22

$$i + j + k$$
$$4 \quad 5 \quad 3$$

$3*5 + 3*2 + 1*1$ ✓

$3*5 + 2*2 + 3*1$ ✓

$2*5 + 5*2 + 2*1$ ✓

$2*5 + 4*2 + 4*1$ ✓

$$(a_0, a_1, \dots) \longmapsto \sum_{i=0}^{\infty} a_i x^i$$

[illegible]

$$F(x) = (1 + x + x^2 + x^3 + x^4)$$

$$(\underbrace{1,1,1,}_{x} 1,1,0,0,1 \ldots) \quad x \to 2x \quad \mapsto (1+2x+4x^2+\ldots$$

$$G(x) = (1 + x + x^2)$$

$$(1,1,1,0,0,0,0,\ldots)$$

$$(F(x) - G(x)) \cdot \frac{1}{x^3} = 1 + x$$

$$(1,1,0,0,\ldots)$$

shift o 3 místa doleva

diferenční rovnice: $a_{n+1} = F_n(a_n)$

$((n+1)! = n!\,(n+1))$

$$F_{n+2} = F_n + F_{n+1}, \quad F_0 = 0, \; F_1 = 1$$

$F(x)$ … vytvořující funkce pro F_n … $(F_0, F_1, F_2, \dots)$

$x\,F(x)$ … $(0, F_0, F_1, F_2, \dots)$

$x^2 F(x)$ … $(0, 0, F_0, F_1, F_2, \dots)$

$x\,F(x) + x^2 F(x) - F(x)$ … $(-F_0, -F_1 + F_0, \overbrace{-F_2 + F_1 + F_0}^{0}, \dots, \underbrace{-F_n + F_{n-1} + F_{n-2}}_{0}, \dots)$

$\Rightarrow (\underset{0}{-F_0}, \underset{-1}{-F_1 + F_0}, 0, \dots, 0, \dots)$

$$\Rightarrow (1 - x - x^2)\,F(x) = x \Rightarrow F(x) = \frac{x}{1 - x - x^2}$$

$$\Rightarrow F(x) = \frac{x}{1-x-x^2} = \frac{A}{x-x_1} + \frac{B}{x-x_2}$$

$$= \frac{a}{1-\lambda_1 x} + \frac{b}{1-\lambda_2 x}$$

$\overline{\text{rovnice}}$ k-tého řádu obdobně

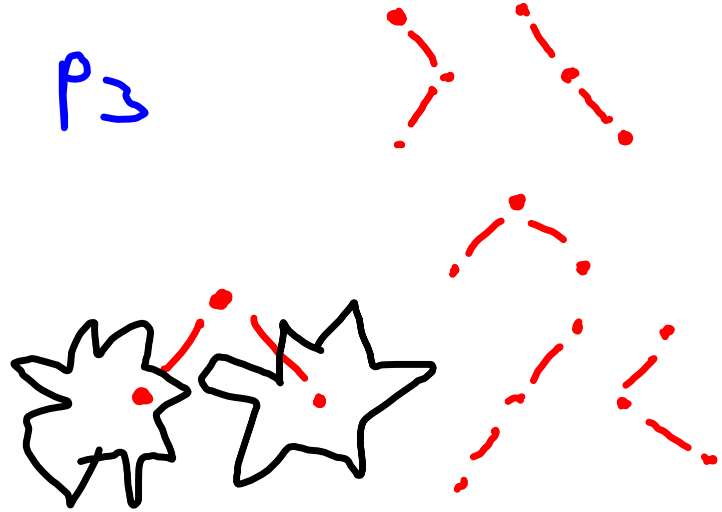
$$\Downarrow$$

$$\boxed{F(x) = \frac{x^{k-1}}{1-a_0 x^{k-1} - \dots - a_{k-1} x}}$$

pro $\boxed{F_{n+k} = a_0 F_n + \dots + a_{k-1} F_{n+k-1}}$

Pěstěné (binární) stromy — kolik je neizomorfních na n vrcholech?

$$p_0 = 1, \quad p_1 = 1, \quad p_2 = 2, \quad p_3$$



$$P(x) = p_0 + p_1 x + p_2 x^2 + \dots$$

$$\boxed{p_n = \sum_{i+j=n-1} p_i \cdot p_j}$$

$$\Sigma = n-1$$

$$p_3 = \sum_{i+j=2} p_i \cdot p_j = p_0 \cdot p_2 + p_1 \cdot p_1 + p_2 \cdot p_0$$

$$= 2 + 1 + 2 = 5$$

$$P(x) \cdot P(x) = (p_0 + p_1 x + p_2 x^2 + \dots)(p_0 + p_1 x + p_2 x^2 + \dots)$$

$$= p_1 x^0 + p_2 x^1 + p_3 x^2 + \dots$$

$$\Rightarrow \quad \underbrace{P(x)} = 1 + x\left(\underbrace{P(x)}\right)^2 \qquad P(x) \to p_0 \text{ pro } x \to 0_+$$

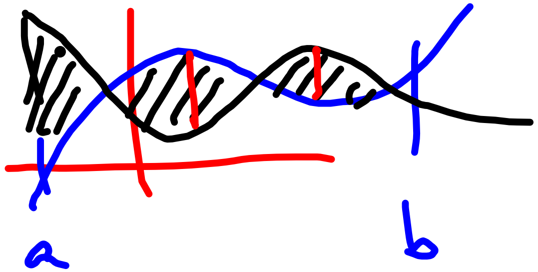
$$\Rightarrow \quad P(x) = \frac{1 \pm \sqrt{1-4x}}{2x} \qquad \Downarrow \quad \oplus \text{ vyhovuje}$$

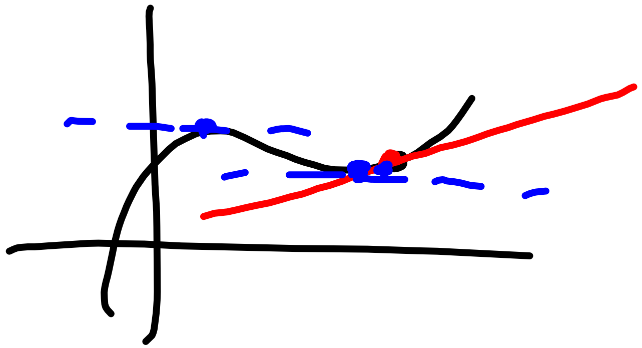
$\Rightarrow$ vyjádříme jako mocninnou řadu $\sqrt{1-4x}$, dosadíme $\dashrightarrow$

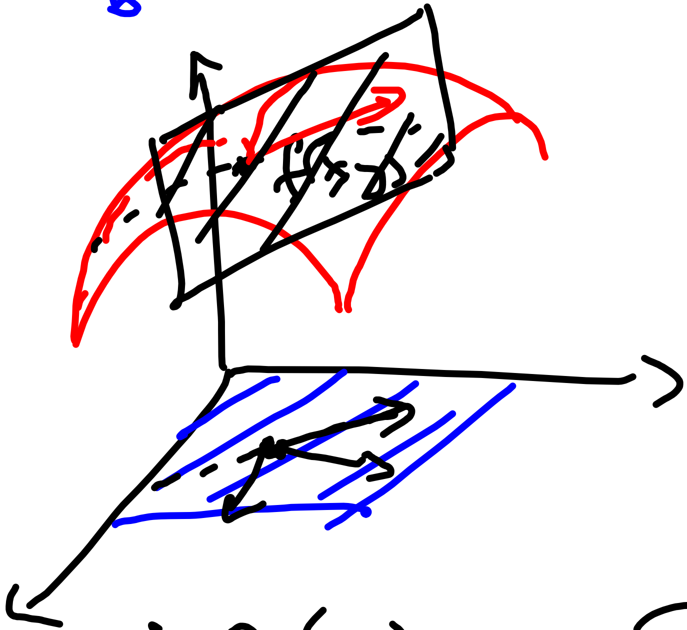
$$\Rightarrow \boxed{p_n = \frac{1}{n+1}\binom{2n}{n}}$$

Catalánova čísla

Rekapitulace :







$$v = v_1 e_1 + v_2 e_2$$

$$Df(v)(x) = v_1 Df(e_1)(x) + v_2 Df(e_2)(x)$$

$$Df(e_1)(x) := \frac{\partial f}{\partial x_1}(x)$$

$\mathbb{R}^1$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!} f''(x_0)(x - x_0)^2$$

$$+ \dots + \frac{1}{k!} f^{(k)}(x_0)(x - x_0)^k + \frac{1}{(k+1)!} f^{(k+1)}(\xi)(x - x_0)^{k+1}$$

$\mathbb{R}^n$

$$d \quad \begin{pmatrix} \frac{\partial f^1}{\partial x^1} & \dots & \frac{\partial f^1}{\partial x^n} \\ \vdots & & \\ \frac{\partial f^n}{\partial x^1} & \dots & \frac{\partial f^n}{\partial x^n} \end{pmatrix} \cdot \begin{pmatrix} x^1 - x^1_0 \\ \vdots \\ x^n - x^n_0 \end{pmatrix}$$

$$f(x) = f(x_0) + D^1(x_0)(x - x_0)$$

$$+ \frac{1}{2!} D^2(x_0)(x - x_0, x - x_0)$$

$$+ \dots$$

$$f : \mathbb{R}^n \to \mathbb{R} \qquad D^1 f = \left(\frac{\partial f}{\partial x^1}, \dots, \frac{\partial f}{\partial x^n} \right)$$

$$D^2 f = \begin{pmatrix} \frac{\partial^2 f}{\partial x^1 \partial x^1} & \frac{\partial^2 f}{\partial x^1 \partial x^2} & \cdots & \frac{\partial^2 f}{\partial x^1 \partial x^n} \\ \frac{\partial^2 f}{\partial x^2 \partial x^1} & \frac{\partial^2 f}{\partial x^2 \partial x^2} & \cdots & \\ \vdots & & & \end{pmatrix}$$

$$D^2 f (x - x_0) = \underbrace{\left(\cdot \cdot \cdot \right)}_{(x-x_0)^T} \underbrace{\begin{pmatrix} \cdot & & \\ & \ddots & \\ & & \cdot \end{pmatrix}}_{\text{Hessián}} \overbrace{\begin{pmatrix} \cdot \\ \vdots \\ \cdot \end{pmatrix}}^{(x-x_0)}$$

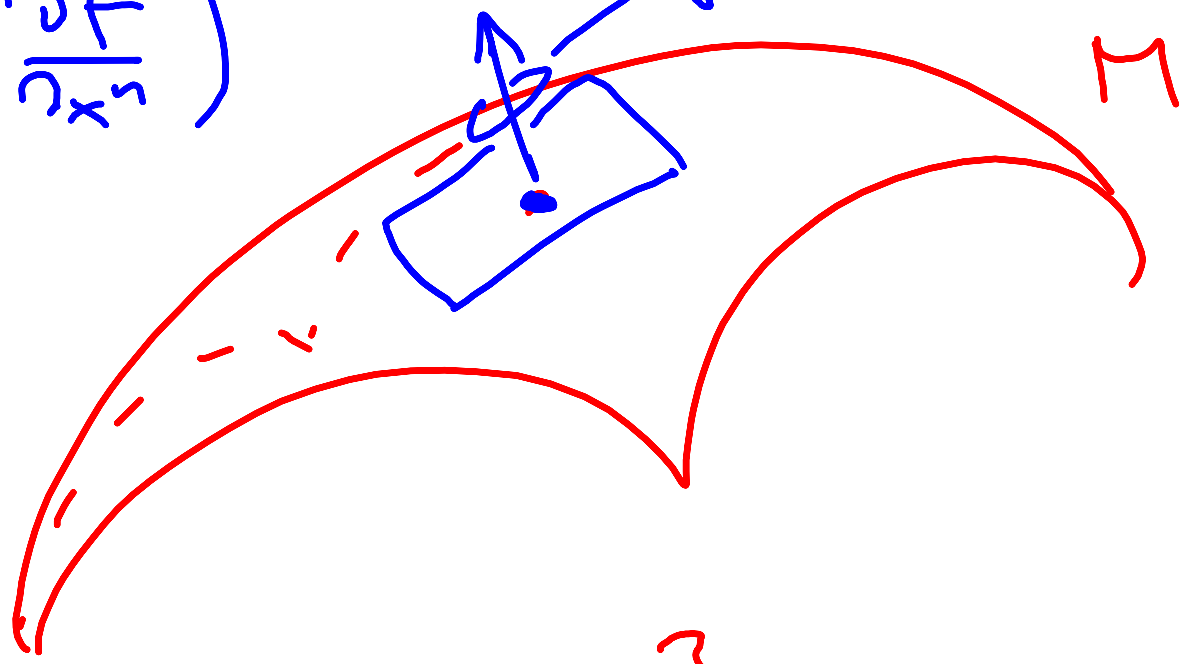
$M = F(x^1, \dots, x^n) = 0$

$\left(\frac{\partial F}{\partial x^1}, \dots, \frac{\partial F}{\partial x^n}\right)$

$\uparrow$

kolmý
vektor
na M

$\operatorname{grad} f = \left(\frac{\partial f}{\partial x^1}, \dots, \frac{\partial f}{\partial x^n}\right)$



$f : \mathbb{R}^3 \to \mathbb{R}$

min f na M

$$y^{(n)} = F(x, y(x), \dots, y^{(n-1)}(x))$$

$$y' = f(x) \cdot g(y)$$

$$\int \frac{y'}{g(y)} = \int f(x)$$