1.) $$\lim_{x \to 0} \frac{e^{-\frac{1}{x^2}}}{x^{100}} =: L$$

a) $$L = \left[\frac{0}{0}\right] \overset{L'H.}{=} \lim_{x \to 0} \frac{e^{-\frac{1}{x^2}} \cdot (-1) \cdot (-2) \cdot x^{-3}}{100 \cdot x^{99}} =$$

$$= \lim_{x \to 0} \frac{2 \cdot e^{-\frac{1}{x^2}}}{100 \cdot x^{102}} = \frac{1}{50} \cdot \lim_{x \to 0} \frac{e^{-\frac{1}{x^2}}}{x^{102}} =$$

b) $$L = \lim_{x \to 0} \frac{x^{-100}}{e^{\frac{1}{x^2}}} = \left[\frac{\infty}{\infty}\right] \overset{L'H}{=}$$

$$= \lim_{x \to 0} \frac{-100 \cdot x^{-101}}{e^{\frac{1}{x^2}} \cdot (-2) \cdot x^{-3}} =$$

$$= 50 \cdot \lim_{x \to 0} \frac{x^{-98}}{e^{\frac{1}{x^2}}} = \left[\frac{\infty}{\infty}\right] \overset{L'H.}{=} \dots$$

$$= c \cdot \lim_{x \to 0} \frac{x^0}{e^{\frac{1}{x^2}}} = \left[\frac{1}{\infty}\right] = \underline{\underline{0}}$$

2.) $y \cdot \log_2 x + \sin y = 0$

$[x_0, y_0] = [1, ?]$, KDE $y \in [\frac{1}{2}, \frac{7}{2}]$.

(i) $\underline{\underline{y_0 = ?}}$ ... $y \cdot \underbrace{\log_2 1}_{=0} + \sin y = 0$

$\sin y = 0$

$\Updownarrow$

$y = k \cdot \pi, \; k \in \mathbb{Z}$

$\wedge \; y \in [\frac{1}{2}, \frac{7}{2}] \Rightarrow$

$\Rightarrow \underline{\underline{y = \pi}}$

(ii) $\underbrace{y\cdot\log_2 x}_{\text{součin}} + \underbrace{\sin y}_{\text{slož. fce}} = 0 \quad \Big/ \frac{d}{dx},\ y = y(x)$

$$\underbrace{y'\cdot\log_2 x + y\cdot\frac{1}{x\cdot\ln 2}} + \underbrace{(\cos y)\cdot y'} = 0$$

$$y'\cdot(\log_2 x + \cos y) = -\frac{y}{x\cdot\ln 2}$$

$$y' = -\frac{y}{x\cdot\ln 2}\cdot\frac{1}{\log_2 x + \cos y}$$

$$\underline{\underline{y' = -\frac{y}{x\cdot\ln 2\cdot(\log_2 x + \cos y)}}} = \Bigg|_{[1,\pi]} = -\frac{\pi}{1\cdot\ln 2\cdot(0-1)} =$$

$$= \underline{\underline{\frac{\pi}{\ln 2}}}$$

(iii) t: $\quad y - y_0 = f'(x_0) \cdot (x - x_0)$

$$y - \pi = \frac{\pi}{\ln 2} \cdot (x - 1)$$

$$t: \frac{\pi}{\ln 2} \cdot x - y + \pi - \frac{\pi}{\ln 2} = 0$$

3.) $f(x) = x^2 \cdot e^{-x}$

a) Dom & H

$Dom(f) = \mathbb{R}$, $H(f) = [0, \infty)$

b) SUDOST / LICHOST

$f(-x) = (-x)^2 \cdot e^{-(-x)} = x^2 \cdot e^{x} \neq \pm f(x)$

$f(x) = x^2 \cdot e^{-x}$ $\qquad -f(x) = -(x^2 \cdot e^{-x})$

$\Rightarrow$ ANI S. ANI L.

c) PRŮSEČÍKY S x, y $\quad (y = x^2 \cdot e^{-x})$

$f \cap y \; \ldots \; x = 0 \Rightarrow y = 0^2 \cdot e^{-0} = 0$

$[0, 0]$

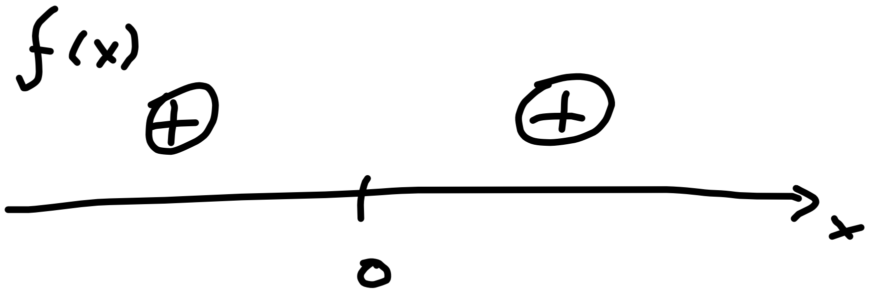
$f \cap x \; \ldots \; y = 0 \Rightarrow 0 = x^2 \cdot e^{-x}$

$\Updownarrow$

$x = 0$

$[0, 0]$

d) KLADNÁ & ZÁP.



$$y = e^{-x} \cdot x^2$$

$$0 = e^{-x} \cdot x^2$$

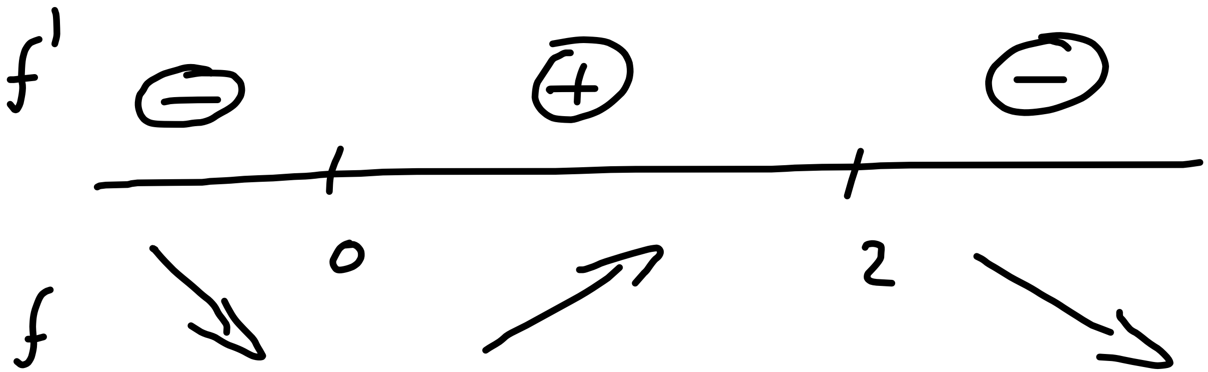
N.B. ... $x = 0$

e) INT. MONOTONIE

$$f'(x) = -e^{-x} \cdot x^2 + e^{-x} \cdot 2x =$$

$$= \underline{x \cdot e^{-x} \cdot (2 - x)} = 0$$

$$\Leftrightarrow \underline{\underline{0,\ 2}} \quad \text{(N.B. } f'\text{)}$$



LOK. MIN.

LOK. MAX.

KLES. $x \in (-\infty; 0] \cup [2, \infty)$

ROST. $x \in [0, 2]$

f) KX / KV

$$f''(x) = \left[e^{-x} \cdot (2x - x^2) \right]' =$$

$$= -e^{-x} \cdot (2x - x^2) + e^{-x} \cdot (2 - 2x) =$$

$$= e^{-x} \cdot (-2x + x^2 + 2 - 2x) = e^{-x} \cdot (x^2 - 4x + 2) = 0$$

$$x^2 - 4x + 2 = 0 \iff x_{1,2} = 2 \pm \sqrt{2}$$

$$f'' \quad \oplus \quad \ominus \quad \oplus$$

$$2 - \sqrt{2} \qquad 2 + \sqrt{2}$$

$$f \quad KX \quad KV \quad KX$$

INFL. B. INFL. B.

$$[x - (2 + \sqrt{2})] \cdot [x - (2 - \sqrt{2})]$$

g) EXTR.: STAC. BODY $\rightarrow$ $f'(x) = 0$

$$\Downarrow$$

$$\underbrace{0,\ 2}$$

$f''(0) = 2 > 0 \Rightarrow$ L. MIN.

$f''(2) = (-2) \cdot e^{-2} < 0 \Rightarrow$ L. MAX.

INFL. B.: $f''(x) = 0$ ... $2 \pm \sqrt{2}$

$f'''(x) = \ldots = e^{-x} \cdot (-x^2 + 6x - 6)$

$f'''(2 \pm \sqrt{2}) \neq 0 \Rightarrow$ INFL. B.

h) A.S.

BEZ SN. – NEJSOU

SE SN.: $y = ax + b$

$$a = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x^2 \cdot e^{-x}}{x} =$$

$$= \lim_{x \to \infty} x \cdot e^{-x} = [\infty \cdot 0] = \lim_{x \to \infty} \frac{x}{e^x} = \left[\frac{\infty}{\infty}\right]$$

$$\overset{(l'H)}{=} \lim_{x \to \infty} \frac{1}{e^x} = \underline{\underline{0}}$$

$$b = \lim_{x \to \infty} [f(x) - ax] =$$

$$= \lim_{x \to \infty} (x^2 \cdot e^{-x} - 0 \cdot x) = \lim_{x \to \infty} \frac{x^2}{e^x} \overset{L'H.}{=}$$

$$= \lim_{x \to \infty} \frac{2.x}{e^x} \overset{L'H}{=} \lim_{x \to \infty} \frac{2}{e^x} = \underline{\underline{0}}$$

AS. SE SM. V $+\infty$ JE $\boxed{y = 0}$

$$a = \lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} x \cdot e^{-x} = -\infty$$

$$-\infty \qquad e^{\infty} = \infty$$

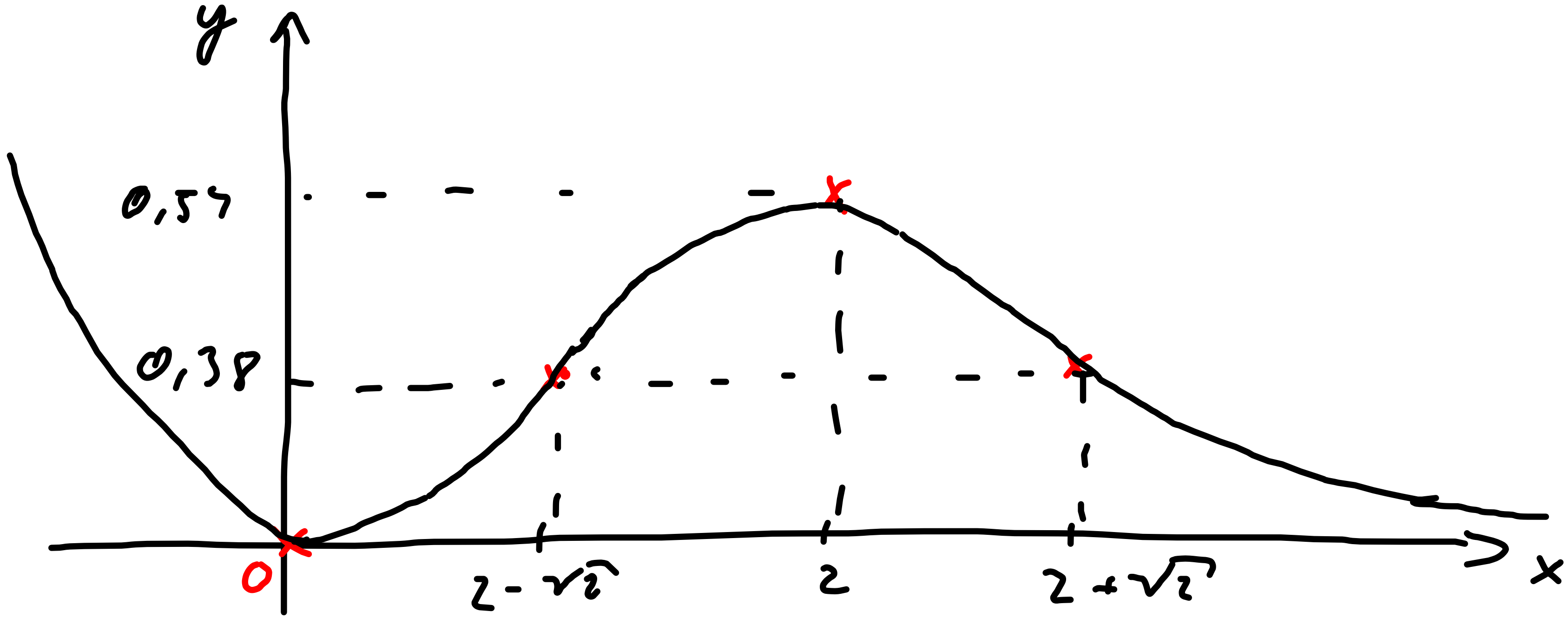
AS. SE SM. V $-\infty$ NENÍ

i) GRAF

$[0,0]$

$$f(2) = \frac{4}{e^2} \doteq 0{,}54$$

$$f(2 \pm \sqrt{2}) \doteq 0{,}38$$



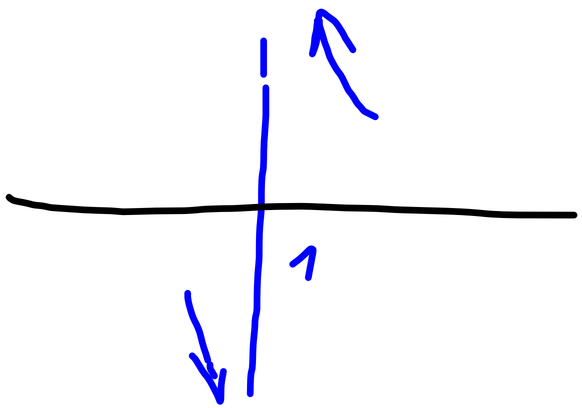
Př.) $f(x) = \dfrac{x}{x^2-1} = \dfrac{x}{\underbrace{(x-1)\cdot(x+1)}}$

ASYMPTOTY

$\mathrm{Dom}(f) = \mathbb{R} \setminus \{1, -1\}$

AS. BEZ SM. - KANDIDÁTI : $x = \pm 1$

$$\lim_{x \to 1^+} \frac{x}{x^2-1} = \left[\frac{1}{0^+} \right] = +\infty$$



$$\lim_{x \to 1^-} \frac{x}{x^2-1} = \left[\frac{1}{0^-} \right] = -\infty$$

$$\lim_{x \to -1^+} \frac{x}{x^2-1} = \infty$$

$$\lim_{x \to -1^-} \frac{x}{x^2-1} = -\infty$$

AS. SE SM.

$$a = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{1}{x^2-1} = 0$$

$$b = \lim_{x \to \pm\infty} \frac{x}{x^2-1} = 0$$

$$\Rightarrow \quad y = 0 \quad v \ \pm\infty$$

Př.: $f(x) = -\dfrac{x^2}{x+1}$

a) $\mathcal{D}om(f) = \mathbb{R} \setminus \{-1\}$

b) $f(x) = 0 \iff x = 0 \Rightarrow$

f: $\oplus$ | -1 | $\ominus$ | 0 | $\ominus$

c) $$f'(x) = \frac{-2x\cdot(x+1) + 1\cdot x^2}{(x+1)^2} = \frac{-x^2-2x}{(x+1)^2} =$$

$$= \frac{-x\cdot(x+2)}{(x+1)^2} = 0 \iff \begin{array}{l} x_1 = 0 \\ x_2 = -2 \end{array}$$

f': $\ominus$ | -2 | $\oplus$ | -1 | $\oplus$ | 0 | $\ominus$

f: $\searrow$ L. min. $\nearrow$ NEDEF. $\nearrow$ L. MAX $\searrow$

d) $f''(x) = \ldots = \dfrac{-2x-2}{(x+1)^4} = \dfrac{-2\cdot(x+1)}{(x+1)^4} =$

$= \dfrac{-2}{(x+1)^3} = 0$ ... NELZE

VŽDY $\neq 0$

f''	$+$		$-$
f	KX	-1	KV

NEDEF.

e) AS. BEZ SM.: $x = -1$

$$\lim_{x \to -1^-} \left(\frac{-x^2}{x+1} \right) = \left[- \frac{1}{0^-} \right] = +\infty \quad \text{JE AS.}$$

$$\lim_{x \to -1^+} \left(\frac{-x^2}{x+1} \right) = \left[- \frac{1}{0^+} \right] = -\infty \quad \text{JE AS.}$$

AS. SE SM.:

$$a = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{-x^2}{x^2+x} = \underline{\underline{-1}}$$

$$b = \lim_{x \to \pm\infty} (f(x) - ax) = \lim_{x \to \pm\infty} \frac{-x^2}{x+1} =$$

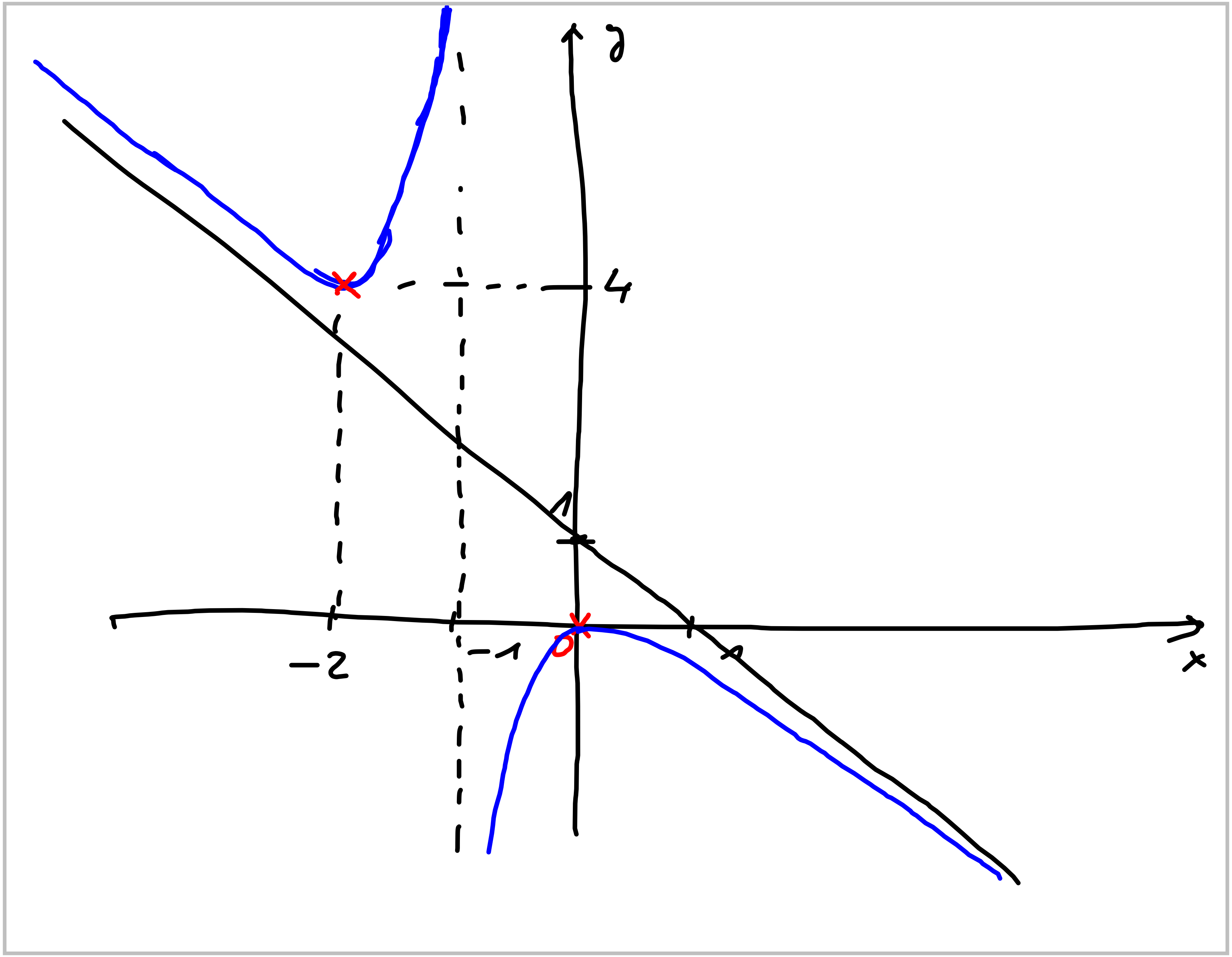
$$= \lim_{x \to \pm\infty} \frac{x}{x+1} = \underline{\underline{1}}$$

AS. SE SM. V $\pm\infty$ JE $y = -x + 1$

f) GRAF

$f(0) = 0$

$f(-2) = 4$


y
4
1
-2
-1
0
1
x