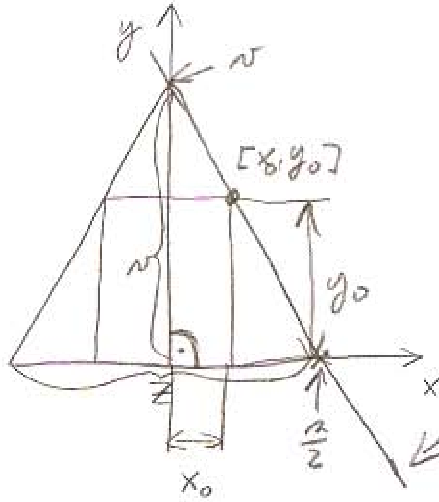
A1/



$y = ax + b$, $[0, v]$, $[\frac{z}{2}, 0]$

$v = b$

$0 = a\frac{z}{2} + b$

$0 = a \cdot \frac{z}{2} + v$

$a = -\frac{2v}{z}$

$y = -\frac{2v}{z} \cdot x + v$

$$S = 2 \cdot x \cdot y = \underbrace{-4\frac{v}{z}x^2 + 2xv}_{= S(x)} \rightarrow MAX$$

$$S'(x) = -8 \cdot \frac{v}{z} \cdot x + 2v = 0$$

$$4\frac{v}{z}x = v$$

$$\boxed{x = \frac{z}{4}}$$

$$\boxed{y = -2\frac{v}{\cancel{z}} \cdot \frac{\cancel{z}}{4} + v = v - \frac{v}{2} = \frac{v}{2}}$$

$$S = 2 \cdot \frac{z}{4} \cdot \frac{v}{2} = \frac{z \cdot v}{4}$$

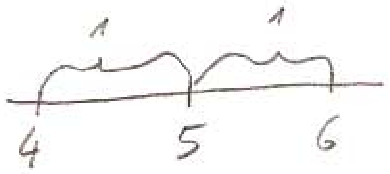
$$S_{\triangle} = \frac{z \cdot v}{2}$$

Tj., max. obdélník pokrýva 1/2 plochy △.

A2/ $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2} \Rightarrow s = 5, \quad a_n = \frac{1}{n^2}$

$$\frac{1}{r} = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{1}{(n+1)^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2+2n+1} = \underline{\underline{1}}$$

$$\Rightarrow \underline{\underline{r = 1}}$$



$x = 4 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ KONV.

$x = 6 \Rightarrow \sum_{n=1}^{\infty} \frac{1^n}{n^2}$ KONV.

$\Rightarrow$ OBOR KONV. $= \underline{\underline{[4, 6]}}$

A3/ $\int \frac{3x+5}{x^2+4x+8}\,dx = \frac{3}{2}\underbrace{\int \frac{2x+4}{x^2+4x+8}\,dx}_{I_1} - \underbrace{\int \frac{1}{x^2+4x+8}\,dx}_{I_2} = ⊛$

$I_1 = \ln|x^2+4x+8| + c_1 = \underline{\ln(x^2+4x+8) + c_1}$

$$I_2 = \int \frac{1}{(x+2)^2+4}\,dx = \frac{1}{4}\int \frac{1}{\left(\frac{x+2}{2}\right)^2+1}\,dx = \left| \begin{array}{l} t = \frac{x+2}{2} \\ dt = \frac{1}{2}\,dx \\ dx = 2\,dt \end{array} \right| =$$

$$= \frac{1}{4}\cdot 2 \cdot \int \frac{1}{t^2+1}\,dt = \frac{1}{2}\,\mathrm{arctg}\, t + c_2 = \underline{\frac{1}{2}\,\mathrm{arctg}\,\frac{x+2}{2} + c_2}$$

$$⊛ = \underline{\frac{3}{2}\cdot \ln(x^2+4x+8) - \frac{1}{2}\,\mathrm{arctg}\,\frac{x+2}{2} + c}$$

A4/ $y^2 = 3x - 1$ $\qquad$ $x^2 + y^2 = 3$

PAR.

$V = [1/3, 0]$ $\qquad$ $S = [0,0]$, $r = \sqrt{3}$

$$\left.\begin{aligned} y^2 &= 3x - 1 \\ x^2 + y^2 &= 3 \end{aligned}\right\} \Rightarrow x^2 + 3x - 1 = 3$$

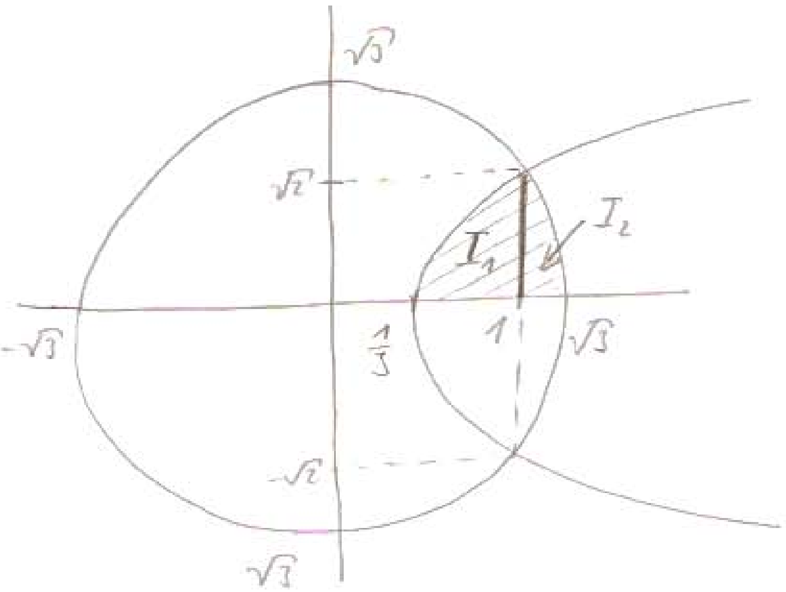
$$x^2 + 3x - 4 = 0$$

$$D = 9 + 16 = 25$$

$$x_{1,2} = \frac{-3 \pm 5}{2} = \begin{cases} = 1 \\ = -4 \end{cases}$$

$x = 1 \Rightarrow y^2 = 3 - 1 = 2$

$y = \pm\sqrt{2}$



$$I_1 = \int_{\frac{1}{3}}^{1} \sqrt{3x-1}\, dx = \left| \begin{matrix} t = 3x - 1 \\ dt = 3dx \\ dx = \frac{1}{3} dt \end{matrix} \middle| \begin{matrix} x = 1 \Rightarrow t = 2 \\ x = \frac{1}{3} \Rightarrow t = 0 \end{matrix} \right| = \frac{1}{3} \int_0^2 \sqrt{t}\, dt =$$

$$= \frac{1}{3} \cdot \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^2 = \frac{1}{3} \cdot \frac{2}{3} \cdot 2^{\frac{3}{2}} = \frac{2}{9} \cdot 2\sqrt{2} = \frac{4\sqrt{2}}{9}$$

$$I_2 = \int_1^{\sqrt{3}} \sqrt{3 - x^2}\, dx = \left| \begin{matrix} x = \sqrt{3} \cdot \sin t \\ dx = \sqrt{3} \cdot \cos t\, dt \end{matrix} \middle| \begin{matrix} x = \sqrt{3} \Rightarrow t = \overbrace{\arcsin 1}^{=: b} \\ x = 1 \Rightarrow t = \underbrace{\arcsin \frac{\sqrt{3}}{3}}_{=: a} \end{matrix} \right| =$$

$$= \int_a^b \sqrt{3 - 3\sin^2 t} \cdot \sqrt{3} \cdot \cos t\, dt = 3 \cdot \int_a^b \sqrt{1 - \sin^2 t} \cdot \cos t\, dt =$$

$$= 3 \cdot \int_a^b \sqrt{\cos^2 t} \cdot \cos t \, dt = 3 \cdot \int_a^b \cos^2 t \, dt =$$

$$= 3 \cdot \int_a^b \frac{1 + \cos 2t}{2} \, dt = 3 \cdot \left[\frac{1}{2} t + \frac{1}{2} \cdot \frac{\sin 2t}{2} \right]_a^b =$$

$$= \frac{3}{2} \cdot \left(\arcsin 1 - \arcsin \frac{\sqrt{3}}{3} \right) + \frac{3}{4} \left[\sin(2 \cdot \arcsin 1) - \right.$$
$$\left. - \sin\left(2 \cdot \arcsin \frac{\sqrt{3}}{3}\right) \right]$$

Bez kalk. se nechá takto.

Plocha kruhu = $\pi r^2 = \underline{\underline{3\pi}}$

Křivka dělí kruh na 2 části v poměru

$$\left[2 \cdot (I_1 + I_2) \right] : \left[3\pi - 2 \cdot (I_1 + I_2) \right],$$

kde I_1 a I_2 jsou vyp výše.

A5/ $\lim_{x\to\frac{\pi}{2}} \left(\frac{\pi}{2}-x\right)\cdot \operatorname{tg} x = \lim_{x\to\frac{\pi}{2}} \frac{\left(\frac{\pi}{2}-x\right)\sin x}{\cos x} = \left\| \frac{0\cdot 1}{0} = \frac{0}{0} \right\| =$

$\overset{L'H}{=} \lim_{x\to\frac{\pi}{2}} \frac{(-1)\cdot \sin x + \left(\frac{\pi}{2}-x\right)\cdot\cos x}{-\sin x} = \frac{(-1)\cdot 1 + 0\cdot 0}{-1} = \underline{\underline{1}}$

A6/ $\underline{\underline{f(x) = x\, e^{-\frac{x^2}{2}}}}$

a) $Dom(f) = \mathbb{R}$

b) per. NENÍ, $f(-x) = (-x)\cdot e^{-\frac{(-x)^2}{2}} =$
$= -x\cdot e^{-\frac{x^2}{2}} = -f(x)$
$\Rightarrow$ <u>LICHÁ</u>

c) Body nesp. <u>nemá</u>.

d) $f(x) = x\cdot e^{-\frac{x^2}{2}} = 0 \Leftrightarrow x = 0$
N. B.

e) $\ominus$ $\oplus$
ZÁP. 0 KL.

f) $f'(x) = 1\cdot e^{-\frac{x^2}{2}} + x\cdot e^{-\frac{x^2}{2}}\cdot\left(-\frac{1}{2}\right)\cdot 2x = e^{-\frac{x^2}{2}} - x^2\cdot e^{-\frac{x^2}{2}} = e^{-\frac{x^2}{2}}(1-x^2) = 0$
$\Updownarrow$
$\underline{x = \pm 1}$

f': $\ominus$ | -1 | $\oplus$ | 1 | $\ominus$
f: $\searrow$ -1 $\nearrow$ 1 $\searrow$
L. MIN. L. MAX.

$f(-1) = (-1)\cdot e^{-\frac{(-1)^2}{2}} = -e^{-\frac{1}{2}} = -\frac{1}{\sqrt{e}}$

$f(1) = 1\cdot e^{-\frac{1^2}{2}} = \frac{1}{\sqrt{e}}$

$\Rightarrow H(f) = \left[-\frac{1}{\sqrt{e}}, \frac{1}{\sqrt{e}}\right]$

g) $f''(x) = e^{-\frac{x^2}{2}}\cdot\left(-\frac{1}{2}\right)\cdot 2x\cdot(1-x^2) +$
$+ e^{-\frac{x^2}{2}}\cdot(-2x) =$
$= -e^{-\frac{x^2}{2}}(x-x^3) - 2x\cdot e^{-\frac{x^2}{2}} = e^{-\frac{x^2}{2}}(-x+x^3-2x) = e^{-\frac{x^2}{2}}(x^3-3x) =$
$= e^{-\frac{x^2}{2}}\cdot x\cdot(x^2-3) = 0 \Leftrightarrow x = 0 \vee x = \pm\sqrt{3}$

f'': $\ominus$ I.B. $\oplus$ I.B. $\ominus$ I.B. $\oplus$
f: (KV) $-\sqrt{3}$ (KX) 0 (KV) $\sqrt{3}$ (KX)

h) AS. bez sm. → NEJSOU ($Dom(f)=\mathbb{R}$)

AS. se sm.: $y = ax + b$

$$a = \lim_{x\to\pm\infty} \frac{x\cdot e^{-\frac{x^2}{2}}}{x} = \lim_{x\to\pm\infty} \frac{1}{e^{\frac{x^2}{2}}} = \underline{\underline{0}}$$

$$b = \lim_{x\to\pm\infty} \left(x\cdot e^{-\frac{x^2}{2}} - 0\cdot x\right) = \lim_{x\to\pm\infty} \frac{x}{e^{\frac{x^2}{2}}} = \underline{\underline{0}}$$

⇒ AS. v $\boxed{-\infty}$ & $\boxed{+\infty}$ je totožná: $\underline{\underline{y=0}}$

i)