řádek	r	N	S
1.	$1/2$	2	$\pi\left(\frac{1}{2}\right)^2 = \pi\cdot\frac{1}{2^2}$
2.	$1/2^2$	2^2	$\pi\cdot\left(\frac{1}{2^2}\right)^2\cdot 2 = \pi\cdot\frac{1}{2^3}$
3.	$1/2^3$	2^3	$\pi\cdot\frac{1}{2^4}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n.$	$1/2^n$	2^n	$\pi\left(\frac{1}{2^n}\right)^2\cdot 2^{n-1} = \pi\cdot\frac{1}{2^{n+1}}$

$$S = \sum_{n=1}^{\infty} \pi\cdot\frac{1}{2^{n+1}} = \pi\cdot\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n+1} = \left| \begin{array}{l} a = \frac{1}{4} \\ q = \frac{1}{2} \end{array} \right| =$$

$$= \pi\cdot\frac{\frac{1}{4}}{1-\frac{1}{2}} = \underline{\underline{\frac{\pi}{2}}}$$

Př.: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots =$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1} \cdot (-1)^n}{(2n+1)!}$$

$\cos x = (\sin x)' = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots =$

$$= \sum_{n=0}^{\infty} \frac{x^{2n} \cdot (-1)^n}{(2n)!}$$

Př.: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\Big/ \frac{d}{dx}$

$(e^x)' = 0 + 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\frac{\cancel{3}x^2}{\cancel{3}\cdot 2 \cdot 1}$

$\left(\frac{x^n}{n!}\right)' = \frac{\cancel{n}\cdot x^{n-1}}{\cancel{n}\cdot (n-1)!} =$

$= \frac{x^{n-1}}{(n-1)!}$

př.: $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \qquad -1 \le x \le 1$

$$f'(x) = 1 - x^2 + x^4 - \dots =$$

$$= \sum_{n=0}^{\infty} (-1)^n \cdot x^{2n} = \left| \begin{array}{l} a = 1 \\ q = -x^2 \end{array} \right| = \underline{\underline{\frac{1}{1+x^2}}}$$

$$f'(x) = \frac{1}{1+x^2} \quad / \int$$

$$f(x) = \operatorname{arctg} x \underline{+ C}$$

$x = 0 \Rightarrow f(0) = 0$

$0 = \operatorname{arctg} 0 + c$

$\underline{\underline{c = 0}}$

$\Rightarrow \underline{\underline{f(x) = \operatorname{arctg} x}}$

Př.: $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2} = \frac{\sin 1}{1} + \frac{\sin 2}{4} + \dots$

$$\sum_{n=1}^{\infty} \left| \frac{\sin n}{n^2} \right| = \frac{|\sin 1|}{1} + \frac{|\sin 2|}{4} + \frac{|\sin 3|}{9} + \dots$$

$$\leq \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots = \sum_{n=1}^{\infty} \frac{1}{n^2}$$

KONV.

MAJORANTA

$\Leftarrow$

Ř. KONV. ABS.

př.:

$$\sum_{n=1}^{\infty} n\cdot(n+1)\cdot x^n = \sum_{n=1}^{\infty} n\cdot\left(x^{n+1}\right)' =$$

$$= \sum_{n=1}^{\infty}\left(n\cdot x^{n+1}\right)' = \left(\sum_{n=1}^{\infty} n\cdot x^{n+1}\right)' =$$

$$= \left(\sum_{n=1}^{\infty} x^2\cdot n\cdot x^{n-1}\right)' = \left(\sum_{n=1}^{\infty} x^2\left(x^n\right)'\right)' =$$

$$= \left(x^2\cdot\sum_{n=1}^{\infty}\left(x^n\right)'\right)' = \left[x^2\cdot\left(\sum_{n=1}^{\infty} x^n\right)'\right]' = \left|\begin{matrix} a = x \\ q = x \end{matrix}\right|$$

$$= \left[x^2 \cdot \left(\frac{x}{1-x} \right)' \right]' = \left[x^2 \cdot \frac{1}{(1-x)^2} \right]' =$$

$$= \left[\frac{x^2}{(1-x)^2} \right]' = \underline{\underline{\frac{2x}{(1-x)^3}}}$$

př.: $$\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{x^{n+1}}{n \cdot (n+1)} =$$

$$= \sum_{n=1}^{\infty} \left[(-1)^{n+1} \frac{1}{n} \int x^{n} \, dx \right] =$$

$$= \sum_{n=1}^{\infty} \left[\int (-1)^{n+1} \frac{1}{n} x^{n} \, dx \right] =$$

$$= \int \left[\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} x^{n} \right] dx =$$

$$= \int \sum_{n=1}^{\infty} \left[(-1)^{n+1} \cdot \int x^{n-1}\, dx \right] dx =$$

$$= \int \left[\int \sum_{n=1}^{\infty} (-1)^{n+1} \cdot x^{n-1}\, dx \right] dx =$$

$$= \int \left[\int (1 - x + x^2 - x^3 + x^4 - \dots)\, dx \right] dx = \left| \begin{array}{l} a = 1 \\ q = -x \end{array} \right|$$

$$= \int \left[\int \frac{1}{1+x}\, dx \right] dx =$$

$$= \int \ln(1+x)\,dx = \left| \begin{array}{ll} u' = 1 & u = x \\ v = \ln(1+x) & v' = \frac{1}{1+x} \end{array} \right| =$$

$$= x \cdot \ln(1+x) - \int \frac{x}{1+x}\,dx =$$

$$= x \cdot \ln(1+x) - \int \frac{x+1-1}{1+x}\,dx =$$

$$= x \cdot \ln(1+x) - \int \left(1 - \frac{1}{1+x}\right) dx =$$

$$= x \cdot \ln(1+x) - x + \ln(1+x) + C =$$

$$= \ln(1+x) \cdot (x+1) - x + C$$

$x = 0 \Rightarrow$ SOUČET $= 0$

$$0 = \ln 1 \cdot 1 - 0 + C \Rightarrow C = 0$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{n \cdot (n+1)} = \ln(1+x) \cdot (x+1) - x$$

$\underline{\underline{\text{zní.:}}}$ $$\frac{1}{x^2-x-12} = \frac{1}{(x-4)\cdot(x+3)} = \frac{A}{x-4} + \frac{B}{x+3} =$$

$$\boxed{= \frac{\frac{1}{7}}{x-4} - \frac{\frac{1}{7}}{x+3}}$$

(i) $$\frac{\frac{1}{7}}{x-4} = \frac{-\frac{1}{7}}{4-x} \cdot \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{-\frac{1}{28}}{1-\frac{x}{4}} =$$

$$= \left| \begin{array}{l} a = -\frac{1}{28} \\ q = \frac{x}{4} \end{array} \right| = -\frac{1}{28} - \frac{x}{28\cdot 4} - \frac{x^2}{28\cdot 4^2} - \dots$$

$$\dots - \frac{x^{n-1}}{28\cdot 4^{n-1}} - \dots$$

$I = (-3, 3)$

(ii) $$\frac{\frac{1}{7}}{3+x} = \frac{\frac{1}{21}}{1-\left(-\frac{x}{3}\right)} = \left| \begin{array}{l} a = \frac{1}{21} \\ q = -\frac{x}{3} \end{array} \right| =$$

$I = (-3, 3)$

$$= \frac{1}{21} - \frac{x}{21 \cdot 3} + \frac{x^2}{21 \cdot 3^2} - \dots + (-1)^{n-1} \cdot \frac{x^{n-1}}{21 \cdot 3^{n-1}} + \dots$$

Celkem:
$$\frac{1}{x^2-x-12} = -\frac{1}{28} - \frac{1}{21} -$$
$$-\left(\frac{1}{28\cdot 4} - \frac{1}{21\cdot 3}\right)\cdot x + \dots =$$
$$= \sum_{n=1}^{\infty}\left(-\frac{x^{n-1}}{28\cdot 4^{n-1}} - (-1)^{n-1}\cdot\frac{x^{n-1}}{21\cdot 3^{n-1}}\right) =$$
$$= \sum_{n=1}^{\infty}\left(\left[(-1)^n\cdot\frac{1}{21\cdot 3^{n-1}} - \frac{1}{28\cdot 4^{n-1}}\right] x^{n-1}\right) =$$

$$= \sum_{n=0}^{\infty} \left[(-1)^{n+1} \cdot \frac{1}{21 \cdot 3^n} - \frac{1}{28 \cdot 4^n} \right] x^n$$

Př.: $\sqrt[3]{70} \approx ?$

$$\sqrt[3]{70} = \sqrt[3]{64+6} = \sqrt[3]{4^3+6} =$$

$$= \sqrt[3]{4^3\cdot\left(1+\frac{6}{64}\right)} = 4\cdot\sqrt[3]{1+\frac{3}{32}} =$$

$$= 4\cdot\left(1+\frac{3}{32}\right)^{\frac{1}{3}}$$

$$\boxed{f(x) = (1+x)^{\frac{1}{3}}}$$

$$x_0 = 0$$

$$x = \frac{3}{32}$$

$$f'(x) = \frac{1}{3}(1+x)^{-\frac{2}{3}}$$

$$f(0) = 1 \qquad f'(0) = \frac{1}{3}$$

$$(1+x)^{\frac{1}{3}} \approx 1 + \frac{1}{3}x$$

$$x = \frac{3}{32} \Rightarrow \sqrt[3]{70} \approx 4 \cdot \left(1 + \frac{1}{3} \cdot \frac{3}{32}\right) =$$

$$= 4 + \frac{1}{8} = \underline{\underline{4{,}125}}$$

Př.: $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$

$\boxed{\cos 10^\circ \approx ?}$ $\qquad 10^\circ \dots \frac{\pi}{18}$

$1 > 10^{-5}$ $\qquad \dfrac{\left(\frac{\pi}{18}\right)^2}{2!} \doteq 0{,}015 > 10^{-5}$

$$\frac{(\pi/18)^4}{4!} \doteq 3{,}866 \cdot 10^{-5} > 10^{-5}$$

$$\frac{(\pi/18)^6}{6!} \doteq 3{,}926 \cdot 10^{-8} < 10^{-5}$$

$$\cos\frac{\pi}{18} \approx 1 - \frac{\left(\frac{\pi}{18}\right)^2}{2!} + \frac{\left(\frac{\pi}{18}\right)^4}{4!} \doteq 0,985$$

$\underline{\underline{\check{r}}}.:$ $y' = (2-y)\cdot \operatorname{tg} x$

$$\frac{dy}{dx} = (2-y)\cdot \operatorname{tg} x$$

$$\int \frac{1}{2-y}\, dy = \int \operatorname{tg} x \, dx \qquad \Big/ \boxed{y \neq 2}$$

$$-\ln|2-y| = -\ln|\cos x| + c_1$$

$$\ln|2-y| = \ln|\cos x| + c_2 \qquad \Big/ \exp.$$

$$|2-y| = e^{|\cos x| + c_2} = e^{|\cos x|} \cdot e^{c_2}$$

$$|2-y| = |\cos x| \cdot c_3 \qquad \Big|\ c_3 > 0$$

$$2-y = \cos x \cdot c \qquad \Big|\ c \gtrless 0$$

$$(c \in \mathbb{R})$$

$$\boxed{y = 2 - c \cdot \cos x}$$

$y = 2$

$$\left.\begin{aligned} 2' &= 0 \\ (2-2)\cdot \operatorname{tg} x &= 0 \end{aligned}\right\} \Rightarrow 2 \text{ je řeš.}$$

volbou $c = 0$

$\underline{\underline{ZK}}$:

$$LS = y' = (2 - c \cdot \cos x)' =$$

$$\underline{\underline{= +c \sin x}}$$

$$PS = (2-y) \cdot \operatorname{tg} x = (2 - 2 + c \cdot \cos x) \cdot \frac{\sin x}{\cos x}$$

$$\underline{\underline{= c \cdot \sin x}}$$

$$\underline{\underline{LS = PS}}$$

Ř.: $$1 + y^2 - xy\,(1+x^2)\cdot y' = 0$$

$$xy\,(1+x^2)\cdot y' = 1 + y^2$$

$$\frac{dy}{dx} = y' = \underbrace{(1+y^2)\cdot\frac{1}{y}} \cdot \underbrace{\frac{1}{x\cdot(1+x^2)}}$$

$$\int \frac{y}{1+y^2}\,dy = \int \frac{1}{x\cdot(1+x^2)}\,dx$$

$$\int \frac{y}{1+y^2}\,dy = \frac{1}{2}\int \frac{2y}{1+y^2}\,dy = \frac{1}{2}\ln(1+y^2) + c_1$$

$$\int \frac{1}{x\cdot(1+x^2)}\,dx = \int \frac{A}{x}\,dx = \int \frac{Bx+C}{1+x^2}\,dx =$$

$$= \int \frac{1}{x}\,dx - \int \frac{x}{1+x^2}\,dx =$$

$$= \ln|x| - \frac{1}{2}\ln(1+x^2) + c_2$$

$$\frac{1}{2}\ln(1+y^2) = \ln|x| - \frac{1}{2}\ln(x^2+1) + c_3$$

$$\ln\sqrt{1+y^2} = \ln|x| - \ln\sqrt{x^2+1} + c_3$$

$$\ln\sqrt{1+y^2} = \ln\frac{|x|}{\sqrt{x^2+1}} + c_3 \quad \Big/ \exp.$$

$$\sqrt{1+y^2} = \frac{|x|}{\sqrt{x^2+1}} \cdot c_4 \quad \Big/ c_4 > 0$$

$$\sqrt{1+y^2} = \frac{x}{\sqrt{x^2+1}} \cdot c \quad \Big/ c \in \mathbb{R}$$