9.8/ $\displaystyle\int_2^{\infty} \frac{dx}{x^2+x-2} = \left| \text{PARC. ZL.} \right| =$

$$= \int_2^{\infty} \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{3}}{x+2}\, dx =$$

$$= \left[\frac{1}{3} \cdot \ln|x-1| - \frac{1}{3} \cdot \ln|x+2| \right]_2^{\infty} =$$

$$= \frac{1}{3} \cdot \lim_{x \to \infty} \left(\ln(x-1) - \ln(x+2) \right) -$$

$$- \frac{1}{3} \cdot \left(\ln 1 - \ln 4 \right) =$$

$$= \frac{1}{3} \cdot \lim_{x \to \infty} \ln \frac{x-1}{x+2} + \frac{1}{3} \cdot 2 \ln 2 =$$

$$= \frac{1}{3} \cdot \lim_{x \to \infty} \ln \frac{1 - \frac{1}{x}}{1 + \frac{2}{x}} + \frac{2}{3} \ln 2 =$$

$$= \frac{1}{3} \cdot \ln 1 + \frac{2}{3} \ln 2 = \underline{\underline{\frac{2}{3} \ln 2}}$$

b) $f(x) = (x+1)\cdot \dfrac{x^2-x+1}{x^2+x-2} = \dfrac{x^3+1}{x^2+x-2}$

na $[2, \infty)$ $\quad f > g$

$$S = \int_2^{\infty} (f-g)\,dx =$$

$$= \int_2^{\infty} \frac{1}{x^2+x-2}\,dx = \left| \text{viz a)} \right| = \underline{\underline{\frac{2}{3} \ln 2}}$$

$(\doteq 0{,}462)$

10.1/

$$\int_0^4 \frac{2x^2+\sqrt{x}}{x}\,dx = \int_0^4 \frac{2x^2}{x}\,dx + \int_0^4 \frac{\sqrt{x}}{x}\,dx$$

$$= 2\cdot\int_0^4 x\,dx + \int_0^4 x^{-\frac{1}{2}}\,dx =$$

$$= 2\cdot\left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^4 =$$

$$= (16-0) + (2\cdot 2-0) = \underline{\underline{20}}$$

$$\int_0^{\frac{3}{2}\pi} \frac{2\cdot \cos x}{1+\sin x}\,dx =$$

$t = \sin x$

$dt = \cos x\, dx$

$x = \frac{3}{2}\pi \Rightarrow t = \sin\frac{3}{2}\pi =$

$= -1$

$x = 0 \Rightarrow t = \sin 0 = 0$

$$= \int_0^{-1} \frac{2}{1+t}\,dt = -2\int_{-1}^{0} \frac{1}{1+t}\,dt =$$

$$= -2\cdot \Big[\ln|1+t|\Big]_{-1}^{0} =$$

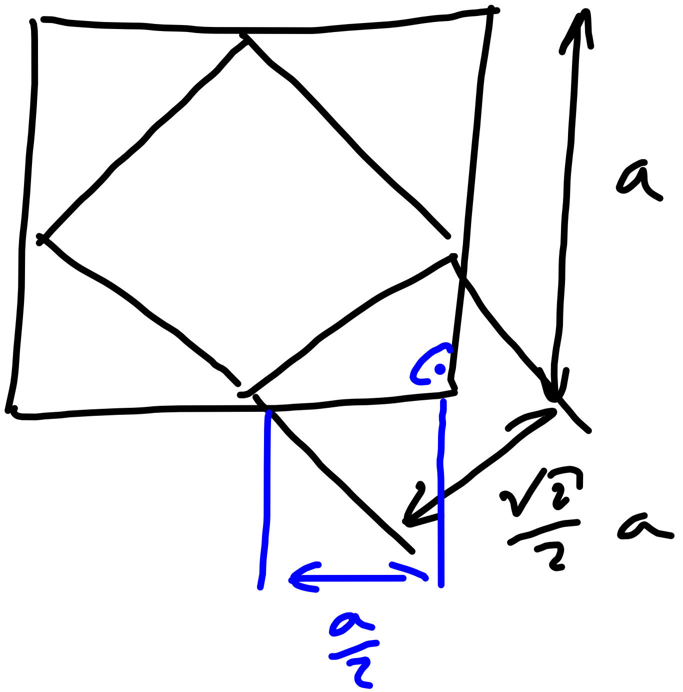
$$= -2 \cdot \left(\ln 1 - \lim_{x \to -1^+} \ln(1+x) \right) =$$

$$= 2 \cdot (-\infty) = \underline{\underline{-\infty}}$$

$$\sum S_n = a^2 + \left(\frac{\sqrt{2}}{2}a\right)^2 + \left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\cdot a\right)^2 + \dots$$

$$= a^2 + \frac{1}{2}a^2 + \frac{1}{4}a^2 + \dots =$$

$$= a^2\cdot\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right) =$$



$$= a^2\cdot\left(\underbrace{\frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \dots}_{\text{geom. ř.}}\right) =$$

$\rightarrow q = \frac{1}{2}$

$$= a^2\cdot\frac{1}{1-\frac{1}{2}} = \underline{\underline{2a^2}}$$

$a = \frac{1}{4}\,m \Rightarrow S = \frac{1}{8}\,m^2$

$$\sum o_n = 4a + 4\left(\frac{\sqrt{2}}{2}a\right) + 4\cdot\left(\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}\cdot a\right) + \dots$$

$$= 4a\cdot\underbrace{\left(1 + \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)^2 + \dots\right)}_{\text{g. ř. s } q = \frac{\sqrt{2}}{2}} =$$

$$= 4a\cdot\frac{1}{1-\frac{\sqrt{2}}{2}} = \underline{\underline{4a\left(2+\sqrt{2}\right)}}$$

$$a = \frac{1}{4}\,m \Rightarrow \quad 2+\sqrt{2}\ m \doteq \underline{\underline{3{,}41\,m}}$$

$$\sum_{0}^{\infty} \frac{1}{(n+1)\cdot 3^n}$$

$$\frac{1}{(n+1)\cdot 3^n} \leq \frac{1}{3^n}$$

KONV.:

geom. $\Rightarrow q = \frac{1}{3}$

$$\sum_{0}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{1-\frac{1}{3}}$$

$\Downarrow$

KONV.

$$\sum_{1}^{\infty} \frac{n^2+1}{n^3} = \sum_{1}^{\infty} \left(\frac{1}{n} + \frac{1}{n^3} \right)$$

$$\frac{1}{n} + \frac{1}{n^3} \geq \frac{1}{n} \quad \text{DIV.}$$

$$\Downarrow$$

DIV.

NUTNÁ PODM. KONV. :

$$\boxed{\lim_{n\to\infty} a_n = 0}$$

$$\sum_{1}^{\infty} \frac{1}{(n+1)\cdot \ln^2(n+1)}$$

$$f(x) = \frac{1}{(x+1)\cdot \ln^2(x+1)}$$ $\boxed{?\text{NEROST.}?}$

$$f'(x) = \frac{0 - \left[1\cdot \ln^2(x+1) + (x+1)\cdot 2\cdot \ln(x+1)\cdot \frac{1}{x+1}\right]}{(x+1)^2\cdot \ln^4(x+1)}$$

$$= \ominus \frac{\ln^2(x+1) + 2\ln(x+1)}{(x+1)^2\cdot \ln^4(x+1)} \leq 0 \Rightarrow \text{NEROST.}$$

$$\int_1^\infty \frac{1}{(x+1)\cdot \ln^2(x+1)}\,dx = \left| \begin{array}{l} t = \ln(x+1) \\ dt = \frac{1}{x+1}\cdot dx \end{array} \right|$$

$$\left| \begin{array}{l} x=\infty \Rightarrow t=\infty \\ x=1 \Rightarrow t=\ln 2 \end{array} \right|$$

$$= \int_{\ln 2}^\infty \frac{1}{t^2}\,dt = \left[-t^{-1} \right]_{\ln 2}^\infty =$$

$$= \lim_{t\to\infty} \left(-\frac{1}{t}\right) + \frac{1}{\ln 2} = \frac{1}{\ln 2} < \infty$$

$\Rightarrow$ ŘADA KONV.

$$\sum_{0}^{\infty} \boxed{\frac{(n+1)!}{2^n \cdot n!}}$$

a_n

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \begin{cases} <1 \\ \Rightarrow K \\ \hline >1 \\ \Rightarrow D \end{cases}$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{(n+2)!}{2^{n+1}\cdot(n+1)!}}{\dfrac{(n+1)!}{2^n\cdot n!}} =$$

$$= \frac{\dfrac{(n+2)\cdot\cancel{(n+1)!}}{2\cdot\cancel{2^n}\cdot\cancel{(n+1)!}}}{\dfrac{(n+1)!}{\cancel{2^n}\cdot n!}} = \frac{\dfrac{n+2}{2}}{\dfrac{n+1}{1}} = \frac{n+2}{2(n+1)}$$

$$\lim_{n\to\infty} \frac{n+2}{2n+2} = \frac{1}{2} < 1 \Rightarrow \underline{\underline{\text{KONV.}}}$$

$$\sum_{25}^{\infty} \frac{n}{2n-1}$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{n+1}{2(n+1)-1}}{\dfrac{n}{2n-1}} = \frac{\dfrac{n+1}{2n+1}}{\dfrac{n}{2n-1}} =$$

$$= \frac{(n+1)\cdot(2n-1)}{(2n+1)\cdot n} = \frac{2n^2+n-1}{2n^2+n}$$

$$\lim_{n\to\infty} \frac{2n^2+n-1}{2n^2+n} = \lim_{n\to\infty} \frac{2+\frac{1}{n}-\frac{1}{n^2}}{2+\frac{1}{n}} = \underline{\underline{1}}$$

$\Rightarrow$ NELZE ROZH.

ALE :

$$\lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2} \neq 0$$

NESPL. NUTNOU PODM.

$\Longrightarrow$ DIV

$$\sum_{0}^{\infty} \frac{3^n}{2^n(2n+1)}$$

$$\frac{a_{n+1}}{a_n} = \frac{\dfrac{\cancel{3^n}\cdot 3}{\cancel{2^n}\cdot 2\cdot(2n+3)}}{\dfrac{\cancel{3^n}}{\cancel{2^n}\cdot(2n+1)}} =$$

$$= \frac{\dfrac{3}{4n+6}}{\dfrac{1}{2n+1}} = \frac{3\cdot(2n+1)}{4n+6} \xrightarrow{n\to\infty} \frac{6}{4} > 1$$

$\Rightarrow$ **DIV.**

$$\sum_{1}^{\infty} \frac{1}{\ln^n(n+1)}$$

$$\boxed{\lim \sqrt[n]{a_n}}$$

$$\sqrt[n]{a_n} = \sqrt[n]{\frac{1}{\ln^n(n+1)}} = \frac{1}{\ln(n+1)}$$

$$\lim_{n \to \infty} \frac{1}{\ln(n+1)} = \left\| \frac{1}{\ln \infty} \right\| = 0 < 1$$

$\Rightarrow$ Konv.

$$\sum_{1}^{\infty} \frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n}$$

$$\sqrt[n]{a_n} = \frac{\left(\frac{n+1}{n}\right)^{n}}{3} = \frac{1}{3} \cdot \left(\frac{n+1}{n}\right)^{n}$$

$$\frac{1}{3}\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{n} = \left| 1^{\infty} \right| = \frac{1}{3} \cdot \lim_{n\to\infty} e^{n \cdot \ln \frac{n+1}{n}} = \circledast$$

$$\lim_{n\to\infty} n \cdot \ln\left(\frac{n+1}{n}\right) = \left| \infty \cdot 0 \right| =$$

$$= \lim_{n\to\infty} \frac{\ln\left(\frac{n+1}{n}\right)}{\frac{1}{n}} = \left| \frac{0}{0} \right| \overset{\text{L'H}}{=}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{\frac{n+1}{n}} \cdot \cancel{\left(-\frac{1}{n^2}\right)}}{\cancel{-\frac{1}{n^2}}} =$$

$$= \lim_{n\to\infty} \frac{n}{n+1} = \underline{\underline{1}}$$

$$(*) = \frac{1}{3}\cdot e = \frac{e}{3} < 1$$

$\Rightarrow$ <u>ŘADA KONV.</u>

$$\sum_{0}^{\infty} (-1)^n \underbrace{\frac{1}{3n-1}}_{a_n} \qquad \sum (-1)^n \cdot a_n$$

ALT. ŘADA : $\lim a_n = 0 \Rightarrow$ KONV.

$$\lim_{n\to\infty} \frac{1}{3n-1} = 0 \Rightarrow \text{ř. konv.}$$

$$\sum_{0}^{\infty} (-1)^n \frac{\sqrt{n}}{n+100}, \quad \lim_{n\to\infty} \frac{\sqrt{n}}{n+100} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{1}{n}}}{1 + \frac{100}{n}} = 0 \Rightarrow \underline{\underline{\text{KONV.}}}$$

$$a_9 = (-1)^9 \cdot \frac{\sqrt{9}}{109} = -\frac{3}{109} \Rightarrow$$

$$\Rightarrow \text{ZBYTEK} < \frac{3}{109}$$

& ZNAM. $-$

$$a_{10000} = (-1)^{10000} \cdot \frac{\sqrt{10\,000}}{10\,100} = \frac{100}{10\,100} =$$

$$= \frac{1}{101} \Rightarrow \text{CHYBA} < \frac{1}{101}$$

& ZNAM. $+$

$$\sum_{0}^{\infty} (-1)^n \left(\frac{2n-1}{3n+2}\right)^n$$

$$\sum_{0}^{\infty} \left| (-1)^n \cdot \left(\frac{2n-1}{3n+2}\right)^n \right| \quad \begin{matrix} \nearrow K \\ \searrow D \end{matrix} \quad ?$$

ODM. KRIT.: $\sqrt[n]{\left(\frac{2n-1}{3n+2}\right)^n} = \frac{2n-1}{3n+2}$

$$\lim_{n\to\infty} \frac{2n-1}{3n+2} = \frac{2}{3} < 1 \Rightarrow \text{KONV.}$$

$\Rightarrow$ ŘADA KONV. ABS.

$$\sum_{0}^{\infty} (-1)^n \frac{1}{n+1}$$

$$\left|(-1)^n \frac{1}{n+1}\right| = \frac{1}{n+1} \qquad \text{DIV.}$$

$$\lim_{n\to\infty} \frac{1}{n+1} = 0 \Rightarrow \text{ALT. Ř. KONV.}$$

$$\Rightarrow \text{KON. RELATIVNĚ}$$

$$\sum_{0}^{\infty} \frac{n\,x}{e^{nx}}$$

$$\left|\frac{n\,x}{e^{nx}}\right| \qquad \text{PODÍL. KRIT.}$$

$$\left|\frac{\frac{(n+1)\cdot \cancel{x}}{e^{(n+1)\cdot x}}}{\frac{n\cancel{x}}{e^{nx}}}\right| = \left|\frac{\frac{n+1}{\cancel{e^{nx}}\cdot e^{x}}}{\frac{n}{\cancel{e^{nx}}}}\right| = \left|\frac{n+1}{n\cdot e^{x}}\right| =$$

$$= \left|\frac{n+1}{n}\right|\cdot e^{-x} \xrightarrow{n\to\infty} 1\cdot e^{-x} = e^{x}$$

$$= \frac{1}{e^x} > 0 \quad \forall x \in \mathbb{R}$$

$$\begin{cases} < 1 & x > 0 & \text{KONV.} \\ > 1 & x < 0 & \text{DIV.} \\ = 1 & x = 0 & \Rightarrow \sum 0 = 0 \end{cases}$$

$\underline{\underline{\text{KON.}}}$

KONV. ABS. PRO $x \in [0, \infty)$

$$\sum (x - x_0)^n \cdot a_n$$

x_0 ... STŘED

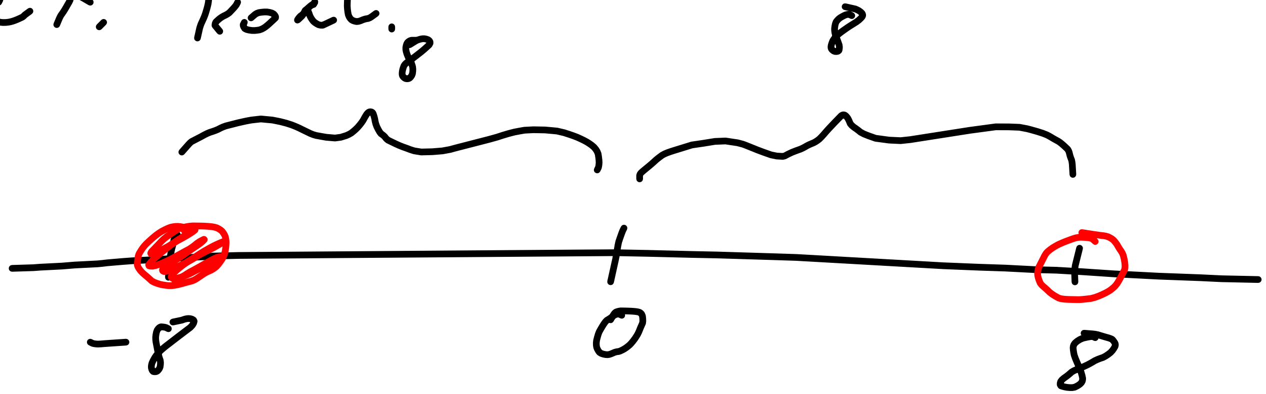
$$\sum_{0}^{\infty} \frac{x^n}{(n+1)\cdot 8^n} = \sum_{0}^{\infty} x^n \cdot \underbrace{\frac{1}{(n+1)\cdot 8^n}}_{a_n}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \frac{\frac{1}{(n+2)\cdot 8\cdot \cancel{8^n}}}{\frac{1}{(n+1)\cdot \cancel{8^n}}} =$$

$$= \lim_{n\to\infty} \frac{n+1}{8n+16} = \frac{1}{8} = \frac{1}{R}$$

$$\Rightarrow \boxed{R = 8}$$

INT. KONV.



$\underline{x=-8}$: $\sum_{0}^{\infty} \frac{(-8)^n}{(n+1)\,8^n} = \sum_{0}^{\infty} \frac{(-1)^n}{n+1}$ KONV.

$\underline{x=8}$: $\sum_{0}^{\infty} \frac{8^n}{(n+1)\cdot 8^n} = \sum_{0}^{\infty} \frac{1}{n+1}$ DIV.

$\Rightarrow$ INT. KON. $= \underline{[-8, 8)}$

$$\sum_{1}^{\infty} n^n (x-5)^n \quad , \quad \underline{\underline{S=5}}$$

$$a_n = n^n$$

$$\lim_{n\to\infty} \sqrt[n]{n^n} = \lim_{n\to\infty} n = \infty = \frac{1}{R}$$

$$\Rightarrow \underline{\underline{R = 0}}$$

KONV. POUZE VE SVÉM STŘEDU

$(I = [5,5])$

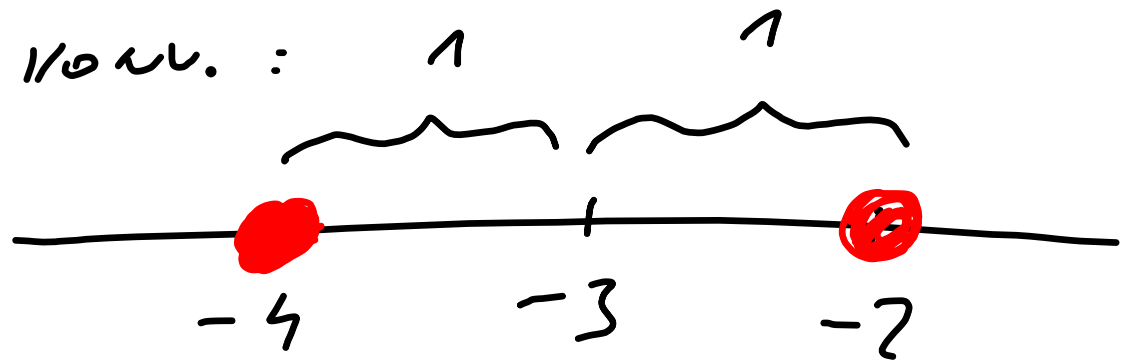
$$\sum_{1}^{\infty} \frac{(x+3)^n}{n^2} \quad , \quad S = -3$$

$$a_n = \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+2n+1} =$$

$$= 1 = \frac{1}{R} \Rightarrow R = 1$$

INT. KONV.:



$\underline{x = -4}$ $\quad \sum \frac{(-1)^n}{n^2}$ $\quad$ ALT. & $\frac{1}{n^2} \to 0$

$\Rightarrow$ KONV.

$\underline{x = -2}$ $\quad \sum \frac{1^n}{n^2} = \sum \frac{1}{n^2}$ $\Rightarrow$ KONV.

INT. KONV. = $\underline{\underline{[-4, -2]}}$