aproximuj $\sqrt{e}$ (5 členů)

— $\sqrt{x}$ ... dosadit e (co dosadit??)

— $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + R_4(x)$

$$e^{\frac{1}{2}} \approx 1 + \frac{1}{2} + \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 + \frac{1}{6}\cdot\left(\frac{1}{2}\right)^3 + \frac{1}{24}\cdot\left(\frac{1}{2}\right)^4$$

$$\approx 1{,}65$$

$\sqrt[5]{245}$

$\sqrt[5]{x} = x^{\frac{1}{5}}$

$(1+x)^{a}$

$(1+x)^{-1} = \frac{1}{1+x} = \sum_{n=1}^{\infty} (-1)^n x^n \qquad |x| < 1$

$(1+x)^{a}$ má T- rozvoj konvergující pro $x \in (-1,1)$.

$(1+x)^{\frac{3}{5}}$

$\sqrt[5]{245} = \sqrt[5]{243+2} = \sqrt[5]{3^5\left(1+\frac{2}{3^5}\right)} = 3 \cdot \sqrt[5]{1+\frac{2}{3^5}} = ?$

$$\sqrt[5]{245} = 3\left(1+\frac{2}{3^5}\right)^{\frac{1}{5}}$$

$$(1+x)^{\frac{1}{5}} \approx 1 + \frac{1}{5}\cdot\frac{2}{3^5} \approx 1{,}002$$

$$\sqrt[5]{245} \approx 3{,}005$$

Vypočtete

$$\lim_{x\to\infty}\left(x - x^2\cdot\ln\left(1+\frac{1}{x}\right)\right)$$

Maclaurin:

$$\ln(1+x) = \sum_{n=\cancel{0}\,1}^{\infty} \frac{(-1)^{n+1}}{n}\cdot x^n \qquad |x|<1$$

$$\lim_{x\to\infty}\left(x - x^2\cdot\left(\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \dots\right)\right) =$$

$$= \lim_{x\to\infty}\left(x - \left(x - \frac{1}{2} + \frac{1}{3x} - \frac{1}{4x^2} + \dots\right)\right) =$$

$$= \lim_{x\to\infty}\left(\frac{1}{2} - \frac{1}{3x} + \dots\right) = \boxed{\frac{1}{2}}$$

$$\int_0^{1/2} \frac{dx}{1+x^4} =$$

$$(1+x^4)^{-1} = 1 - x^4 + x^8 - x^{12} + x^{16} - \dots \qquad |x|<1$$

$$\frac{1}{1-q} = 1 + q + q^2 + \dots \qquad |q|<1$$

$$\int_0^{1/2} \frac{dx}{1+x^4} = \int_0^{1/2} \sum_{n=0}^{\infty} (-x^4)^n \, dx$$

$$= \sum_{n=0}^{\infty} \int_0^{1/2} (-x^4)^n \, dx =$$

$$= \left[x - \frac{x^5}{5} + \frac{x^9}{9} - \frac{x^{13}}{13} + \dots \right]_0^{1/2} \doteq \,!$$

$$= \frac{1}{2} - \frac{1}{5}\left(\frac{1}{2}\right)^5 + \frac{1}{9}\cdot\left(\frac{1}{2}\right)^9 - \frac{1}{13}\left(\frac{1}{2}\right)^{13} + \dots$$

chyba $< 10^{-4}$

$$|R_3| < |a_4| = \frac{1}{13}\cdot\frac{1}{2^{13}} < \cancel{\frac{1}{10^4}} \; \frac{1}{10^5}$$

neboť

$$13\cdot 2^{13} = \underbrace{13\cdot 8}_{>10^2}\cdot\underbrace{2^{10}}_{>10^3} > 10^5$$

$$\int_0^{1/2} \frac{dx}{1+x^4} \approx 0{,}4940$$