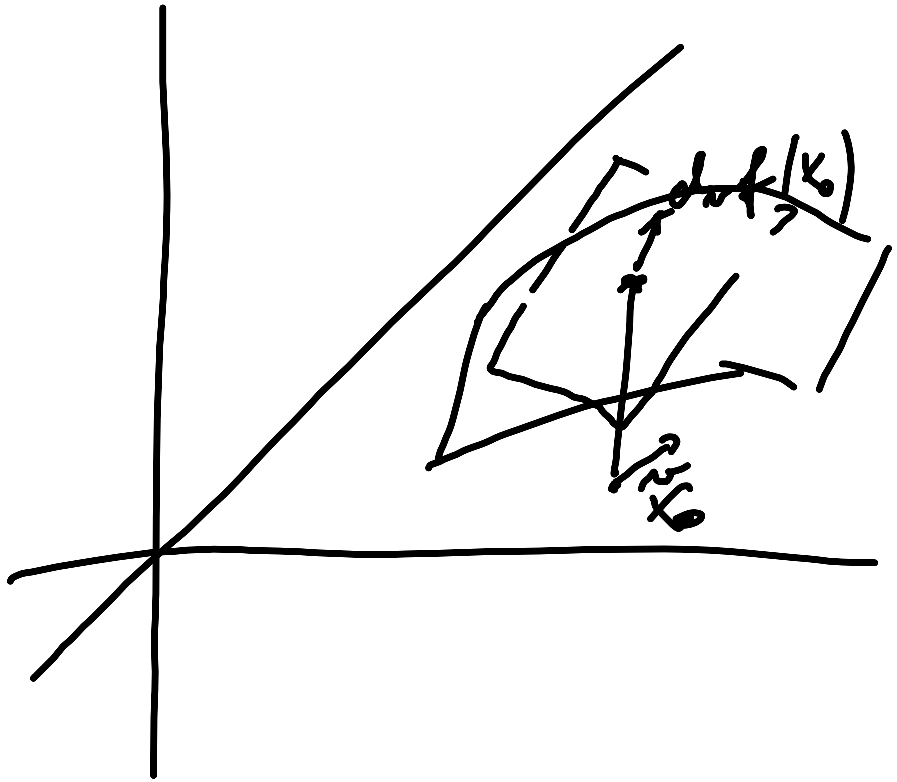


př.

$$f(x,y) = \begin{cases} \boxed{\dfrac{xy}{x^2+y^2}} & \text{pro } (x,y) \neq (0,0) \\ 0 & \text{pro } (x,y) = (0,0) \end{cases}$$

Spočtěme parc. derivace:

$$\frac{\partial f(x,y)}{\partial x} = f_x(x,y) = \frac{y(x^2+y^2) - xy(2x)}{(x^2+y^2)^2} = \frac{y^3 - x^2 y}{(x^2+y^2)^2}$$

$$f_y(x,y) = \frac{x^3 - y^2 x}{(x^2+y^2)^2}$$

V bodě $(0,0)$:

$$f_x(0,0) = \lim_{h\to 0} \frac{1}{h}\left(f(0+h,0) - f(0,0)\right) = \lim_{h\to 0} \frac{1}{h}\left(0 - 0\right) = 0$$

Parciální derivace existují na okolí bodu $[0,0]$, ale nejsou spojité:

$$\lim_{t\to 0} f(0+t, 0+2t) = \frac{t\cdot(2t)}{(t^2+4t^2)} = \frac{2}{5}$$

$$\lim_{t\to 0} f(0+t, 0+t) = \frac{t\cdot t}{(t^2+t^2)} = \frac{1}{2}$$

Má-li funkce $f: \mathbb{R}^n \to \mathbb{R}$ extrém v bodě x_0, pak je $df = 0$.

Dk.: Obměnou: nechť $df \neq 0$, pak ex. $v \in E^n$: $df(v) \neq 0$. BÚNO $df(v) > 0$, potom $df(-v) < 0$

$$\frac{\partial f}{\partial x_1} : \mathbb{R}^n \to \mathbb{R}$$

$$f_z = x^2 \cos(yz)\cdot y$$

$$f_{zx} = 2x\cos(yz)\cdot y$$

$$f_{zxy} = 2x\left(-\sin(yz)\cdot z\cdot y + \cos(yz)\right)$$

$$\frac{\partial\left(\frac{\partial f}{\partial x_1}\right)}{\partial x_2} = f_{x_1 x_2} = \frac{\partial^2 f}{\partial x_1 \partial x_2}$$

Př.

$$f(x,y,z) = x^2\cdot\sin(yz)$$

$$f_x(x,y,z) = 2x\cdot\sin(yz)$$

$$f_{xy}(x,y,z) = 2x\cos(yz)\cdot z$$

$$f_{xyz}(x,y,z) = 2x\left(-\sin(yz)\cdot y\cdot z + \cos(yz)\right)$$

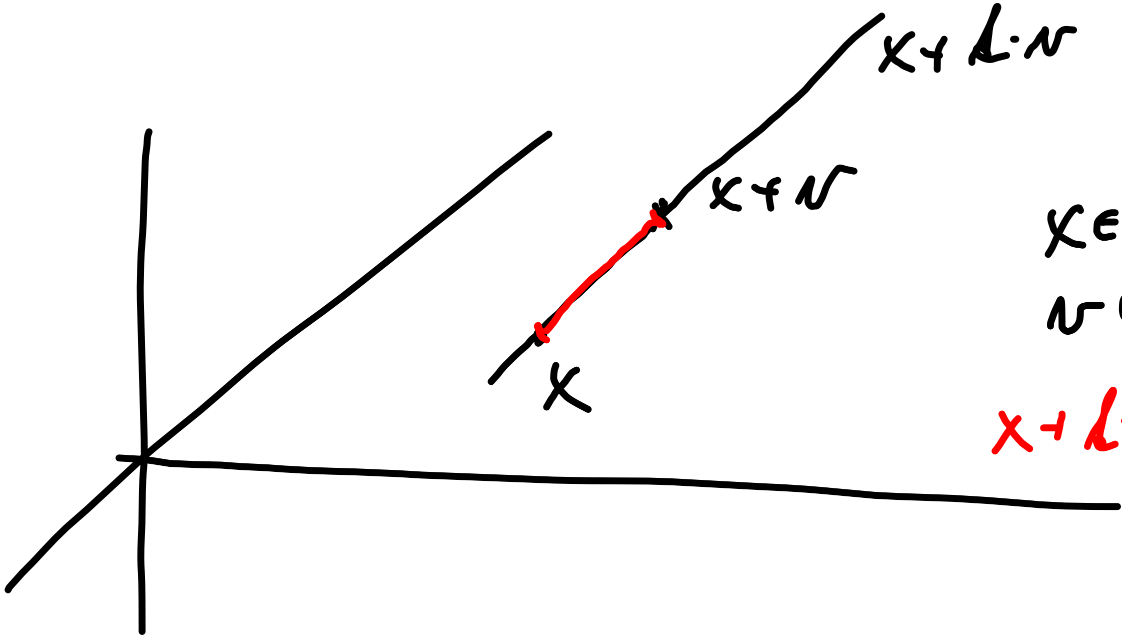
$$\alpha(t) = f(x(t), y(t))$$

$$\beta(t) = f(x_0, y_0) + t\left(\frac{\partial f}{\partial x}(x_0, y_0)\,\xi + \frac{\partial f}{\partial y}(x_0, y_0)\,\eta\right) +$$

$$+ \frac{1}{2} t^2 \left(f_{xx}(x_0, y_0)\,\xi^2 + 2 f_{xy}(x_0, y_0)\,\xi\eta \right.$$

$$\left. + f_{yy}(x_0, y_0)\,\eta^2 \right)$$

$$Hf : \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$



$\lambda \in \mathbb{R}$

$x \in \mathbb{R}^n$

$v \in E^n$

$x + \lambda \cdot v, \quad \lambda \in (0,1)$

$f(x,y) = \sin(x)\cos y$

$f_x(x,y) = \cos(x)\cos y(y)$

$f_y(x,y) = -\sin(x)\sin(y)$

$v^T H v > 0$

$h(v) = v^T H v$

H je positiv. definitní

$\Leftrightarrow$ $(-H)$ je negat. definitní

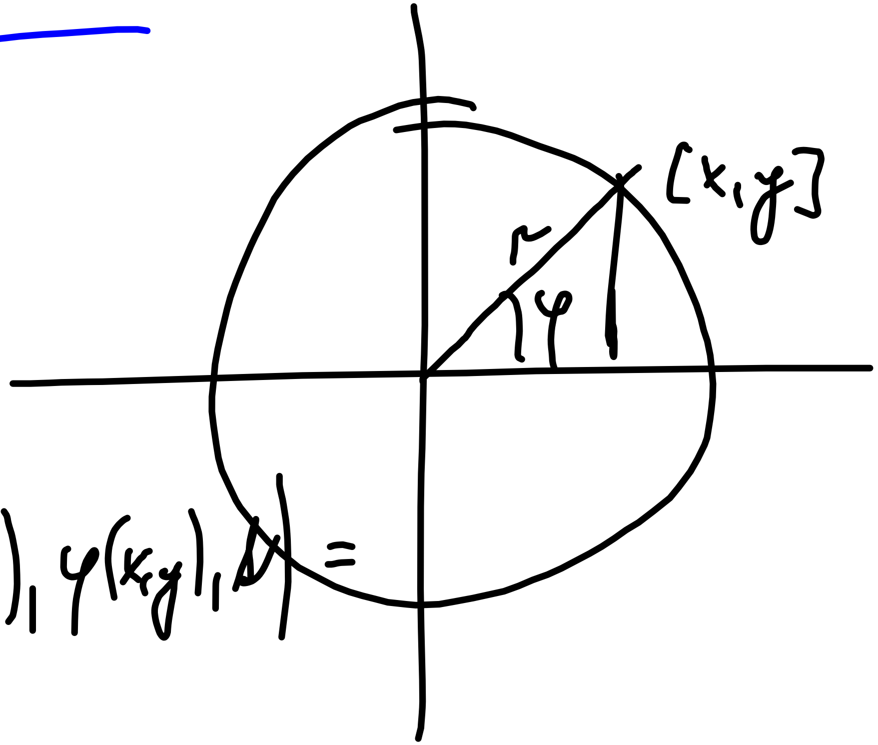
$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & & & \vdots \\ a_{n1} & \dots & \dots & a_{nn} \end{pmatrix}$$

$\varphi: \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}^2 \setminus \{(0,0)\}$

$x = r\cos\varphi$

$y = r\cdot\sin\varphi$

$$(f\circ g)'(t) = f'(\boxed{g(t)})\cdot g'(t)$$



$$\frac{\partial g}{\partial x}(x, y, t) = \frac{\partial g}{\partial x}\left(r(x,y), \varphi(x,y), t\right) =$$