$$\int x^2\cdot(5-x)^3\,dx = \int x^2\cdot(125-3\cdot25\cdot x+$$

$$+3\cdot5\cdot x^2-x^3)\,dx =$$

$$=\int(125x^2-75x^3+15x^4-x^5)\,dx=$$

$$=125\frac{x^3}{3}-75\frac{x^4}{4}+15\frac{x^5}{5}-\frac{x^6}{6}+C$$

$$\int \frac{x+1}{\sqrt{x}}\,dx = \int\left(\frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\right)dx =$$

$$= \int\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$= \frac{2}{3}\cdot x\sqrt{x} + 2\cdot\sqrt{x} + C$$

$$\int (2^x + 3^x)^2 \, dx =$$

$$= \int (2^{2x} + 2 \cdot 2^x \cdot 3^x + 3^{2x}) \, dx =$$

$$= \int (2^2)^x \, dx + 2 \cdot \int (2 \cdot 3)^x \, dx + \int (3^2)^x \, dx =$$

$$= \int 4^x \, dx + 2 \cdot \int 6^x \, dx + \int 9^x \, dx =$$

$$= \frac{4^x}{\ln 4} + 2 \cdot \frac{6^x}{\ln 6} + \frac{9^x}{\ln 9} + \underline{\underline{C}}$$

$$\int (1 + \sin x + \cos x)\, dx =$$

$$= x - \cos x + \sin x + C$$

$$(u \cdot v)' = u'v + uv' \quad / \int$$

$$\int (u \cdot v)' = \int u'v + \int uv'$$

$$\underline{\underline{u \cdot v}} = \int u'v + \int uv'$$

$$\Rightarrow \int u'v = uv - \underline{\underline{\int uv'}}$$

$$\int x \cdot \sin x \, dx = \left| \begin{matrix} u' = \sin x & u = -\cos x \\ v = x & \underbrace{v' = 1} \end{matrix} \right| =$$

$$= \underbrace{-x \cdot \cos x}_{u \cdot v} - \int 1 \cdot (-\cos x) \, dx =$$

$$u \cdot v \quad - \int u v'$$

$$= -x \cdot \cos x + \int \cos x \, dx =$$

$$= -x \cdot \cos x + \sin x + C$$

$$\int x^2 \cdot e^x \, dx = \left| \begin{matrix} u' = e^x & u = e^x \\ v = x^2 & v' = 2x \end{matrix} \right| =$$

$$= x^2 \cdot e^x - \int 2x e^x \, dx =$$

$$= x^2 \cdot e^x - 2 \int x \cdot e^x \, dx = \left| \begin{matrix} u' = e^x & u = e^x \\ v = x & v' = 1 \end{matrix} \right| =$$

$$= x^2 \cdot e^x - 2 \cdot \left[x \cdot e^x - \int 1 \cdot e^x \, dx \right] =$$

$$= x^2 \cdot e^x - 2 \cdot x \cdot e^x + 2 \cdot e^x + c = \underline{\underline{e^x \cdot (\sim) + c}}$$

$$\int \operatorname{arctg} x \, dx = \int 1 \cdot \operatorname{arctg} x \, dx =$$

$$= \left| \begin{array}{ll} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \dfrac{1}{1+x^2} \end{array} \right| =$$

$$= x \cdot \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx =$$

$$= x \cdot \operatorname{arctg} x - \frac{1}{2} \cdot \ln(1+x^2) + c$$

$$\int \frac{f'}{f} = \ln|f| + c$$

$$\underbrace{\int \frac{\ln x}{x}\,dx}_{I} = \left| \begin{array}{ll} u' = \frac{1}{x} & u = \ln x \\ v = \ln x & v' = \frac{1}{x} \end{array} \right| =$$

$$= \underbrace{\ln^2 x}_{+C} - \underbrace{\int \frac{\ln x}{x}\,dx}_{I}$$

$$I = \ln^2 x - I$$

$$2I = \ln^2 x$$

$$I = \frac{\ln^2 x}{2} + C$$

$$\int f(x)\,dx = \left| \begin{array}{l} x = \varphi(t) \\ dx = \varphi'(t)\,dt \end{array} \right| =$$

$$= \int f(\varphi(t)) \cdot \varphi'(t)\,dt$$

$$\int (2x+5)^{10}\,dx = \left| \begin{array}{l} t = 2x+5 \\ dt = 2\,dx \\ dx = \frac{1}{2}\,dt \end{array} \right| =$$

$$= \int t^{10} \cdot \frac{1}{2}\,dt = \frac{1}{2} \int t^{10}\,dt =$$

$$= \frac{1}{2} \cdot \frac{t^{11}}{11} + C = \frac{(2x+5)^{11}}{22} + C$$

$$\int \frac{\ln x}{x}\,dx = \int \ln x \cdot \frac{1}{x}\,dx =$$

$$= \left| \begin{matrix} t = \ln x \\ dt = \frac{1}{x}\,dx \end{matrix} \right| = \int t\,dt =$$

$$= \frac{t^2}{2} + c = \frac{\ln^2 x}{2} + c$$

$$\int \sin\sqrt{x}\,dx = \left| \begin{array}{l} t = \sqrt{x} \\ t^2 = x \\ 2t\,dt = \underline{\underline{dx}} \end{array} \right| =$$

$$= \int \sin t \cdot 2t\,dt = 2 \cdot \int t \cdot \sin t\,dt =$$

$$= \left| \begin{array}{ll} u' = \sin t & u = -\cos t \\ v = t & v' = 1 \end{array} \right| = 2 \cdot \left[-t \cdot \cos t + \int \cos t\,dt \right]$$

$$= -2 \cdot t \cdot \cos t + 2 \sin t + C = 2 \cdot \left(\sin\sqrt{x} - \sqrt{x} \cdot \cos\sqrt{x} \right) + C$$

$$\int \frac{1}{x \cdot \ln^2 x}\,dx = \left| \begin{array}{l} t = \ln x \\ dt = \frac{1}{x}\,dx \end{array} \right| =$$

$$= \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = \frac{t^{-1}}{-1} + C =$$

$$= -\frac{1}{\ln x} + C$$

$$\int \frac{\operatorname{arctg}\sqrt{x}}{\sqrt{x}\cdot(1+x)}\,dx = \left|\begin{aligned} t &= \operatorname{arctg}\sqrt{x} \\ dt &= \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}} \\ 2\,dt &= \frac{1}{\sqrt{x}\cdot(1+x)} \end{aligned}\right| =$$

$$= \int t\cdot 2\,dt = 2\cdot\frac{t^2}{2} + C =$$

$$= \underline{\underline{(\operatorname{arctg}\sqrt{x})^2 + C}}$$

$$\int x^{n} \cdot \ln x \, dx = \qquad \boxed{n \neq -1}$$

$$= \left| \begin{array}{ll} u' = x^{n} & u = \dfrac{x^{n+1}}{n+1} \\ v = \ln x & v' = \dfrac{1}{x} \end{array} \right| =$$

$$= \ln x \cdot \frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} \cdot \frac{1}{x} \, dx =$$

$$= \frac{x^{n+1}}{n+1} \cdot \ln x - \frac{1}{n+1} \cdot \int x^{n} \, dx =$$

$$= \frac{x^{n+1}}{n+1} \cdot \ln x - \frac{1}{n+1} \cdot \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{x}{1+x^4}\,dx = \left| \begin{aligned} t &= x^2 \\ dt &= 2x\,dx \\ x\,dx &= \tfrac{1}{2}\,dt \end{aligned} \right| =$$

$$\Downarrow$$

$$\frac{1}{2}\,dt$$

$$= \int \frac{1}{1+t^2} \cdot \frac{1}{2}\,dt = \frac{1}{2} \cdot \operatorname{arctg} t + C =$$

$$= \frac{\operatorname{arctg} x^2}{2} + C$$

$$\int \cos^5 x \cdot \sqrt{\sin x}\, dx = \left| \begin{array}{l} t = \sin x \\ dt = \cos x\, dx \end{array} \right| =$$

$$= \int \cos^4 x \cdot \sqrt{t}\, dt = \int (1 - \sin^2 x)^2 \cdot \sqrt{t}\, dt =$$

$$= \int (1 - t^2)^2 \cdot \sqrt{t}\, dt = \int t^{\frac{1}{2}} \cdot (1 - 2t^2 + t^4)\, dt$$

$$= \int \left(t^{\frac{1}{2}} - 2 \cdot t^{\frac{5}{2}} + t^{\frac{9}{2}} \right) dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2 \cdot \frac{t^{\frac{7}{2}}}{\frac{7}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}}$$

$$= \frac{2}{3} \cdot (\sin x)^{\frac{3}{2}} - \frac{4}{7} (\sin x)^{\frac{7}{2}} + \frac{2}{11} \cdot (\sin x)^{\frac{11}{2}} + C$$

$$\int x \cdot e^{-x}\,dx = \left| \begin{matrix} u' = e^{-x} & u = -e^{-x} \\ v = x & v' = 1 \end{matrix} \right| =$$

$$= -x \cdot e^{-x} + \int e^{-x}\,dx =$$

$$= -x e^{-x} + \left(-e^{-x}\right) + C =$$

$$= -e^{-x} \cdot (x+1) + C$$

I.

$$\frac{A}{x-a}$$

$$\int \frac{3}{x-2}\,dx = \left| \begin{array}{l} t = x-2 \\ dt = dx \end{array} \right| = \int \frac{3}{t}\,dt =$$

$$= 3 \cdot \ln|t| + c = \underline{\underline{3 \cdot \ln|x-2| + c}}$$

$$\text{II.}\qquad \frac{A}{(x-a)^n}, \quad n \geq 2$$

$$\int \frac{3}{(x-2)^3}\,dx = \left| \begin{array}{l} t = x-2 \\ dt = dx \end{array} \right| =$$

$$= \int \frac{3}{t^3}\,dt = 3 \cdot \int t^{-3}\,dt =$$

$$= 3 \cdot \frac{t^{-2}}{-2} + C = \underline{\underline{-\frac{3}{2\cdot(x-2)^2} + C}}$$

III. $\dfrac{Ax+B}{x^2+px+q}$ (diskr. < 0 tj. $p^2 < 4q$)

$$\int \frac{3x+5}{x^2+4x+8}\,dx = \left| \begin{array}{l} \text{CHCI} \\ \text{NAHOŘE} \\ \text{DERIVACI} \\ \text{JMENOVATELE} \end{array} \right| =$$

$$= \frac{3}{2} \underbrace{\int \frac{2x+4}{x^2+4x+8}\,dx}_{I_1} - \underbrace{\int \frac{1}{x^2+4x+8}\,dx}_{I_2} = (*)$$

$$I_1 = \int \frac{2x+4}{x^2+4x+8}\,dx = \ln|x^2+4x+8| + C_1 =$$

$$= \ln(x^2+4x+8) + C_1$$

$$I_2 = \int \frac{1}{x^2+4x+8}\,dx = \int \frac{1}{(x+2)^2-4+8}\,dx =$$

$$= \int \frac{1}{(x+2)^2+4}\,dx = \int \frac{1}{4\cdot\left[\frac{(x+2)^2}{4}+1\right]}\,dx =$$

$$= \frac{1}{4}\cdot\int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}\,dx = \left| \begin{array}{l} t = \frac{x+2}{2} \\ dt = \frac{1}{2}\,dx \\ dx = 2\,dt \end{array} \right| =$$

$$= \frac{1}{4}\cdot\int \frac{2}{t^2+1}\,dt = \frac{1}{2}\cdot\int\frac{1}{t^2+1}\,dt =$$

$$= \frac{1}{2}\,\mathrm{arctg}\,\frac{x+2}{2} + c_2$$

$$\circledast = \frac{3}{2} \cdot \ln(x^2+4x+8) - \frac{\operatorname{arctg}\frac{x+2}{2}}{2} + C$$

IV. $\displaystyle \frac{Ax+B}{(x^2+px+q)^n}$ $\quad (n \geq 2,\ p^2 < 4q)$

$$\int \frac{3x+5}{(x^2+4x+8)^3}\,dx =$$

$$= \frac{3}{2} \cdot \underbrace{\int \frac{2x+4}{(x^2+4x+8)^3}\,dx}_{I_1} - \underbrace{\int \frac{1}{(x^2+4x+8)^3}\,dx}_{I_2}$$

$\parallel$

$(*)$

$$I_1 = \int \frac{2x+4}{(x^2+4x+8)^3}\,dx = \left| \begin{array}{l} t = x^2+4x+8 \\ dt = (2x+4)\,dx \end{array} \right| =$$

$$= \int \frac{1}{t^3}\,dt = \int t^{-3}\,dt =$$

$$= \frac{t^{-2}}{-2} + C_1 = -\frac{1}{2\cdot(x^2+4x+8)^2} + C_1$$

$$I_2 = \int \frac{1}{(x^2+4x+8)^3}\,dx = \int \frac{1}{\left[(x+2)^2+4\right]^3}\,dx =$$

$$= \int \frac{1}{\left[4\cdot\left(\left(\frac{x+2}{2}\right)^2+1\right)\right]^3}\,dx = \left| \begin{array}{l} t = \frac{x+2}{2} \\ dt = \frac{1}{2}\,dx \\ dx = 2\,dt \end{array} \right|$$

$$= \frac{1}{4^3}\cdot\int \frac{2}{(t^2+1)^3}\,dt =$$

$$= \frac{1}{32}\cdot\int \frac{1}{(t^2+1)^3}\,dt$$

$$K_{n+1} = \frac{1}{2n} \cdot \left[\frac{t}{(1+t^2)^n} + (2n-1) K_n \right]$$

$$\boxed{K_3 = \int \frac{1}{(1+t^2)^3}\, dt = ?}$$

$$K_1 = \int \frac{1}{1+t^2}\, dt = \operatorname{arctg} t \;(+c)$$

$$\boxed{K_2 = \frac{1}{2} \cdot \left[\frac{t}{1+t^2} + \operatorname{arctg} t \right]}$$

$n+1=2$

$n=1$

$$K_3 = \dots = \frac{1}{4}\cdot\frac{t}{(1+t^2)^2} + \frac{3}{8}\,\frac{t}{1+t^2} + \frac{3}{8}\operatorname{arctg} t$$

$(+c)$

$\circledast$ = viz ~~PREZEF~~ PREZENTACE

$$\int \frac{5\cdot \ln x}{x\cdot(\ln^3 x+\ln^2 x-2)}\,dx = \left| \begin{array}{l} t=\ln x \\ dt=\frac{1}{x}\,dx \end{array} \right| =$$

$$= \int \frac{5\cdot t}{t^3+t^2-2}\,dt = (*)$$

$$t^3+t^2-2=0$$

	1	1	0	-2
1	1	2	2	0 ✓

$$\Rightarrow (x-1)\cdot(x^2+2x+2) \to D=-4<0$$

$$\frac{5t}{t^3+t^2-2} = \frac{A}{t-1} + \frac{Bt+C}{t^2+2t+2} \quad \Big/ \cdot (t^3+t^2-2)$$

$$\vdots$$

$$A = 1, \quad B = -1, \quad C = 2$$

$$\circledast = \underbrace{\int \frac{1}{t-1}\,dt}_{I_1} + \underbrace{\int \frac{-t+2}{t^2+2t+2}\,dt}_{I_2} = \circledast\circledast$$

$= DÚ$ (viz přez.)