

$$1.) \lim_{x \rightarrow 0} \frac{e^{-\frac{1}{x^2}}}{x^{100}} =: L$$

$$2.) L = \left[\frac{0}{0} \right] \stackrel{L'H.}{=} \lim_{x \rightarrow 0} \frac{e^{-\frac{1}{x^2}} \cdot (-1) \cdot (-2) \cdot x^{-3}}{100 \cdot x^{99}} =$$

$$= \lim_{x \rightarrow 0} \frac{2 \cdot e^{-\frac{1}{x^2}}}{100 \cdot x^{102}} = \frac{1}{50} \cdot \lim_{x \rightarrow 0} \frac{e^{-\frac{1}{x^2}}}{x^{102}} = \text{X}$$

$$b) L = \lim_{x \rightarrow 0} \frac{x^{-100}}{e^{\frac{1}{x^2}}} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \dots$$

$$= \lim_{x \rightarrow 0} \frac{-100 \cdot x^{-101}}{e^{\frac{1}{x^2}} \cdot (-2) \cdot x^{-3}} =$$

$$= 50 \cdot \lim_{x \rightarrow 0} \frac{x^{-98}}{e^{\frac{1}{x^2}}} = \left[\frac{\infty}{\infty} \right] \stackrel{L'H}{=} \dots$$

$$= C \cdot \lim_{x \rightarrow 0} \frac{x^0}{e^{\frac{1}{x^2}}} = \left[\frac{1}{\infty} \right] = 0$$

$$2.) \quad y \cdot \log_2 x + \sin y = 0$$

$$[x_0, y_0] = [1, ?], \quad \text{KOD} \quad y \in \left[\frac{1}{2}, \frac{3}{2} \right].$$

$$(i) \quad \underline{\underline{y_0 = ?}}$$

...

$$y \cdot \underbrace{\log_2 1}_{=0} + \sin y = 0$$

$$\sin y = 0$$

↕

$$y = h \cdot \pi, \quad h \in \mathbb{Z}$$

$$\wedge y \in \left[\frac{1}{2}, \frac{3}{2} \right] \Rightarrow$$

$$\Rightarrow \underline{\underline{y = \pi}}$$

$$(ii) \quad \underbrace{y \cdot \log_2 x + \sin y}_{\text{součet}} = 0 \quad \left| \frac{d}{dx}, y=y(x) \right.$$

SLOŽ. FCE

$$\underbrace{y' \cdot \log_2 x + y \cdot \frac{1}{x \cdot \ln 2}}_{\text{}} + \underbrace{(\cos y) \cdot y'}_{\text{}} = 0$$

$$y' \cdot (\log_2 x + \cos y) = - \frac{y}{x \cdot \ln 2}$$

$$y' = - \frac{y}{x \cdot \ln 2} \cdot \frac{1}{\log_2 x + \cos y}$$

$$\underline{\underline{y'}} = - \frac{y}{x \cdot \ln 2 \cdot (\log_2 x + \cos y)} \Bigg|_{[1, \pi]} = - \frac{\pi}{1 \cdot \ln 2 \cdot (0 + 1)}$$

$$= \underline{\underline{\frac{\pi}{\ln 2}}}$$

$$\text{(iii) t: } y - y_0 = f'(x_0) \cdot (x - x_0)$$

$$y - \pi = \frac{\pi}{h_2} \cdot (x - 1)$$

$$t: \frac{\pi}{h_2} \cdot x - y + \pi - \frac{\pi}{h_2} = 0$$

$$3.) \quad \underline{\underline{f(x) = x^2 \cdot e^{-x}}}$$

a) Dom & H

$$\text{Dom}(f) = \mathbb{R}, \quad H(f) = [0, \infty)$$

b) sudost / lichost

$$f(-x) = (-x)^2 \cdot e^{-(-x)} = x^2 \cdot e^x \neq \pm f(x)$$

$$f(x) = x^2 \cdot e^{-x} \quad -f(x) = -(x^2 \cdot e^{-x})$$

\Rightarrow Ani S. Ani L.

5) Průsečíky s x, y ($g = x^2 \cdot e^{-x}$)

$$f \cap y \dots x = 0 \Rightarrow y = 0^2 \cdot e^{-0} = 0$$

$$[0, 0]$$

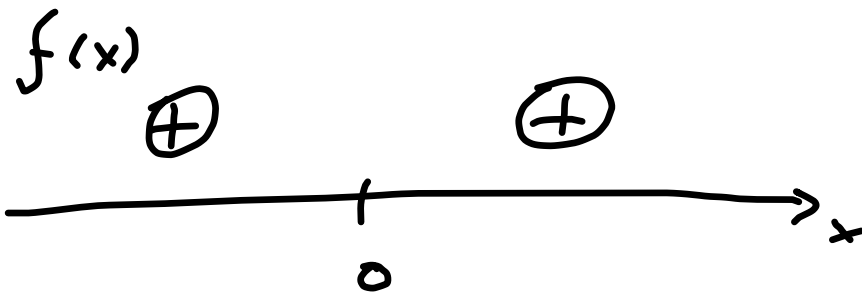
$$f \cap x \dots y = 0 \Rightarrow 0 = x^2 \cdot e^{-x}$$

$$\uparrow$$

$$x = 0$$

$$[0, 0]$$

d) KLADUNA' & ZA'P.



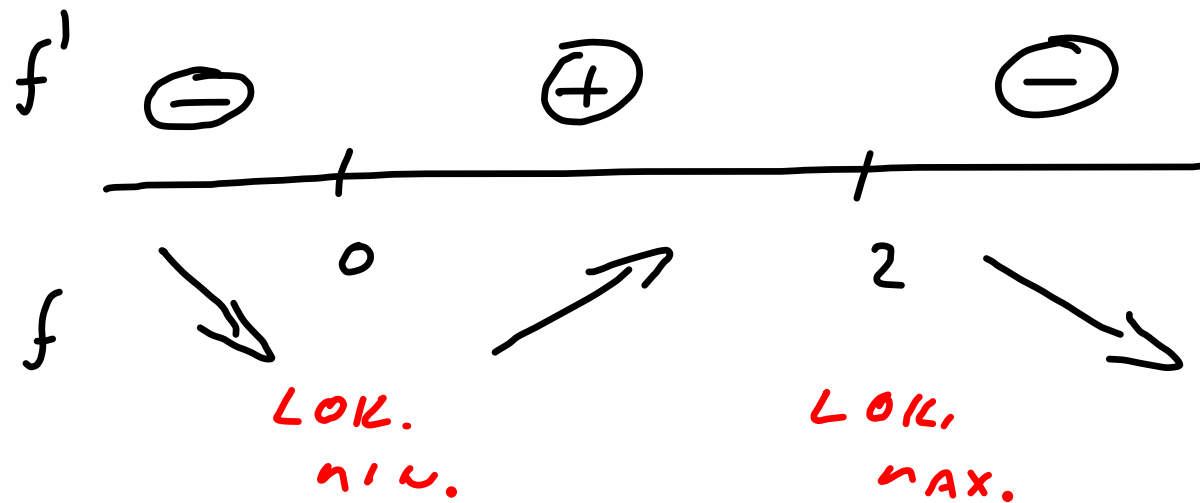
$$\begin{cases} y = e^{-x} \cdot x^2 \\ 0 = e^{-x} \cdot x^2 \\ \text{n. b.} \dots x = 0 \end{cases} \leftarrow$$

e) INT. POLOŽENIE

$$f'(x) = -e^{-x} \cdot x^2 + e^{-x} \cdot 2x =$$

$$= \underline{x \cdot e^{-x} \cdot (2 - x)} = 0$$

$$\Leftrightarrow \underline{\underline{0, 2}} \quad (\text{n. b. } f')$$



KLAS. $x \in (-\infty, 0] \cup [2, \infty)$

POST. $x \in [0, 2]$

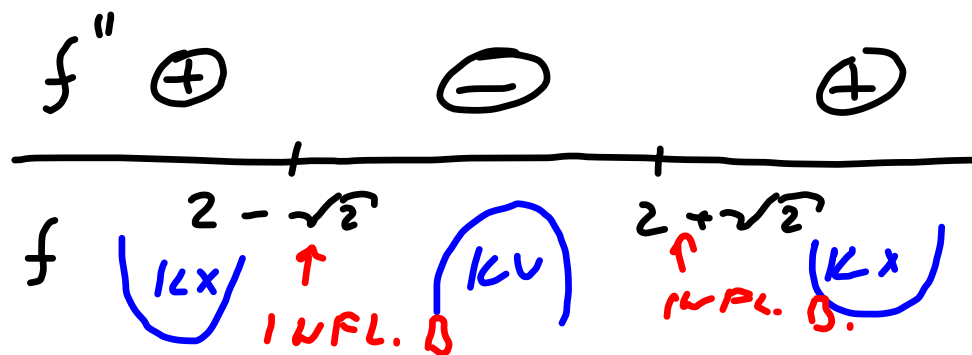
$$f) \quad \underbrace{(kx)} / \underbrace{(kv)}$$

$$f''(x) = [e^{-x} \cdot (2x - x^2)]' =$$

$$= -e^{-x} \cdot (2x - x^2) + e^{-x} \cdot (2 - 2x) =$$

$$= e^{-x} \cdot (\underline{-2x} + \underline{x^2} + \underline{2} - \underline{2x}) = e^{-x} \cdot (x^2 - 4x + 2) \stackrel{!}{=} 0$$

$$x^2 - 4x + 2 = 0 \quad \Leftrightarrow \quad x_{1,2} = \underline{\underline{2 \pm \sqrt{2}}}$$



$$\begin{aligned} & [x - (2 + \sqrt{2})] \cdot \\ & \cdot [x - (2 - \sqrt{2})] \end{aligned}$$

$$g) \text{ EXTR. : } \text{STAC. BODY} \rightarrow f'(x) = 0$$

$$\Downarrow \\ \underbrace{0, 2}$$

$$f''(0) = 2 > 0 \Rightarrow \text{L. MIN.}$$

$$f''(2) = (-2) \cdot e^{-2} < 0 \Rightarrow \text{L. MAX.}$$

$$\text{INFL. B. : } f''(x) = 0 \dots 2 \pm \sqrt{2}$$

$$f'''(x) = \dots = e^{-x} \cdot (-x^2 + 6x - 6)$$

$$f'''(2 \pm \sqrt{2}) \neq 0 \Rightarrow \underline{\underline{\text{INFL. B.}}}$$

h) AS.

BEZ SN. -

NEJSOY

SE SN.:

$$y = ax + b$$

$$a = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^2 \cdot e^{-x}}{x} =$$

$$= \lim_{x \rightarrow \infty} x \cdot e^{-x} = [\infty \cdot 0] = \lim_{x \rightarrow \infty} \frac{x}{e^x} = \left[\frac{\infty}{\infty} \right]$$

(1)

$$\downarrow$$
$$= \lim_{x \rightarrow \infty} \frac{1}{e^x} = \underline{\underline{0}}$$

$$b = \lim_{x \rightarrow \infty} [f(x) - ax] =$$

$$= \lim_{x \rightarrow \infty} (x^2 \cdot e^{-x} - 0 \cdot x) = \lim_{x \rightarrow \infty} \frac{x^2}{e^x} \stackrel{L'H.}{=} \downarrow$$

$$= \lim_{x \rightarrow \infty} \frac{2 \cdot x}{e^x} \stackrel{L'H.}{=} \downarrow \lim_{x \rightarrow \infty} \frac{2}{e^x} = \underline{\underline{0}}$$

AS. SE Sa. \vee $+\infty$ JE $\underbrace{y=0}$

$$a = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} x \cdot e^{-x} = -\infty$$

\downarrow \downarrow
 $-\infty$ $e^a = \infty$

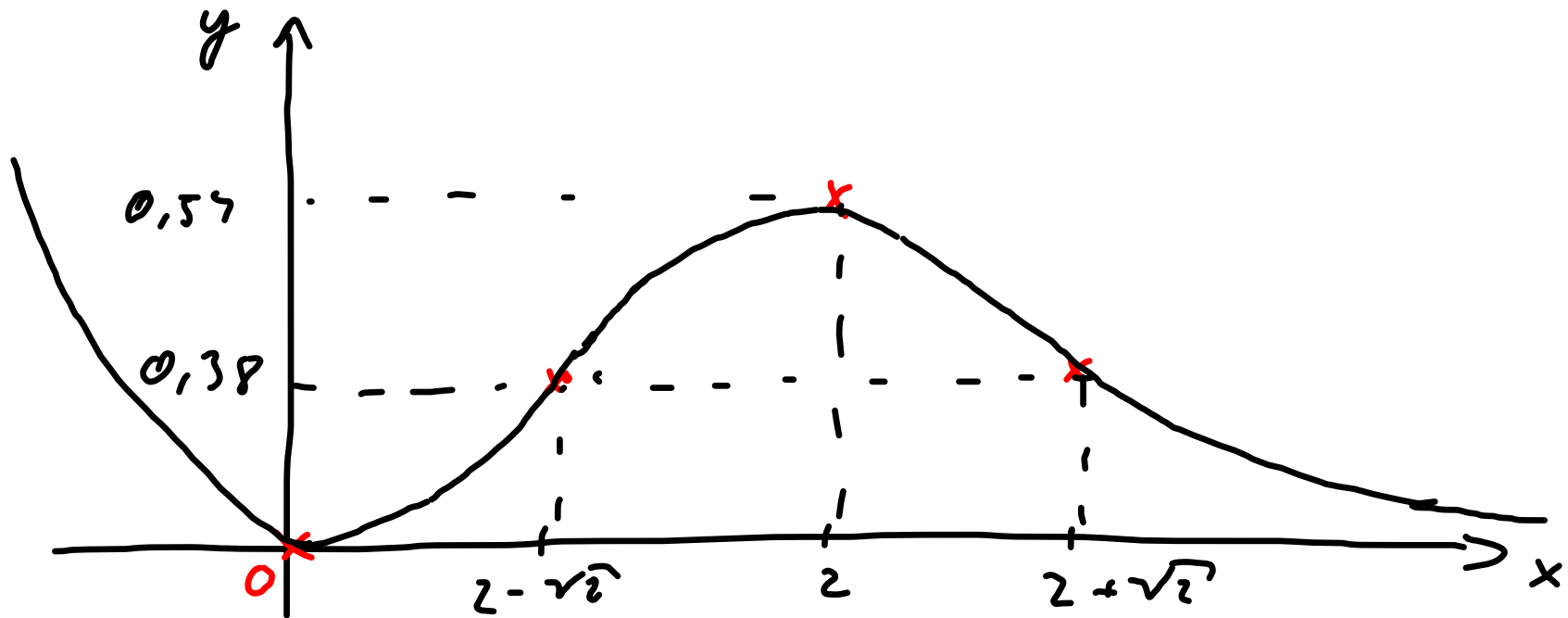
AS. SE SN. \vee $-\infty$ NEKI'

c) GRAF

$[0,0]$

$$f(2) = \frac{4}{e^2} \approx 0,54$$

$$f(2 \pm \sqrt{2}) \approx 0,38$$



$$\underline{P\grave{a}.)} \quad f(x) = \frac{x}{x^2-1} = \frac{x}{\underbrace{(x-1) \cdot (x+1)}}$$

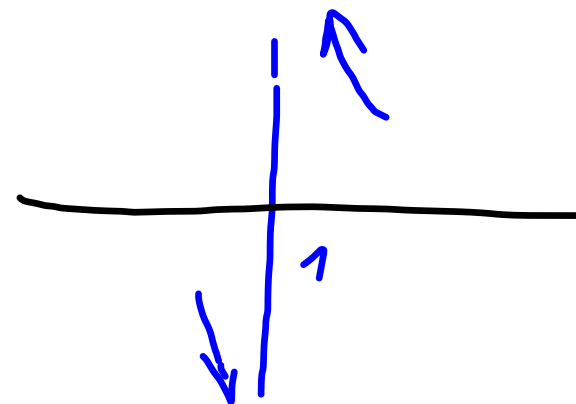
ASYNPTOTY

$$\text{Dom}(f) = \mathbb{R} \setminus \{1, -1\}$$

AS. BĚŽ. SM. - KALDI DA'IT' : $x = \pm 1$

$$\lim_{x \rightarrow 1^+} \frac{x}{x^2-1} = \left[\frac{1}{0^+} \right] = +\infty$$

$$\lim_{x \rightarrow 1^-} \frac{x}{x^2-1} = \left[\frac{1}{0^-} \right] = -\infty$$



$$\lim_{x \rightarrow -1^+} \frac{x}{x^2 - 1} = \infty$$

$$\lim_{x \rightarrow -1^-} \frac{x}{x^2 - 1} = -\infty$$

AS. SE SM.

$$a = \lim_{x \rightarrow \pm\infty} \frac{|x|}{x} = \lim_{x \rightarrow \pm\infty} \frac{1}{x^2 - 1} = 0$$

$$b = \lim_{x \rightarrow \pm\infty} \frac{x}{x^2 - 1} = 0$$

$$\Rightarrow \begin{matrix} y = 0 \\ x \pm \infty \end{matrix}$$

Př.:| $f(x) = -\frac{x^2}{x+1}$

⇒ $\text{Dom}(f) = \mathbb{R} \setminus \{-1\}$

Ⓟ $f(x) = 0 \Leftrightarrow x = 0 \Rightarrow f \oplus \ominus \ominus$

A horizontal number line with tick marks at -1 and 0. Above the line, the sign of the function is indicated: a circled plus sign (+) is above the interval (-∞, -1), a circled minus sign (⊖) is above the interval (-1, 0), and another circled minus sign (⊖) is above the interval (0, ∞).

Ⓟ $f'(x) = \frac{-2x \cdot (x+1) + 1 \cdot x^2}{(x+1)^2} = \frac{-x^2 - 2x}{(x+1)^2} =$

$= \frac{-x \cdot (x+2)}{(x+1)^2} = 0 \Leftrightarrow \begin{matrix} x_1 = 0 \\ x_2 = -2 \end{matrix}$

f' $\ominus \oplus \oplus \ominus$

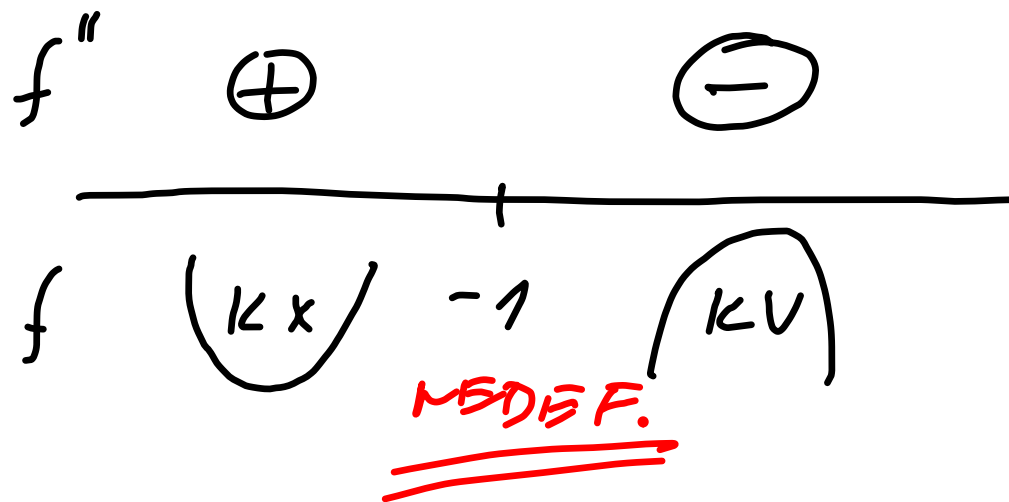
A horizontal number line with tick marks at -2, -1, and 0. Above the line, the sign of the derivative is indicated: a circled minus sign (⊖) is above the interval (-∞, -2), a circled plus sign (+) is above the interval (-2, -1), another circled plus sign (+) is above the interval (-1, 0), and a circled minus sign (⊖) is above the interval (0, ∞).

$f \rightarrow \begin{matrix} -2 \\ \text{L. min.} \end{matrix} \rightarrow \begin{matrix} -1 \\ \text{NEDEF.} \end{matrix} \rightarrow \begin{matrix} 0 \\ \text{L. max.} \end{matrix} \rightarrow$

$$d) f''(x) = \dots = \frac{-2x - 2}{(x+1)^2} = \frac{-2 \cdot (x+1)}{(x+1)^2} =$$

$$= \frac{-2}{(x+1)^3} = 0 \quad \dots \quad \underline{\underline{\text{NELZE}}}$$

$\text{VĚD} \neq 0$



e) AS. BEZ sn.: $x = -1$

$$\lim_{x \rightarrow -1^-} \left(\frac{-x^2}{x+1} \right) = \left[-\frac{1}{0^-} \right] = +\infty \quad \text{JE AS.}$$

$$\lim_{x \rightarrow -1^+} \left(\frac{-x^2}{x+1} \right) = \left[-\frac{1}{0^+} \right] = -\infty \quad \text{JE AS.}$$

AS. SE sn.:

$$a = \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = \lim_{x \rightarrow \pm\infty} \frac{-x^2}{x^2+x} = \underline{\underline{-1}}$$

$$b = \lim_{x \rightarrow \pm\infty} (f(x) - ax) = \lim_{x \rightarrow \pm\infty} \frac{-x^2}{x+1} =$$

$$= \lim_{x \rightarrow \pm\infty} \frac{x}{x+1} = \underline{\underline{1}}$$

AS. SE SN. $\sim \pm\infty$ JF

$$y = -x + 1$$

f) GRAF

$$f(0) = 0$$

$$f(-2) = 4$$

