$= \lim_{x\to 0^+} \frac{-x}{\frac{\cos x}{\cancel{\sin x}}\cdot \sin^{\cancel{2}} x} = \lim_{x\to 0^+} \frac{-x}{\cos x\cdot\sin x} =$$

$$= \left[\frac{0}{0}\right] \overset{\text{L'H.}}{=} \lim_{x\to 0^+} \frac{-1}{-\sin^2 x + \cos^2 x} \overset{\text{DOS.}}{=} \frac{-1}{1} = \underline{\underline{-1}}$$

$$(*) = \lim_{x\to 0^+} e^{\underbrace{\frac{\ln \cot x}{\ln x}}} = e^{-1} = \frac{1}{e}$$

(vi) $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} = \left[1^{\infty}\right] =$

$$= \lim_{x\to 0} e^{\frac{\ln \frac{\sin x}{x}}{x^2}} = (*)$$

$$\lim_{x \to 0} \frac{\ln \frac{\sin x}{x}}{x^2} = \left[\frac{0}{0} \right] \overset{l'H.}{=}$$

$$= \lim_{x \to 0} \frac{\frac{\cancel{x}}{\sin x} \cdot \frac{\cos x \cdot x - \sin x \cdot 1}{x^{\cancel{2}}}}{2x} =$$

$$= \lim_{x \to 0} \frac{x \cdot \cos x - \sin x}{2x^2 \cdot \sin x} = \left[\frac{0 \cdot 1 - 0}{2 \cdot 0 \cdot 0} \right] \overset{l'H.}{=}$$

$$= \lim_{x \to 0} \frac{1 \cdot \cancel{\cos x} + \cancel{x} \cdot (-\sin x) - \cancel{\cos x}}{4\cancel{x} \cdot \sin x + 2x^{\cancel{2}} \cdot \cos x} =$$

$$= \lim_{x \to 0} \frac{-\sin x}{2 \cdot (2 \cdot \sin x + x \cdot \cos x)} = \left[\frac{0}{0}\right] \overset{\text{L'H.}}{=}$$

$$= \frac{1}{2} \cdot \lim_{x \to 0} \frac{-\cos x}{2 \cdot \cos x + 1 \cdot \cos x + x \cdot (-\sin x)} =$$

$$= \frac{1}{2} \cdot \lim_{x \to 0} \frac{-\cos x}{3 \cos x - x \cdot \sin x} = \left[\frac{1}{2} \cdot \frac{-1}{3 \cdot 1 - 0 \cdot 0}\right] =$$

$$= \frac{1}{2} \cdot \left(-\frac{1}{3}\right) = -\frac{1}{6}$$

$$\boxed{* = e^{-\frac{1}{6}}}$$

(vii) $\lim\limits_{x \to 1^-} \left(\cos \frac{\pi}{2} x\right)^{\ln x} = \left[0^0\right] =$

$= \lim\limits_{x \to 1^-} e^{\ln x \cdot \ln\left(\cos \frac{\pi}{2} x\right)} = (*)$

$\lim\limits_{x \to 1^-} \ln x \cdot \ln\left(\cos \frac{\pi}{2} x\right) = \left[0 \cdot \infty\right] =$

$= \lim\limits_{x \to 1^-} \dfrac{\ln\left(\cos \frac{\pi}{2} x\right)}{\frac{1}{\ln x}} = \left[\dfrac{\infty}{\infty}\right] \overset{\text{L'H.}}{=}$

$$= \lim_{x \to 1^-} \frac{\frac{1}{\cos \frac{\pi}{2} x} \cdot \left(\cancel{-} \sin \frac{\pi}{2} x\right) \cdot \frac{\pi}{2}}{(\cancel{-}1) \cdot \frac{1}{\ln^2 x} \cdot \frac{1}{x}} =$$

$$= \frac{\pi}{2} \cdot \underbrace{\lim_{x \to 1^-} \left(x \cdot \sin \frac{\pi}{2} x\right)}_{\to 1} \cdot \lim_{x \to 1^-} \frac{\ln^2 x}{\cos \frac{\pi}{2} x} =$$

$$= \frac{\pi}{2} \cdot \lim_{x \to 1^-} \frac{\ln^2 x}{\cos \frac{\pi}{2} x} = \left[\frac{0}{0}\right] \overset{\text{L'H.}}{=}$$

$$= \frac{\pi}{2} \cdot \lim_{x \to 1^-} \frac{2 \cdot \ln x \cdot \frac{1}{x}}{-\sin \frac{\pi}{2} x \cdot \frac{\pi}{2}} = \left[\frac{\pi}{2} \cdot \frac{2 \cdot 0 \cdot 1}{-1 \cdot \frac{\pi}{2}}\right] = \underline{\underline{0}}$$

$$\circledast = e^0 = \underline{\underline{1}}.$$

$s(t) = -(t-3)^2 + 16 \quad , \quad [m, s]$

určit: (i) rychlost v $t_0 = 0$

(ii) t, s, kde se těl. zastaví

(iii) v, a, s ... 1 s po zast.

(iv) KDY SE TĚL. VRÁTÍ

$$s(t) = -(t-3)^2 + 16$$

$$v(t) = s'(t) = -2(t-3)$$

$$a(t) = v'(t) = s''(t) = -2$$

(i) $v(0) = -2 \cdot (-3) = \underline{\underline{6}} \quad [m \cdot s^{-1}]$

(ii) $v(t) = 0, \quad t = ?$

$$-2 \cdot (t-3) = 0 \iff \underline{\underline{t = 3}}$$

$$\underline{\underline{s(3) = 16}}$$

(iii) $t = 4 \Rightarrow v(4) = -2 \cdot 1 = \underline{\underline{-2}}$

$s(4) = -1 + 16 = \underline{\underline{15}}$

$a(4) = \underline{\underline{-2}}$

(iv) $s(0) = -9 + 16 = \underline{\underline{7}}$

$s(t) = \underbrace{-(t-3)^2 + 16 = 7}$

$(t-3)^2 = 9$

$|t-3| = 3$

$t - 3 = \pm 3 \Rightarrow t = 0$

$t = 6$