$I = [2, 5]$, $f(x) = x^2 - 3x + 5$

(i) $D = \{2; 2{,}5; 3; 4; 5\}$

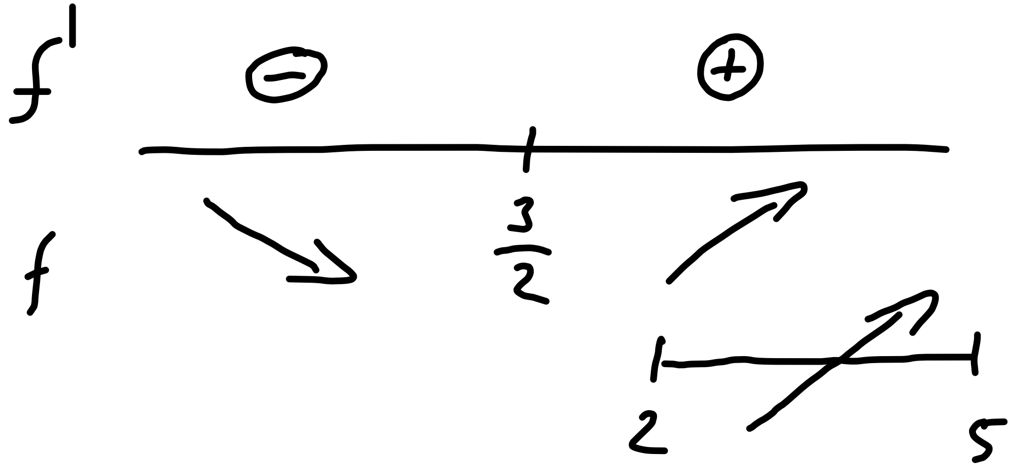
$$\boxed{s(D, f)} = \sum_{k=1}^{n} m_k \cdot (x_k - x_{k-1}) =$$

$$= \sum_{k=1}^{4} m_k \cdot (x_k - x_{k-1}) =$$

$$= 3 \cdot (2{,}5 - 2) + 3{,}75 \cdot (3 - 2{,}5) + 5 \cdot (4 - 3) +$$

$$+ 9 \cdot (5 - 4) = \ldots = \boxed{\frac{139}{8}} = \underline{17{,}375}$$

$$f'(x) = 2x - 3 = 0 \Leftrightarrow x = \frac{3}{2}$$



$$f(2) = 4 - 6 + 5 = 3$$

$$f(2,5) = 3,75$$

$$f(3) = 5$$

$$f(4) = 9$$

$$f(5) = 15$$

$$\boxed{S(D,f)} = \sum_{k=1}^{4} M_k \cdot (x_k - x_{k-1}) =$$

$$= 3{,}75 \cdot (2{,}5 - 2) + 5 \cdot (3 - 2{,}5) + 9 \cdot (4 - 3) +$$

$$+ 15 \cdot (5 - 4) = \dots = \boxed{\frac{227}{8}} = 28{,}375$$

(ii) $\displaystyle\int (x^2 - 3x + 5)\,dx = \frac{x^3}{3} - 3\cdot\frac{x^2}{2} + 5x + C =$

$$= \frac{x^3}{3} - \frac{3x^2}{2} + 5x + C$$

(iii) SP. $\Rightarrow$ 1 LT.

$$\int_2^5 f(x)\,dx = \int_2^5 (x^2 - 3x + 5)\,dx =$$

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 5x\right]_2^5 =$$

$$= \left(\frac{5^3}{3} - \frac{3\cdot 5^2}{2} + 5\cdot 5\right) - \left(\frac{2^3}{3} - \frac{3\cdot 2^2}{2} + 5\cdot 2\right) =$$

$$\boxed{= 22{,}5}$$

(iv) $\operatorname{av} f = \frac{1}{b-a} \cdot \int_a^b f(x)\,dx =$

$$= \frac{1}{5-2} \cdot \int_2^5 (x^2-3x+5)\,dx =$$

$$= \frac{1}{3} \cdot 22{,}5 = \underline{\underline{7{,}5}}$$

(n) SP. $\Rightarrow$ ANO

$$f(x_0) = 7{,}5 \quad , \quad x_0 = ?$$

$$x^2 - 3x + 5 = 7{,}5$$

$$\underbrace{x^2 - 3x - 2{,}5 = 0}$$

$$D = 19$$

$$x_{1,2} = \frac{3 \pm \sqrt{19}}{2} = \begin{cases} \dfrac{3+\sqrt{19}}{2} \doteq 3{,}68 \in I \\ \dfrac{3-\sqrt{19}}{2} \notin I \end{cases}$$

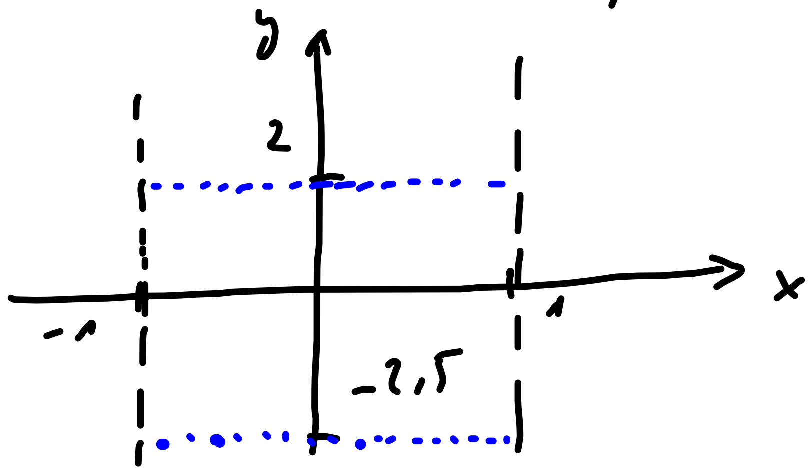
$$\Rightarrow \quad x_0 = \frac{3+\sqrt{19}}{2}$$

2)

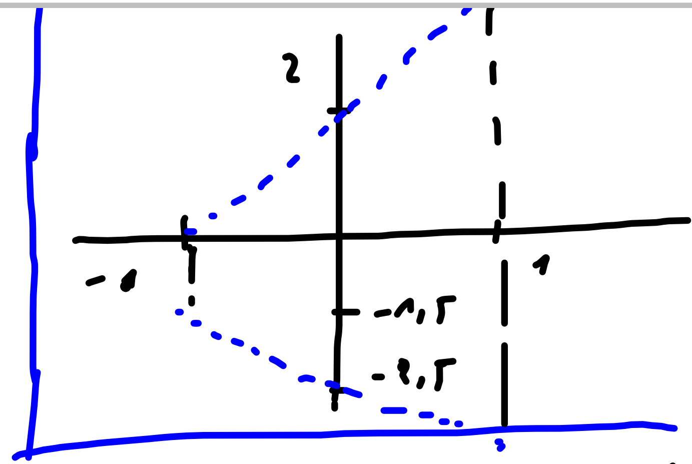
$$\underline{\int_a^b} f := \sup \{ s(D,f),\ D \in \mathbb{D} \}$$

$$\overline{\int_a^b} f := \inf \{ S(D,f),\ D \in \mathbb{D} \}$$

$$f(x) = \begin{cases} 2, & x \in \mathbb{Q} \\ -2{,}5, & x \in \mathbb{I} \ (= \mathbb{R} \setminus \mathbb{Q}) \end{cases}$$



$$f(x) = \begin{cases} 2x+2, & x \in \mathbb{Q} \\ -x-2{,}5, & x \in \mathbb{I} \end{cases}$$



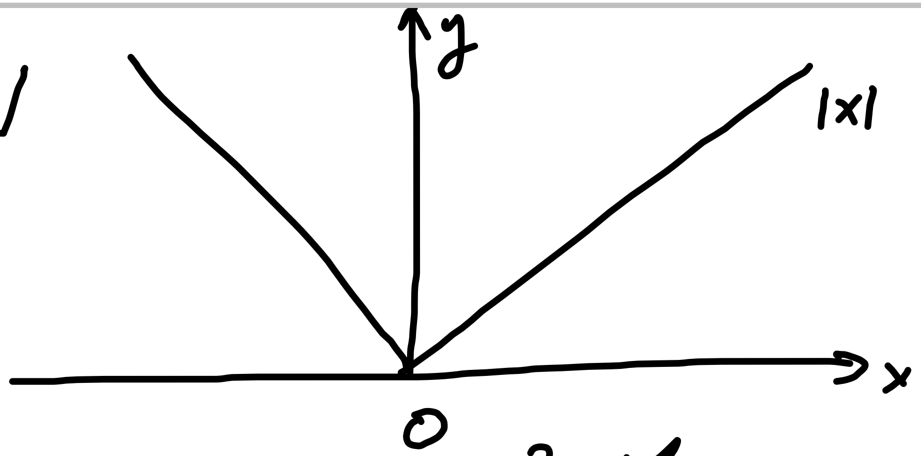
$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (2x+2)\,dx = \left[2\cdot\frac{x^2}{2} + 2x\right]_{-1}^{1} =$$

$$= (1+2) - (1-2) = 3 - 1 + 2 = \underline{\underline{4}}$$

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (-x-2{,}5)\,dx = \left[-\frac{x^2}{2} - 2{,}5x\right]_{-1}^{1} =$$

$$= \left(-\frac{1}{2} - 2{,}5\right) - \left(-\frac{1}{2} + 2{,}5\right) = -3 - 2 = \underline{\underline{-5}}$$

3/



$$s(D_n, f) = \sum_{k=1}^{2n+1} m_k \cdot (x_k - x_{k-1}) =$$

$$= \left|-\frac{n-1}{n}\right| \cdot \Big(\underbrace{-\frac{n-1}{n} + 1}_{\frac{-n+1+n}{n} = \frac{1}{n}}\Big) + \left|-\frac{n-2}{n}\right| \cdot \Big(\underbrace{-\frac{n-2}{n} + \frac{n-1}{n}}_{\frac{-n+2+n-1}{n} = \frac{1}{n}}\Big) + \vdots$$

$$= \left|-\frac{n-1}{n}\right| \cdot \frac{1}{n} + \left|-\frac{n-2}{n}\right| \cdot \frac{1}{n} + \dots =$$

$$= \frac{1}{n}\cdot\left(\underset{\left[-\frac{n-1}{n},\,-1\right]}{\frac{n-1}{n}} + \frac{n-2}{n} + \dots + \underset{\left[-\frac{2}{n},\,-\frac{1}{n}\right]}{\frac{1}{n}} + \underset{\left[-\frac{1}{n},\,0\right]}{0} + \underset{\left[0,\,\frac{1}{n}\right]}{0} + \frac{1}{n} + \dots\right.$$

$$\left.\dots + \frac{n-1}{n}\right) =$$

$$= \frac{1}{n^2}\cdot 2\cdot\Big(1 + 2 + 3 + \dots + (n-2) + (n-1)\Big) =$$

$$= \frac{\cancel{2}}{n^{\cancel{2}}}\cdot\frac{(n-1)\cdot\cancel{n}}{\cancel{2}} = \boxed{\frac{n-1}{n}}$$

$$1+2+3+4+5+\ldots+98+99+100$$

$$100+99+98+97+96+\ldots+3+2+1$$

$$101+101+\ldots 101+101+101$$

$$100\cdot 101$$

$$\text{SOUČET }(1\sim 100)=\frac{100\cdot 101}{2}=\frac{10100}{2}$$

$$S(D_n, f) = |-1| \cdot \left(-\frac{n-1}{n} + 1\right) + \dots =$$

$$= \frac{1}{n} \cdot \Big(|-1| + \left|-\frac{n-1}{n}\right| + \dots + \underset{[-\frac{1}{n}, 0]}{\underset{\uparrow}{\left|-\frac{1}{n}\right|}} + \underset{[0, \frac{1}{n}]}{\underset{\uparrow}{\left|\frac{1}{n}\right|}} + \dots$$

$$\dots + \left|\frac{n-1}{n}\right| + |1| \Big) =$$

$$= \frac{1}{n} \cdot 2 \cdot \frac{1}{n} \cdot \big(n + (n-1) + (n-2) + \dots + 1 \big) =$$

$$= \frac{\cancel{2}}{n^{\cancel{2}}} \cdot \frac{\cancel{n} \cdot (n+1)}{\cancel{2}} = \boxed{\frac{n+1}{n}}$$

(i) $\int_0^1 \operatorname{arctg} x \, dx = \int_0^1 1 \cdot \operatorname{arctg} x \, dx =$

$$= \left| \begin{array}{ll} u' = 1 & u = x \\ v = \operatorname{arctg} x & v' = \frac{1}{1+x^2} \end{array} \right| =$$

$$= \left[x \cdot \operatorname{arctg} x \right]_0^1 - \int_0^1 \frac{x}{1+x^2} \, dx = \left| \begin{array}{l} t = 1+x^2 \\ dt = 2x\,dx \\ x\,dx = \frac{1}{2} dt \\ \hline x = 1 \\ \Rightarrow t = 1+1^2 \\ \quad = 2 \\ x = 0 \\ \Rightarrow t = 1 \end{array} \right.$$

$$= \left(1 \cdot \frac{\pi}{4} - 0\right) - \int_1^2 \frac{1}{t} \cdot \frac{1}{2} \, dt =$$

$$= \frac{\pi}{4} - \frac{1}{2} \cdot \int_1^2 \frac{1}{t}\, dt = \frac{\pi}{4} - \frac{1}{2} \cdot \left[\ln t \right]_1^2 =$$

$$= \frac{\pi}{4} - \frac{1}{2} \cdot \left(\ln 2 - \ln 1 \right) =$$

$$= \frac{\pi}{4} - \frac{\ln 2}{2}$$

(ii) $\displaystyle\int_1^2 x\cdot\sqrt{1+x^2}\,dx = \left| \begin{array}{l} t^2 = 1+x^2 \\ 2t\,dt = 2x\,dx \\ x\,dx = t\,dt \end{array} \right| \begin{array}{l} x=2 \\ t^2 = 2^2+1 = 5 \\ t = \sqrt{5} \\ \hline x = 1 \\ t^2 = 2 \\ t = \sqrt{2} \end{array}$

$$= \int_{\sqrt{2}}^{\sqrt{5}} t\cdot t\;dt = \int_{\sqrt{2}}^{\sqrt{5}} t^2\,dt = \left[\frac{t^3}{3}\right]_{\sqrt{2}}^{\sqrt{5}} =$$

$$= \frac{5\sqrt{5}}{3} - \frac{2\sqrt{2}}{3} = \underline{\underline{\frac{1}{3}\cdot\left(5\sqrt{5}-2\sqrt{2}\right)}}$$

(iii) $\displaystyle\int_0^1 \sqrt{2-x^2}\,dx = \left| \begin{array}{l} x = \sqrt{2}\cdot\cos t \\ dx = \sqrt{2}\cdot(-\sin t)\,dt \\ \hline x = 1 \Rightarrow t = \frac{\pi}{4} \\ x = 0 \Rightarrow t = \frac{\pi}{2} \end{array} \right| =$

$$= \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \sqrt{2-2\cdot\cos^2 t}\cdot\sqrt{2}\cdot(-1)\cdot\sin t\,dt =$$

$$= +\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{2\cdot(1-\cos^2 t)}\cdot\sqrt{2}\cdot\sin t\,dt =$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{2} \cdot \sqrt{\underbrace{1 - \cos^2 t}_{\sin^2 t}} \cdot \sqrt{2} \cdot \sin t \; dt =$$

$$= 2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin t \cdot \sin t \; dt = 2 \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 t \; dt =$$

$$= 2 \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2t}{2} \; dt = \cancel{2} \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cancel{\tfrac{1}{2}} \; dt - \cancel{2} \cdot \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cancel{\tfrac{1}{2}} \cos 2t \; dt$$

$$= \left[t \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2t \, dt =$$

$$= \frac{\pi}{4} - \left[\frac{\sin 2t}{2} \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$= \frac{\pi}{4} - \left[\frac{0}{2} - \frac{1}{2} \right] = \frac{\pi}{4} + \frac{1}{2} = \frac{\pi + 2}{4}$$

(i) $\int \sin^3 x \cdot \cos^4 x \, dx =$

$$= \int \underbrace{\sin^2 x}_{1-\cos^2 x} \cdot \cos^4 x \cdot \sin x \, dx = \left| \begin{array}{l} t = \cos x \\ \underbrace{-dt = \sin x \, dx} \end{array} \right|$$

$$= -\int (1-t^2) \cdot t^4 \, dt = \int (t^6 - t^4) \, dt =$$

$$= \frac{t^7}{7} - \frac{t^5}{5} + C = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C$$

(ii) $\int \sin^5 x \, dx = \int (1-\cos^2 x)^2 \cdot \sin x \, dx =$

$$= \left| \begin{array}{l} t = \cos x \\ -dt = \sin x \, dx \end{array} \right| = -\int (1-t^2)^2 \, dt =$$

$$= -\int (1 - 2t^2 + t^4) \, dt =$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{\cos^5 x}{5} + C$$

(iii) $$\int e^{-x^3}\cdot x^2\,dx = \left|\begin{array}{l} t=-x^3 \\ dt=-3x^2\,dx \\ x^2\,dx=-\frac{1}{3}\,dt \end{array}\right| =$$

$$= \int e^t\cdot\left(-\frac{1}{3}\right)dt = -\frac{1}{3}\cdot e^t + C =$$

$$= \underline{\underline{-\frac{1}{3}\cdot e^{-x^3}+C}}$$

(i) $\int \cos^3 x \cdot \sin x \, dx = \left| \begin{array}{l} t = \cos x \\ -dt = \sin x \, dx \end{array} \right| =$

$= -\int t^3 \, dt = -\dfrac{\cos^4 x}{4} + C$

$= \left| \begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array} \right| = \int (1 - t^2) \cdot t \, dt =$

$= \int (t - t^3) \, dt = \dfrac{\sin^2 x}{2} - \dfrac{\sin^4 x}{4} + \tilde{C}$

(ii) $\displaystyle\int \frac{1+\cos^2 x}{1+\cos 2x}\,dx = \int \frac{1+\cos^2 x}{1+\cos^2 x-\sin^2 x}\,dx =$

$$= \int \frac{1+\cos^2 x}{1+\cos^2 x-1+\cos^2 x}\,dx =$$

$$= \int \frac{1+\cos^2 x}{2\cdot\cos^2 x}\,dx = \frac{1}{2}\int \frac{1}{\cos^2 x}\,dx + \frac{1}{2}\int 1\,dx =$$

$$= \frac{1}{2}\,\mathrm{tg}\,x + \frac{x}{2} + C$$

(iii) $\int 2 \cdot \sin^2 \frac{x}{2} \, dx =$

$= \int \left(\sin^2 \frac{x}{2} + \sin^2 \frac{x}{2} \right) dx =$

$= \int \left(\sin^2 \frac{x}{2} + 1 - \cos^2 \frac{x}{2} \right) dx =$

$= \int \left[1 - \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \right) \right] dx =$

$= \int (1 - \cos x) \, dx = \underline{\underline{x - \sin x + C}}$

$$\text{(i)} \int_0^{\frac{\pi}{2}} \cos x \, dx = \left[\sin x \right]_0^{\frac{\pi}{2}} = 1 - 0 = \underline{\underline{1}}$$

$$\text{(ii)} \int_0^{\ln 2} \frac{x}{e^x} \, dx = \left| \begin{array}{ll} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{array} \right| =$$

$$= \left[-x \cdot e^{-x} \right]_0^{\ln 2} + \int_0^{\ln 2} e^{-x} \, dx =$$

$$= \left[\left(-\ln 2 \cdot e^{\ln \frac{1}{2}} - 0 \cdot e^0 \right) \right] + \left[-e^{-x} \right]_0^{\ln 2} =$$

$$= -\frac{\ln 2}{2} + \left(-e^{\ln 2^{-1}} + e^0 \right) = \underline{\underline{-\frac{\ln 2}{2} - \frac{1}{2} + 1}}$$

(iii) $\int_0^1 x\cdot(2-x^2)^5\,dx = \left| \begin{array}{l} t=2-x^2 \\ dt=-2x\,dx \\ x\,dx=-\frac{1}{2}\,dt \end{array} \right| \left. \begin{array}{l} x=1 \\ t=1 \\ \hline x=0 \\ t=2 \end{array} \right|$

$$= -\frac{1}{2}\int_2^1 t^5\,dt = \frac{1}{2}\int_1^2 t^5\,dt = \frac{1}{2}\cdot\left[\frac{t^6}{6}\right]_1^2 =$$

$$= \frac{1}{2}\cdot\frac{1}{6}\cdot\left(2^6-1^6\right) = \underline{\underline{\frac{63}{12}}}$$

(iv) $\displaystyle\int_1^{e^8} \frac{dx}{x\cdot\sqrt{\ln x+1}} = \left| \begin{array}{l|l} t = \ln x + 1 & x = e^8 \\ dt = \frac{1}{x}\,dx & t = 8+1 = 9 \\ \hline & x = 1 \\ & t = 0+1 = 1 \end{array} \right| =$

$$= \int_1^9 \frac{1}{\sqrt{t}}\,dt = \int_1^9 t^{-\frac{1}{2}}\,dt =$$

$$= \left[\frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^9 = \left[2\sqrt{t} \right]_1^9 = 6 - 2 = \underline{\underline{4}}$$