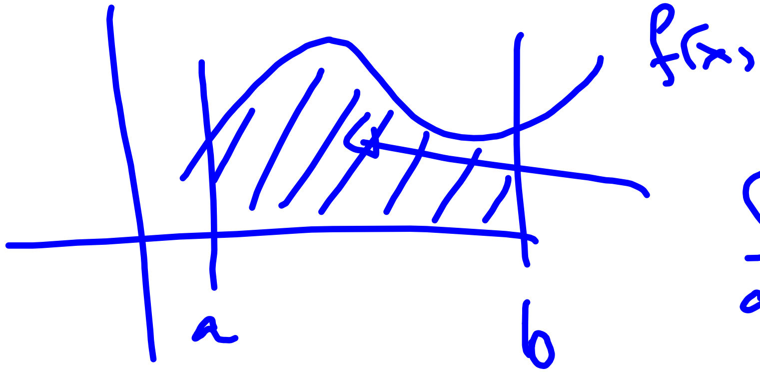
$f: \mathbb{R} \longrightarrow \mathbb{R}$ $\qquad f(t) = 3t^2 + t + 1$

$f: \mathbb{R}^m \longrightarrow \mathbb{R}^n$ $\qquad f(x,y) = x^3 y + y^2 + 3$

$m=2, \; n=1$



$$\int_a^b f(x) \in \mathbb{R}$$

Nejjednodušší: křivky

$c(t): \mathbb{R} \longrightarrow \mathbb{R}^n$ $\qquad n = 2, 3, 4, \ldots$

$$c(t) = \big(c_1(t),\ c_2(t),\ c_3(t)\big)$$

$c: \mathbb{R} \to \mathbb{R}^3$ souřadnice

Příklad 1:

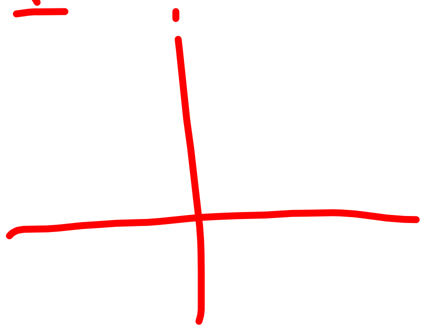
$c_1(t)$ $c_2(t)$

$$c(t) = \left(\ln t,\ t^2/(t-1)\right)$$

definiční obor: průnik po jednotlivé souřadnice

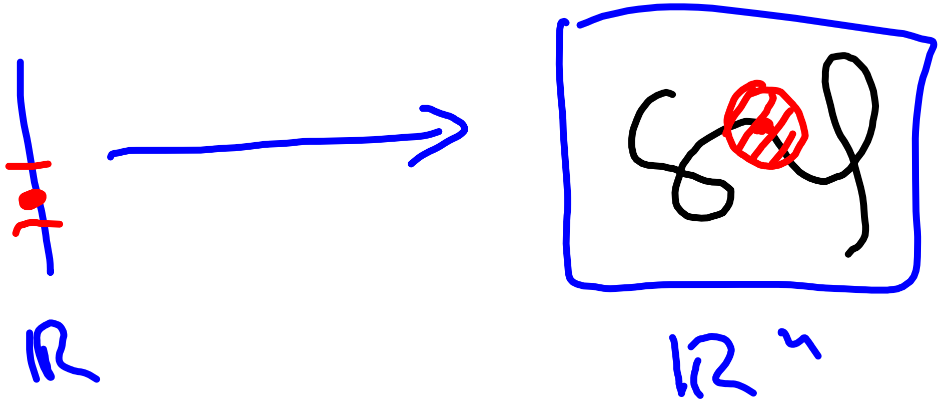
c_1: $t > 0$ c_2: $t \neq 1$

$\Rightarrow (0, \infty) \setminus \{1\}$



$c(t) = \left(\ln t,\ t^2/(t-1)\right)$

spojitost, limita v bodě, (nevlastní), ...



vzdálenost v $\mathbb{R}^n$

$|(x_1,\dots,x_n)-(y_1,\dots,y_n)| = \sqrt{\sum (x_i-y_i)^2}$

$\lim_{t\to 0_+} c(t) = (-\infty, 0)$

odhady:

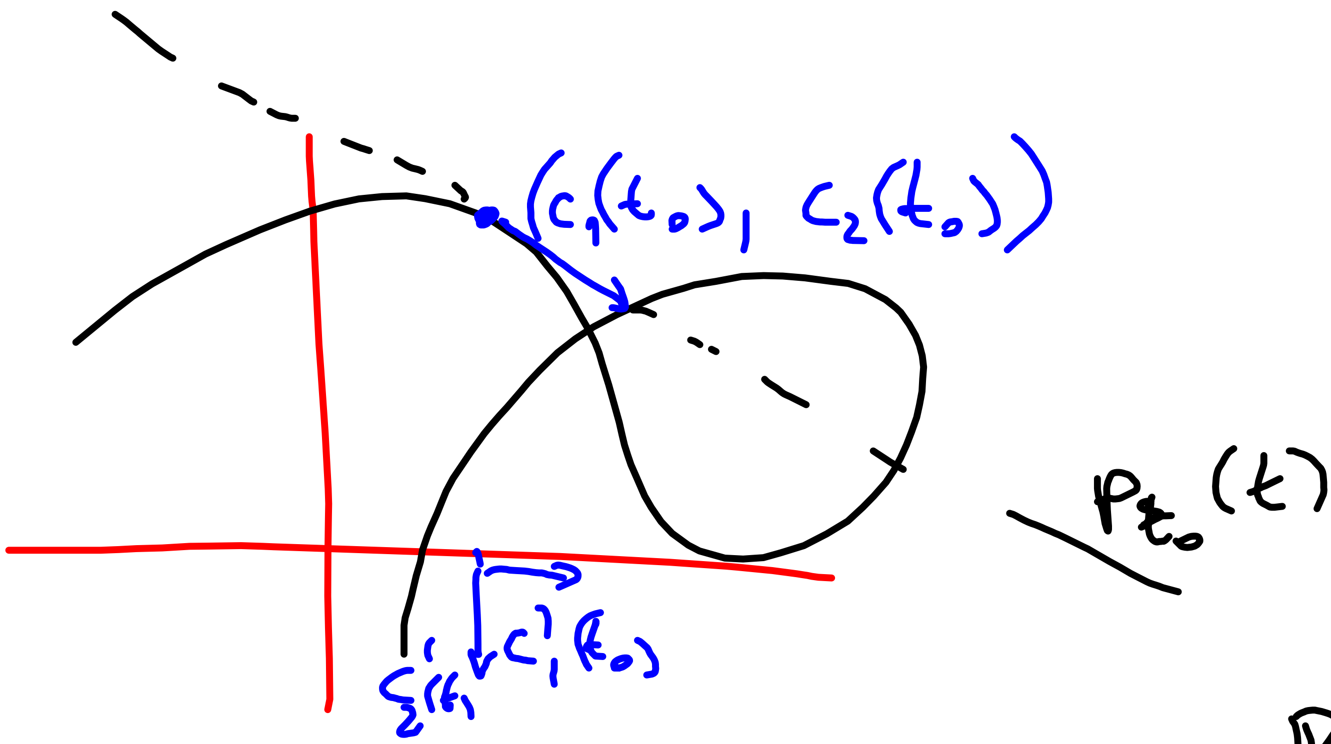
$$f(y) = f(x) + f'(\xi)(y-x)$$

$$|f(y)-f(x)| \leq \max_{\xi\in(a,b)} f'(\xi)\,|y-x|$$

derivace: $c'(t) = (c_1'(t), c_2'(t))$

$$c'(t) = \left(\ln'(t), \frac{d}{dt}\left(t^2/(t-1)\right)\right)$$

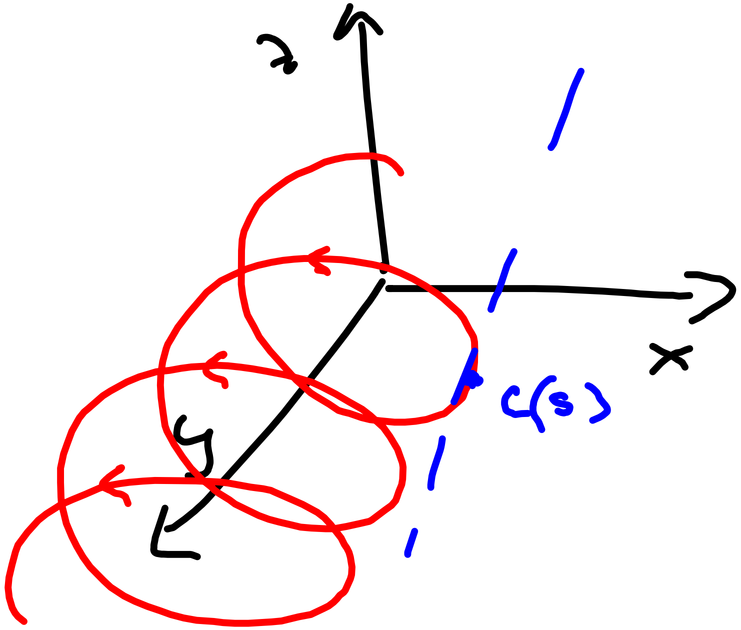
$$= \left(\frac{1}{t}, \frac{2t(t-1)-t^2}{(t-1)^2}\right)$$



$\mathbb{R}^2 \ni (x, y)$

$$c'(s) = \left(\frac{1}{s}, \frac{s^2 - 2s}{(s-1)^2} \right)$$

tečna v $c(s)$: $\left\{ \begin{array}{l} x = \ln s + t \cdot \frac{1}{s} \\ y = \frac{s^2}{s-1} + t \cdot \frac{s(s-2)}{(s-1)^2} \end{array} \right\}$



úlohe 1

$(s,t) \mapsto p_s(t)$

$\mathbb{R}^2 \to \mathbb{R}^3$

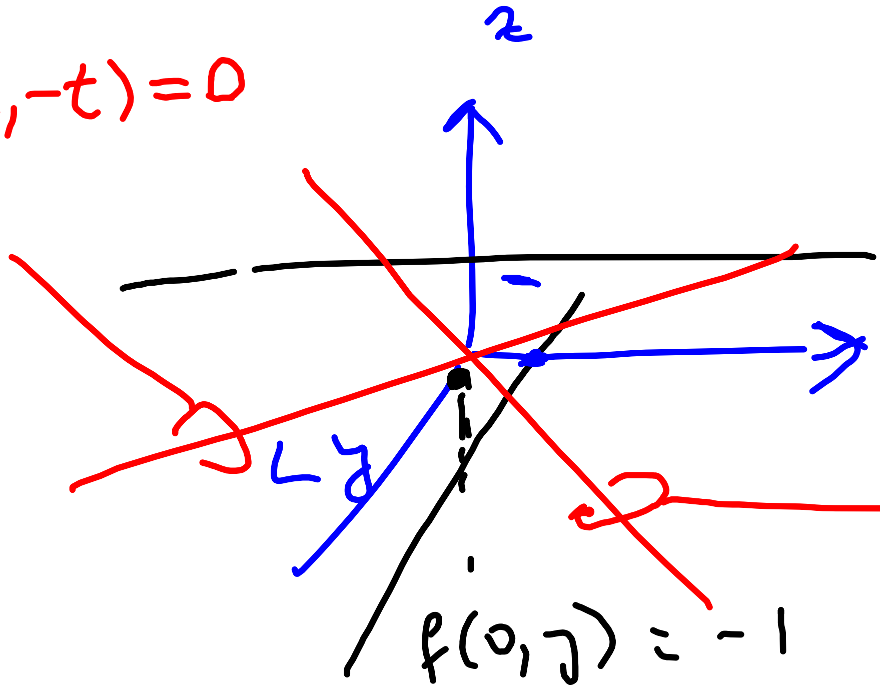
$\mathbb{R}^m \to \mathbb{R}^n$

1 funkce ve více proměnných

$f: \mathbb{R}^2 \to \mathbb{R}$

$$\boxed{f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & x,y \in \mathbb{R}^2 \setminus \{0,0\} \\ 0 & \text{pro } (x,y) = (0,0) \end{cases}}$$

$f(t,-t) = 0$



$f(x,0) = 1$

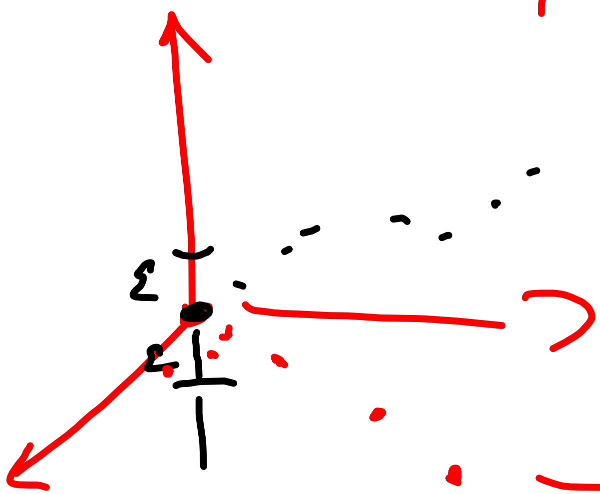
$f(0,y) = -1$

$f(t,t) = 0$

spojitost, limity, ...

je $f(x,y)$ spojitá v $(0,0)$?

spojitá je když umíme najít pro dané $\varepsilon > 0$ další okolí $(0,0)$ tak, že $|f(x,y)|$ na něm $< \varepsilon$



$$\frac{x^2-y^2}{x^2+y^2} \longrightarrow \begin{matrix} 0 \\ -1 \\ +1 \end{matrix}$$

$\Rightarrow$ <u>není spojitá v 0.</u>

$$f(x,y) = x^2 + xy + 3 + y^{356}$$

výpočet limit pro spojité (resp. kde existuje) stačí podél křivky.

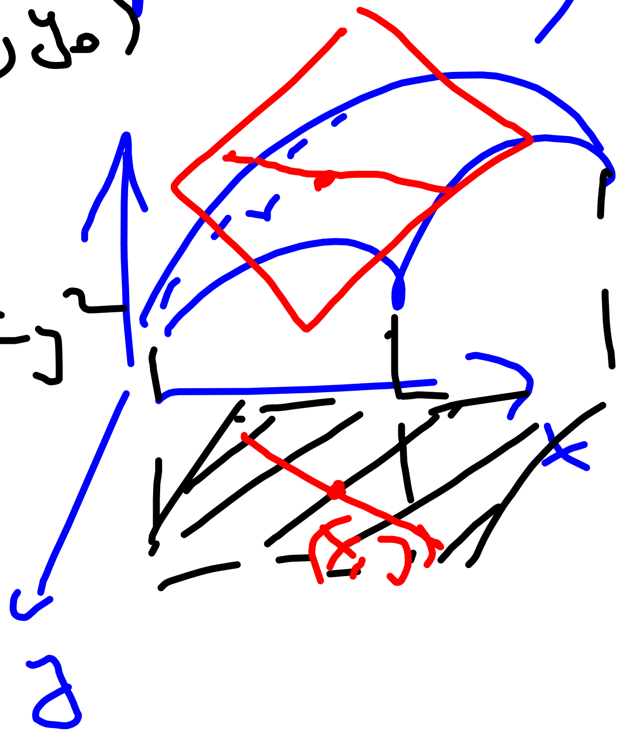
Např.

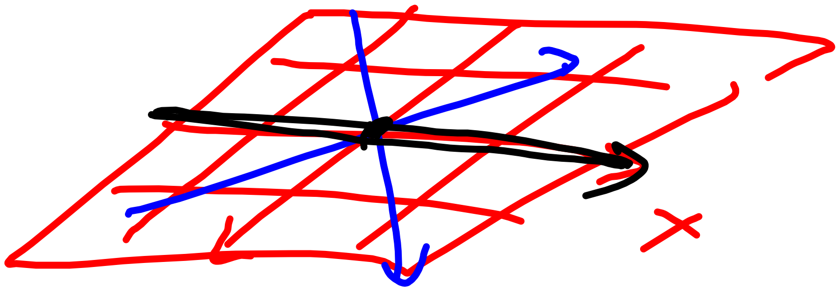
$$\lim_{(x,y)\to(3,0)} f(x,y) = \lim_{x\to 3} f(x,0)$$

$$= \lim_{x\to 3} x^2 + 3 = 12$$

Derivace ? (lineární přiblížení)

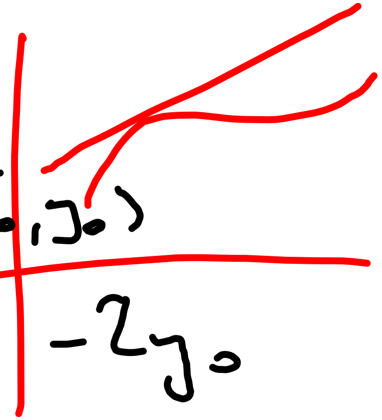
$$f(x,y) = 7 - x^2 - y^2$$





$$\frac{d}{dt}\Big|_0 f(x_0+t, y_0) = -2x_0 = \frac{\partial f}{\partial x}\Big|_{(x_0,y_0)}$$

$$\frac{\partial f}{\partial y} = \frac{d}{dt}\Big|_0 \big(f(x_0, y_0+t)\big) = -2y_0$$



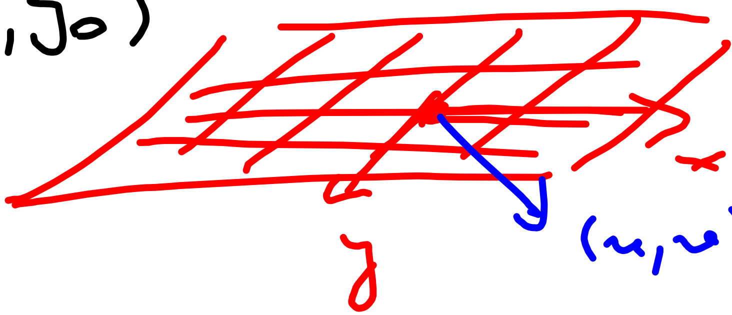
Obecná derivace ve směru:

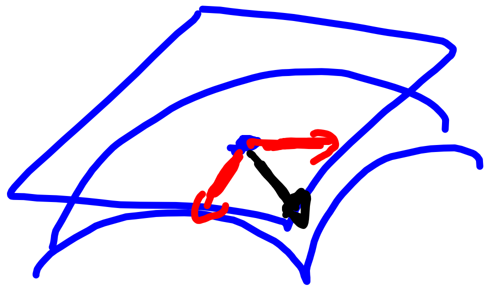
$$f(x,y) \qquad \underbrace{(x_0,y_0)}_{\text{bod v } \mathbb{R}^2} + t\cdot\underbrace{(u,v)}_{\text{libovolný vektor v } \mathbb{R}^2 \; w}$$

$$\left.\frac{d}{dt}\right|_0 f(x_0+tu,\, y_0+tv) =$$

$$= u\cdot\frac{\partial f}{\partial x}(x_0,y_0) + v\,\frac{\partial f}{\partial y}(x_0,y_0)$$

$$= d_w f(x_0,y_0)$$





Příklad:

$$f(x,y) = (\ln x)\cdot y$$

směrová derivace $\omega = (2,1)$ v $(x_0,y_0) = (1,1)$

1) $f(1+2t,\ 1+t) = \ln(1+2t)\cdot(1+t)$

$$\frac{d}{dt}\Big|_0 = \left(\underbrace{\frac{1}{1+2t}\cdot 2}_{\text{derivace 1. směr. činitele}}\cdot\underbrace{(1+t)}_{\text{2. vedení}} + \ln(1+2t)\cdot 1\right)\Bigg|_{t=0} = 2.$$

2) $$2\cdot\frac{\partial f}{\partial x}\Big|_{(1,1)} + 1\cdot\frac{\partial f}{\partial y}\Big|_{(1,1)} = \left(2\frac{1}{x}\cdot y + \ln x\right)\Big|_{(1,1)} = 2$$

<u>tečná rovina</u> : - prochází $(x,y,f(x_0,y_0))$
- obsahuje vektory $\left(1,0,\frac{\partial f}{\partial x}\right)$, $\left(0,1,\frac{\partial f}{\partial y}\right)$

$$f(x,y) = \sin x \cdot \cos y$$

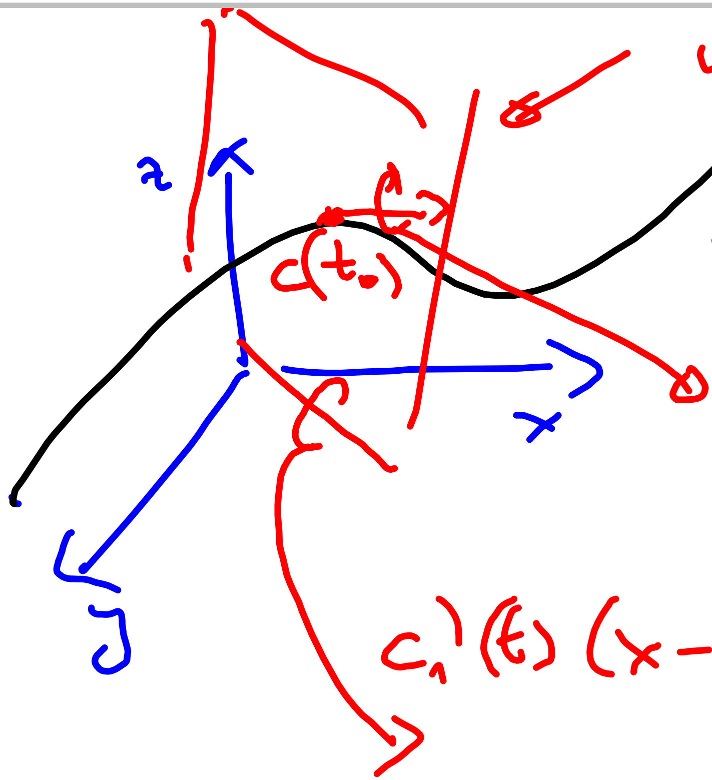
$$f(x,y) = \ln x \cdot y$$

$$\frac{\partial f}{\partial x} = \frac{1}{x}\cdot y \quad \Big| \quad \frac{\partial f}{\partial y} = \ln x$$

tj. <u>tečná rovina v $f(x,y)$ je</u>

$$\boxed{(x,y,\ln x\cdot y) + t\cdot\left(1,0,\frac{y}{x}\right) + s\cdot(0,1,\ln x)}$$

normální rovina



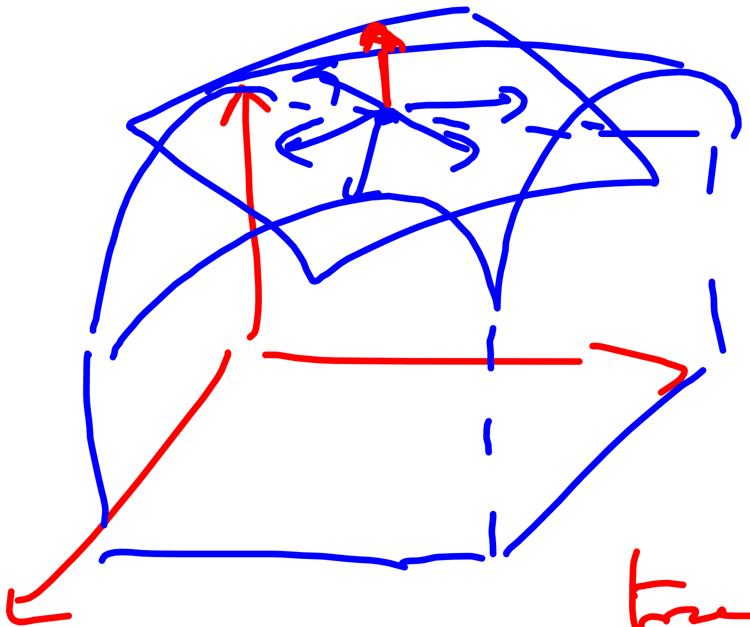
$$c(t) = (c_1(t), c_2(t), c_3(t))$$

$$c'(t) = (c_1'(t), c_2'(t), c_3'(t))$$

tečný vektor

$$c_1'(t)(x-x_0) + c_2'(t)(y-y_0) + c_3'(t)(z-z_0)$$

$$(x_0, y_0, z_0) = c(t_0)$$



tečná rovina:

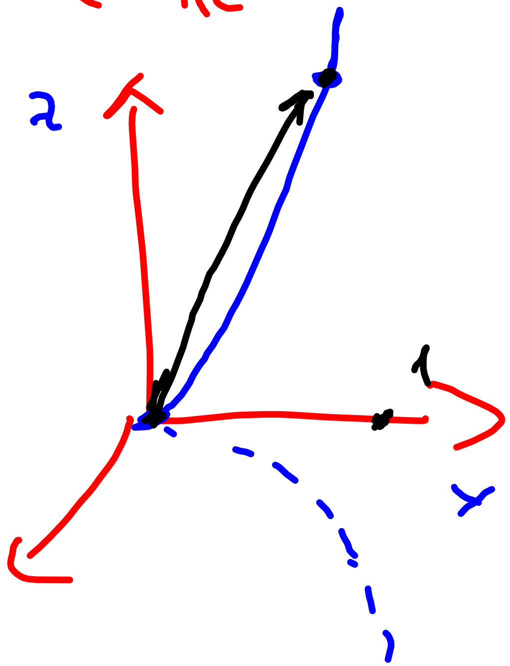
je-li v parametrizaci

tvaru $\alpha(x-x_0)+\beta(y-y_0)$

$+\gamma(z-z_0)$

$\Rightarrow$ normálový vektor je (α, β, γ)

$$c(t) = (t, t^2, t^3) \in \mathbb{R}^3$$



$$\int_0^1 c(t)\,dt =$$

$$\left(\int_0^1 t\,dt,\ \int_0^1 t^2\,dt,\ \int_0^1 t^3\,dt\right)$$

$$= \left(\left[\frac{t^2}{2}\right]_0^1,\ \left[\frac{1}{3}t^3\right]_0^1,\ \left[\frac{1}{4}t^4\right]_0^1\right)$$

$$= \left(\frac{1}{2},\ \frac{1}{3},\ \frac{1}{4}\right)$$