$$f_x = 3x^2 - 12y + 48 = 0$$

$$f_y = +12x + 2by = 0$$

$$x = -\frac{2}{12} b y$$

$$3 \cdot \frac{4}{12^2} b^2 y^2 - 12y + 48 = 0$$

$$D = \sqrt{12^2 - 4 \cdot \frac{3 \cdot 4}{12^2} \cdot 4 \cdot 12 b^2}$$

$$= \sqrt{144 - 16 b^2} \Rightarrow b = 3$$

$f_{xx} = 6x$

$f_{xy} = -12$

$f_{yy} = -26$

$\sim\sim\sim \ominus$

$$\begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$$

② $F(x,y,z) = x^2+y^2+z^2-1=0$

$T(x,y,z) = 1+xy+yz$

$$\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}\right) = \lambda \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)$$

$$F_x = 2x \qquad F_y = 2y \qquad F_z = 2z$$

$$T_x = y \qquad T_y = z + x \qquad T_z = y$$

$$y = 2\lambda x, \quad z + x = 2\lambda y, \quad y = 2\lambda z$$

$$\lambda \neq 0, \quad x = z \Rightarrow 2z = 2\lambda y$$

$$\Rightarrow 2\lambda^2 = 1 \Rightarrow \lambda = \pm\sqrt{1/2}$$

$$\Rightarrow z = \pm 1/2, \quad x = \pm 1/2$$

$\dashrightarrow$ 2 řešení $\dashrightarrow$ dosazení.

③ $\boxed{F(x,y) = \boxed{xy}\sin\left(\frac{\pi}{2}xy\right) = 1}$

$y \overset{?}{=} f(x) \qquad F(x, f(x)) = 1, \quad \boxed{f(1) = 1}$

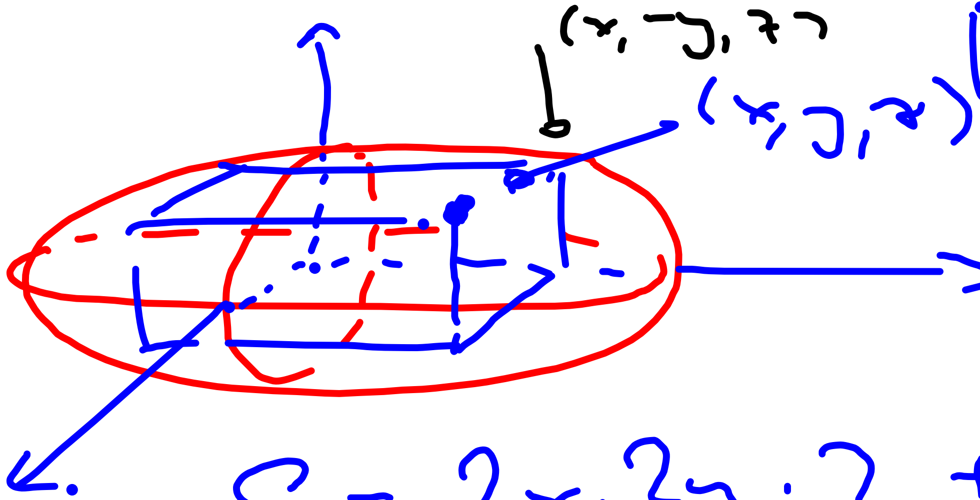
$F(1,1) = 1\cdot 1\cdot \sin(\pi/2) = 1$ ✓

$F_y(1,1) = \left. x\cdot\sin\left(\frac{\pi}{2}xy\right) + x y\cdot\cos\left(\pi/2\, xy\right)\cdot\frac{\pi}{2}x \right|_{(1,1)}$

$= 1 + 0 = 1 \neq 0$ ✓

$\frac{d}{dx}\left(F(x, y(x))\right) = F_x + F_y\cdot y'(x) = 0$

$\Rightarrow y'(x) = -\frac{F_x}{F_y} = -1$



$\boxed{x \ge 0,\ y \ge 0,\ z \ge 0}$

$$F = x^2 + 3y^2 + 3z^2 - 1$$

$$S = 2x \cdot 2y \cdot 2 + 8 \cdot xz + 8 \cdot yz$$
$$= 8(xy + xz + yz)$$

$F_x = 2x$ $\quad$ $F_y = 6y$ $\quad$ $F_z = 6z$

$S_x = 8(y+z)$ $\quad$ $S_y = 8(x+z)$ $\quad$ $S_z = 8(x+y)$

$\Rightarrow 4(y+z) = \lambda x$ $\quad$ $4(x+z) = 3\lambda y$ $\quad$ $4(x+y) = 3\lambda z$

$\lambda = 0$ ~~dá vš~~echno nulové

$\lambda \neq 0$:

$$\underbrace{\underset{\geq 0}{y} + \underset{\geq 0}{z} = \lambda \overset{\geq 0}{x}}$$

$$\underbrace{x + z = 3\lambda y}_{\Downarrow} \qquad x + y = 3\lambda z$$

$$-\lambda x + y + z = 0$$

$$\begin{aligned} \ominus \quad & x - 3\lambda y + z = 0 \\ \oplus \quad & x + y - 3\lambda z = 0 \end{aligned} \Bigg\} \Rightarrow (3\lambda + 1) y - (3\lambda + 1) z = 0$$

$\lambda = -1/3$ nelze $\qquad \underline{\underline{y = z}}$

$\Downarrow$

nejhorší

$$\Rightarrow \lambda x - 2y = 0$$

$$x + (1 - 3\lambda) y = 0 \quad / \cdot \lambda$$

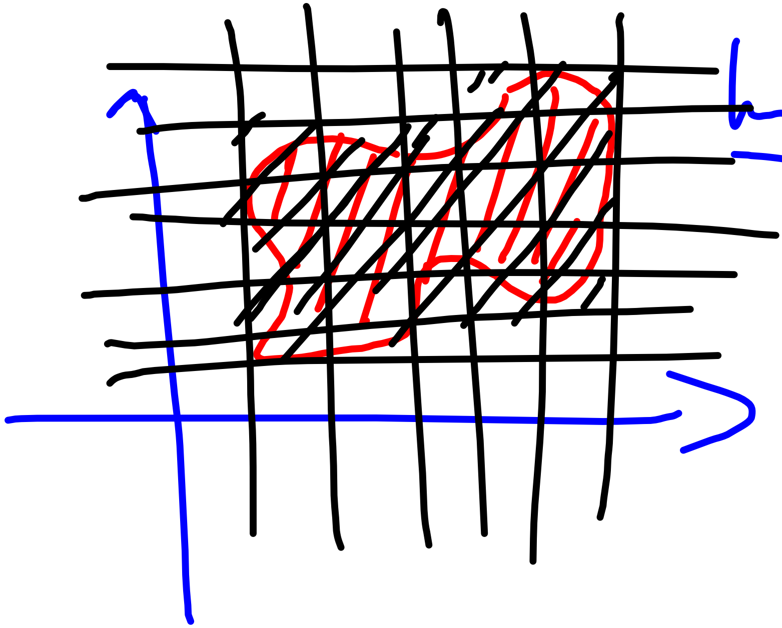
$$(\lambda(1 - 3\lambda) - 2) y = 0 \Rightarrow (2 + \lambda - 3\lambda^2) = 0$$

$$\Rightarrow \lambda = \frac{-1 \pm \sqrt{1 + 24}}{-6} = \begin{cases} +1 \\ \cancel{-\frac{2}{3}} \end{cases}$$

$x = 2y \qquad y = z \qquad S = 8(xy + xz + yz)$

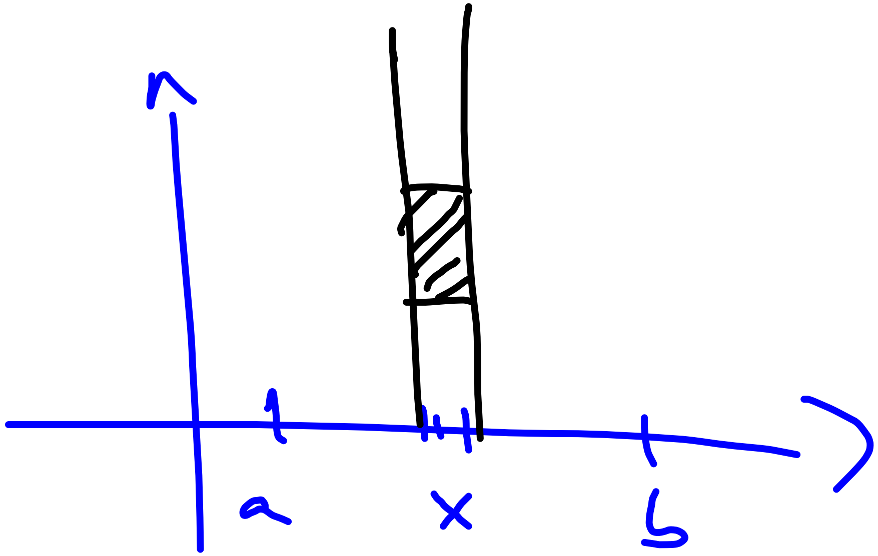
$$4y^2 + 3y^2 + 3y^2 = 1 \Rightarrow y = \sqrt{\frac{1}{10}}$$

$$S = 8\left(\frac{2}{10} + \frac{2}{10} + \frac{1}{10}\right) = 4$$

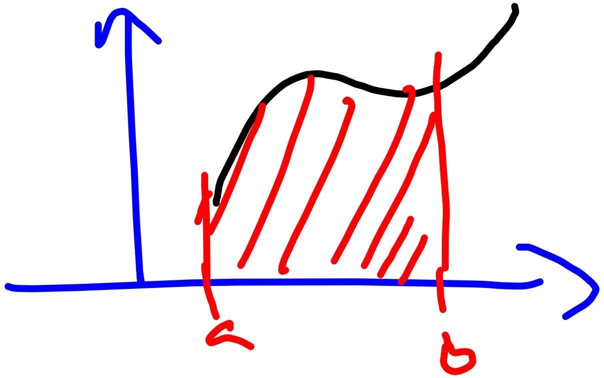


Integrování

$$\int_M f(x_1, \dots, x_n)\, dx_1 \dots dx_n$$

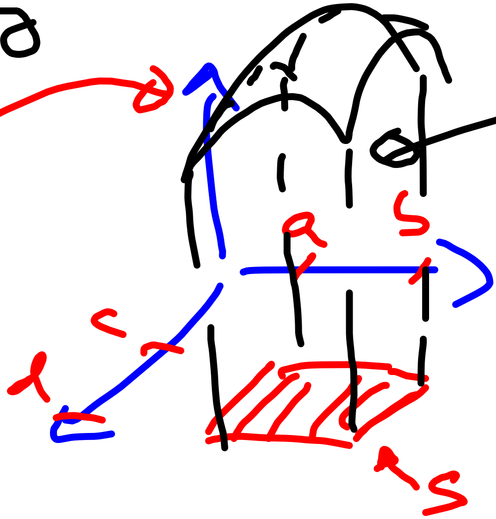


$$\int_a^b f(t)\,dt$$



$$\int_a^b \left(\int_c^d f(x,y)\,dx \right) dy$$

průnik



objem ~

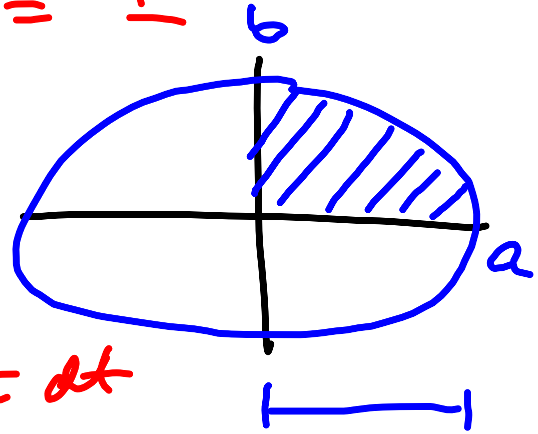
$$\int_S z(x,y)\,dx\,dy$$

Úloha 2.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

jednodušší: plocha

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

$$S = 4 \cdot b \int_0^a \sqrt{1 - \frac{x^2}{a^2}}\, dx \quad \begin{vmatrix} x = a \sin t \\ dx = a \cos t\, dt \end{vmatrix}$$

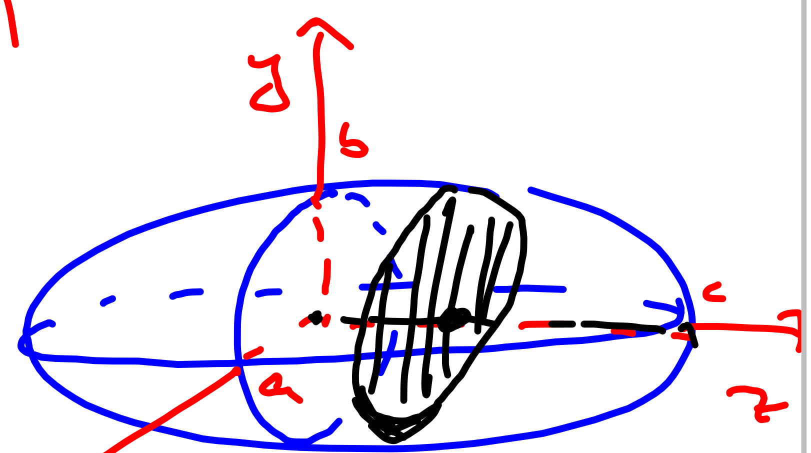
$$0 \quad \pi/2$$

$$= 4 \cdot b \cdot a \int_0^{\pi/2} \cos t \cos t\, dt$$

$$= [\text{chceš t, skočíš t}] = 4 \cdot \frac{1}{2} ab \left[x + \sin x \cos x\right]_0^{\pi/2}$$

$$= \frac{4}{2} \cdot \frac{1}{2} ab \cdot \pi = \pi a b$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



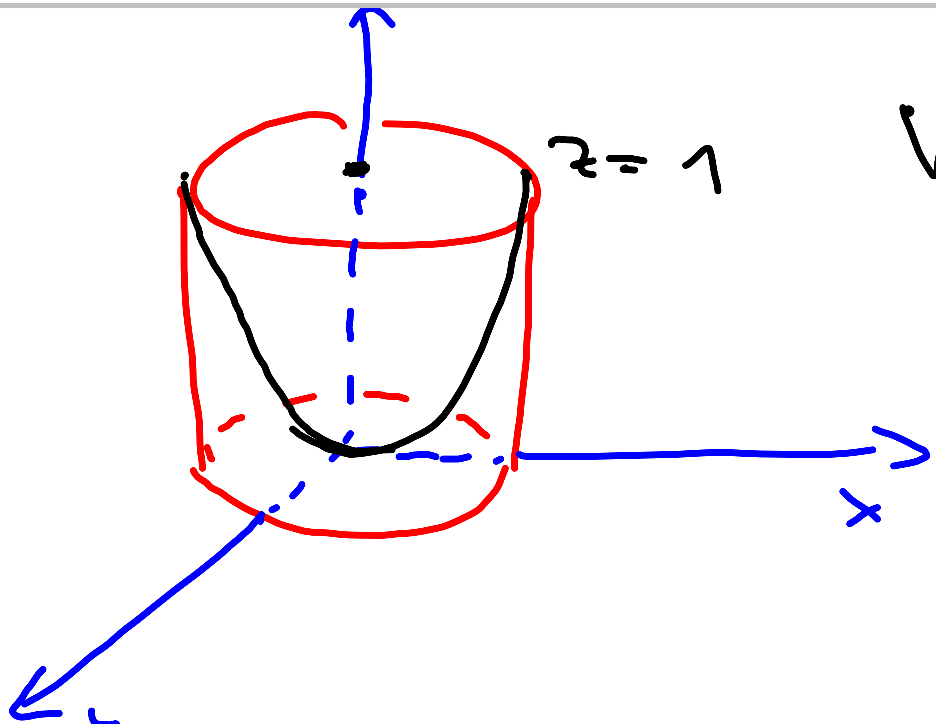
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\left(1-\frac{z^2}{c^2}\right)$$

$$\Downarrow$$

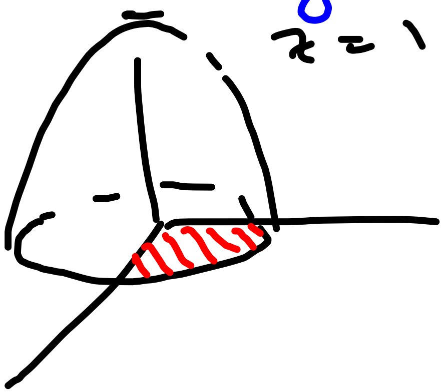
$$\boxed{\frac{x^2}{\underbrace{\left(1-\frac{z^2}{c^2}\right)a^2}_{\bar{a}^2}}+\frac{y^2}{\underbrace{\left(1-\frac{z^2}{c^2}\right)b^2}_{\bar{b}^2}}=1}$$

$$V=2\int_0^c \bar{a}\cdot\bar{b}\cdot\pi\,dz=2ab\pi\int_0^c\left(1-\frac{z^2}{c^2}\right)dz$$

$$=2ab\pi\left[z-\frac{1}{3}\frac{z^3}{c^2}\right]_0^c=\underline{\underline{\frac{4}{3}abc\pi}}$$



$V_{válec} = \pi$



$z = 1 - x^2 - y^2$

$$4 \cdot \int_0^1 \left(\int_0^{\sqrt{1-x^2}} z(x,y)\, dy \right) dx$$

$$\frac{1}{4}V = \int_0^1 \int_0^{\sqrt{1-x^2}} (1-x^2-y^2)\, dy\, dx$$

$$= \int_0^1 \left[y(1-x^2) - \frac{1}{3}y^3 \right]_0^{\sqrt{1-x^2}}$$

$$= \int_0^1 (1-x^2)^{3/2} - \frac{1}{3}(1-x^2)^{3/2}\, dx$$

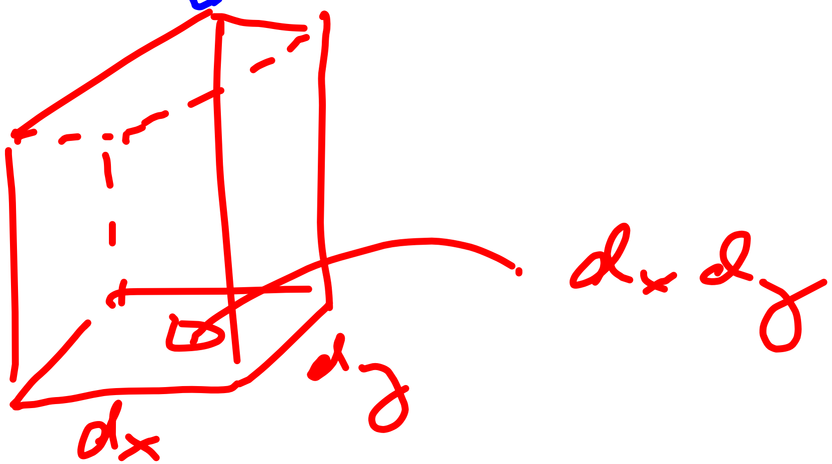
$$= \frac{2}{3}\int_0^1 (1-x^2)^{3/2}\, dx \Rightarrow$$ (dost složit)

$$V = \frac{8}{3}\left[\underbrace{\frac{x}{8}(5-2x^2)\sqrt{1-x^2}}_{\to 0} + \frac{3}{8}\arcsin x \right]_0^1$$

$$= \frac{8}{3}\cdot\frac{3}{8}\cdot\frac{\pi}{2} = \underline{\underline{\frac{\pi}{2}}}$$

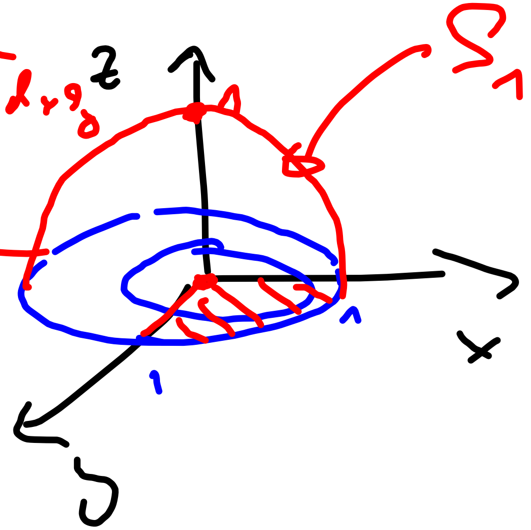
$$z - f(x,y) = 0$$

$$\sqrt{1 + (f_x)^2 + (f_y)^2}\, d_x\, d_y$$



$$z = 1-(x^2+y^2)^{3/2}, \quad z \ge 0$$

$$S_1 = 4\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1+(z_x)^2+(z_y)^2}\,dx\,dy$$



$$= 4\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1+9(x^2+y^2)x^2+9(x^2+y^2)y^2}\,dx\,dy$$

$$= 4\cdot\int_0^1 \int_0^{\sqrt{1-x^2}} \sqrt{1+9(x^2+y^2)^2}\,dx\,dy$$

$$z_x = -\frac{3}{\cancel{2}}\sqrt{x^2+y^2}\cdot \cancel{2}x$$

$$z_y = -\frac{3}{\cancel{2}}\sqrt{x^2+y^2}\cdot \cancel{2}y$$

$$x = r\cdot\cos\varphi \qquad y = r\cdot\sin\varphi$$

$$\begin{vmatrix} x_r & x_\varphi \\ y_r & y_\varphi \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & r\cos\varphi \end{vmatrix} = r$$

$$dx\,dy = r\,dr\,d\varphi$$

$$S = 4\int_0^{\pi/2}\int_0^1 \sqrt{1+9r^4}\cdot r\,dr\,d\varphi$$

$$= 2\pi \int_0^1 \sqrt{1+9r^4}\cdot r\,dr$$

$$r^2 = t$$

$$2r\,dr = dt$$

$$= \pi\int_0^1 \sqrt{1+9\underbrace{t^2}_{(3t)^2}}\,dt \qquad \begin{array}{|l} 3t = s \\ 3\,dt = ds \end{array}$$

$$= \frac{\pi}{3}\int_0^3 \sqrt{1+s^2}\,ds = \frac{\pi}{3}\left[\frac{s}{2}\sqrt{1+s^2}+\frac{1}{2}\ln\left|s+\sqrt{1+s^2}\right|\right]_0^3$$

$$= \frac{\pi}{3}\cdot\left[\frac{3}{2}\sqrt{10}+\frac{1}{2}\ln\left(3+\sqrt{10}\right)\right]$$