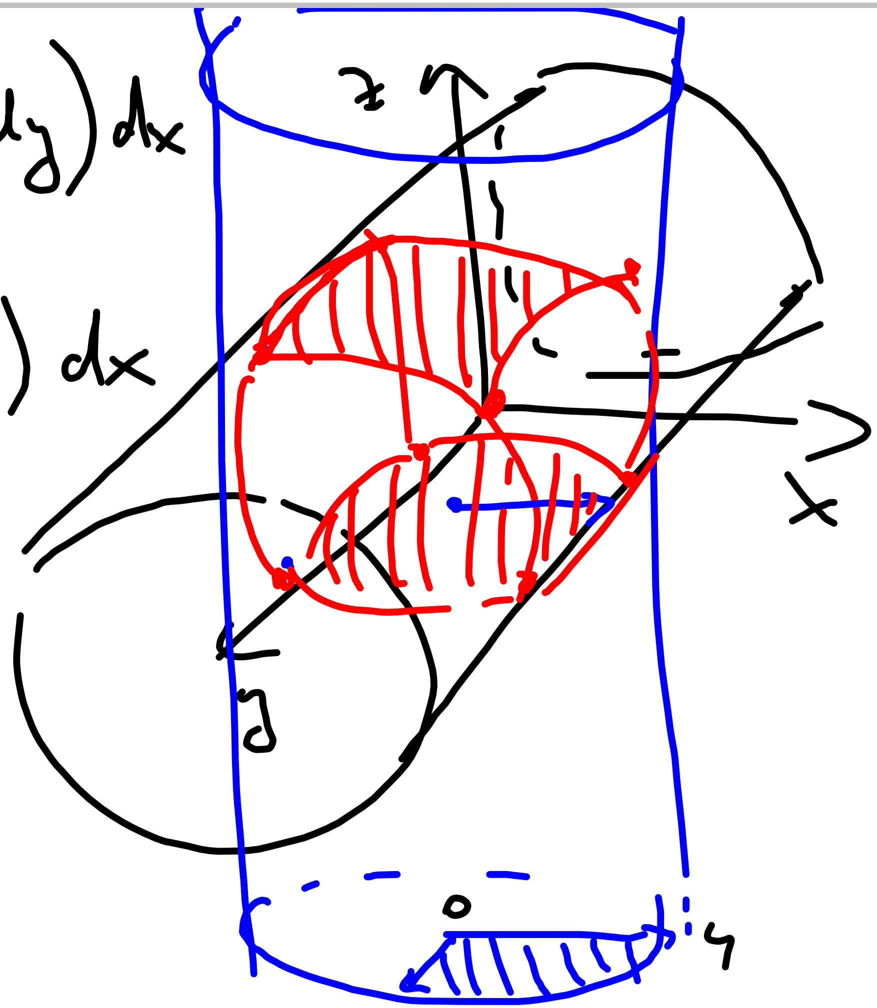
$$P = 4 \cdot 2 \cdot \int_0^4 \left(\int_0^{\sqrt{16-x^2}} \sqrt{1+z_x^2+z_y^2}\, dy \right) dx$$

$$= 8 \cdot \int_0^4 \left(\int_0^{\sqrt{16-x^2}} \sqrt{1+\frac{x^2}{16-x^2}}\, dy \right) dx$$

$$= 8 \cdot \int_0^4 \int_0^{\sqrt{16-x^2}} \frac{\sqrt{16}\, dy}{\sqrt{16-x^2}}\, dx$$

$$= 32 \cdot \int_0^4 \left[y \cdot \frac{1}{\sqrt{16-x^2}} \right]_0^{\sqrt{16-x^2}} dx$$

$$= 32 \cdot \int_0^4 dx = \underline{\underline{128}}$$

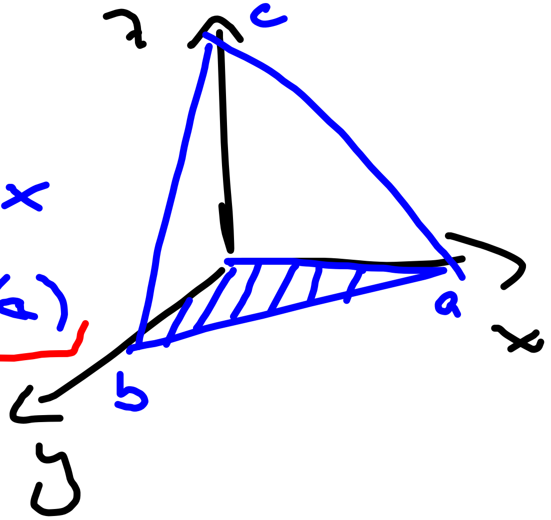


$$z = \sqrt{16-x^2}$$

$$z_x = \frac{1}{\cancel{2}} \frac{-\cancel{2}x}{\sqrt{16-x^2}}$$

$$z_y = 0$$

$$z = c\left(1 - x/a - y/b\right)$$



$$V = c\int_0^a \int_0^{b(1-x/a)} \left(1 - x/a - y/b\right)\, dy\, dx$$

$$= c\int_0^a \left[\left(1 - x/a\right) y - \frac{1}{2} y^2/b\right]_0^{b(1-x/a)} dx$$

$$x/a + y/b = 1$$

$$= \frac{1}{2} c \int_0^a \left(1 - x/a\right)^2 b\, dx$$

$$1 - x/a = t$$

$$-dx = \underline{a\,dt}$$

$$= \frac{1}{2} bc \int_1^0 (-1)\, t^2 a\, dt$$

$$= \frac{1}{2} abc \left[\frac{1}{3} t^3\right]_0^1$$

$$= \frac{1}{6} abc$$

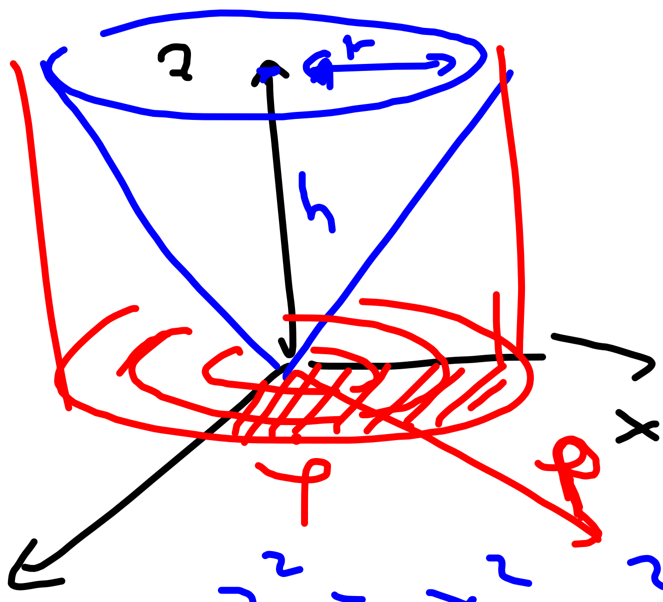
$$V_{\text{vně}} = 4\int_0^r \int_0^{\sqrt{r^2-x^2}} \frac{h}{r}\sqrt{x^2+y^2}\, dy\, dx$$

$$x, y, z \longmapsto \rho, \phi, z$$

$$\underbrace{dx\,dy\,dz}_{\phi} \longmapsto \underbrace{\rho\, d\rho\, d\varphi\, dz}_{\varphi}$$

$$V = 4\int_0^{\pi/2}\int_0^{r}\int_{\frac{h}{r}\rho}^{h} \rho\, dz\, d\rho\, d\phi$$

$$= \frac{1}{3}\pi h r^2$$



$$z^2 = x^2 + y^2$$

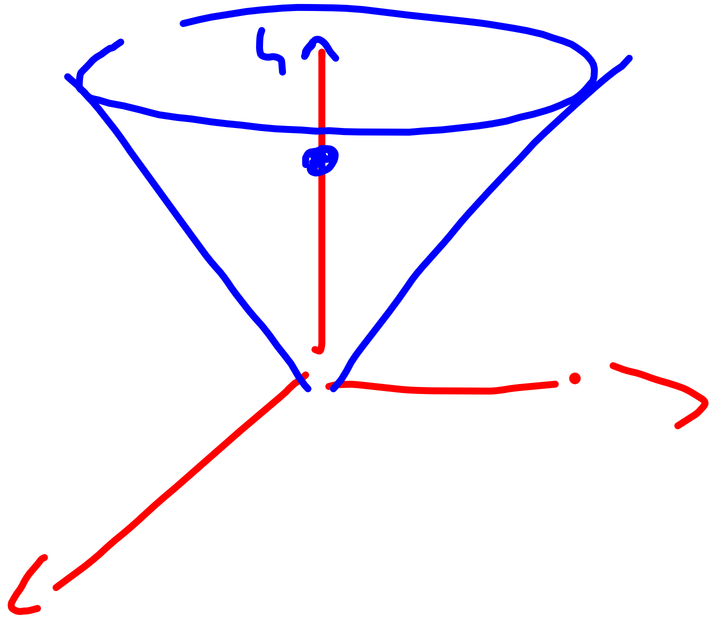
$$z^2 = a^2(\underbrace{x^2+y^2}_{r^2})$$

$$= \left(\frac{h}{r}\right)^2 (x^2+y^2)$$

$$\boxed{z = \frac{h}{r}\sqrt{x^2+y^2}}$$

těžiště

$$z_{\text{těžiště}} = \frac{1}{V} \int_{\text{kužel}} z \, dV$$



$$= \frac{1}{V} \int_0^{\pi/2} \int_0^{r} \int_{\frac{h}{r}\rho}^{h} z \rho \, dz \, d\rho \, d\phi = \underline{\underline{\tfrac{3}{4} h}}$$

Diferenciální operátory:

$$f(x) \mapsto \boxed{F\left(f(x), \frac{df}{dx}(x), \ldots, \frac{d^k f}{dx}(x)\right) = ?}$$

$F(x, y) = x^2 y \qquad x = f(t)$

$F(f(t), f'(t)) = f^2(t) \cdot f'(t)$

$y = f(x) \qquad \boxed{y' = F(x, f(x))}$

$$\boxed{y' = \frac{1 + y^2}{1 + x^2}} \longleftarrow \text{najděte}$$

$y = y(x)$

$$y'(x) = f(x)\cdot g(y)$$

$$\frac{dy}{dx} = f(x)\cdot g(y)$$

$$\underbrace{\int \frac{dy}{g(y)}}_{G(y)} = \underbrace{\int f(x)\cdot dx}_{F(x)}$$

separované proměnné

ODE 1. řádu

$$G(y) - F(x) = 0$$

implicitní zadání řešení

$$\left(\frac{d}{dx}\left(G(y) - F(x)\right) = \frac{1}{g(y)}\cdot y'(x) - f(x) = 0\right)$$

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \qquad \int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\operatorname{arctg} y = \operatorname{arctg} x + C \quad / \ \operatorname{tg}$$

$$y = \operatorname{tg}(\operatorname{arctg} x + C)$$

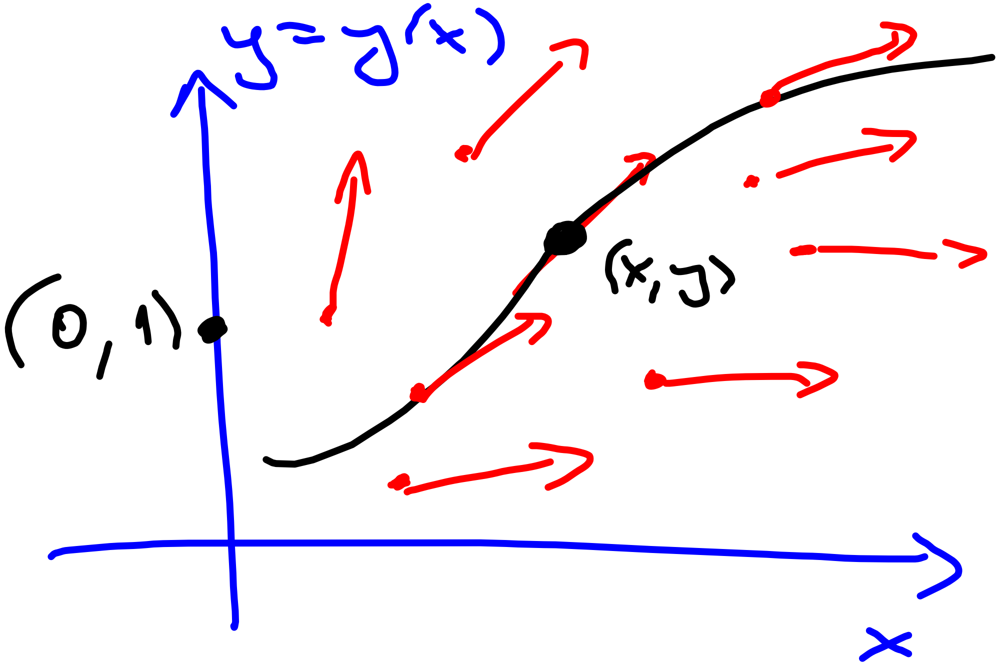
$$= \frac{x + \underbrace{\operatorname{tg} C}_{= D}}{1 - x \cdot \operatorname{tg} C} = \frac{x + D}{1 - D \cdot x}$$

$$y' = \frac{(1 - D\cancel{x}) + (\cancel{x} + D)D}{(1 - D\cdot x)^2} = \frac{1 + D^2}{(1 - Dx)^2}$$

$$\frac{y^2+1}{x^2+1} = \frac{\frac{(x+D)^2}{(1-Dx)^2}+1}{x^2+1} = \frac{\frac{(x+D)^2+(1-Dx)^2}{(1-Dx)^2}}{x^2+1}$$

$$= \frac{x^2+\cancel{2Dx}+D^2+1-\cancel{2Dx}+D^2x^2}{(x^2+1)(1-Dx)^2}$$

$$= \frac{\cancel{(x^2+1)}(D^2+1)}{\cancel{(x^2+1)}(1-Dx)^2}$$



$$y' = F(x, y)$$

$$y = y(t)$$

$$x(t) = t$$

$$y(x_0) = y_0$$

$$y(x) = \frac{\cancel{x} + D}{(1 - xD)} \qquad y(0) = 1$$

$$\cancel{0} + D = 1 \Rightarrow D = 1$$

$$\boxed{y(x) = \frac{1 + x}{1 - x}}$$

② rychlost vytékání v (m^3/s)

$V = 2000\ m^3$

$\frac{m}{V} v\, dt = dm$

$d_{tot} = 10\ (g/m^3)$

$m = m(t)$

$\frac{dm}{dt} = -\frac{m}{V} \cdot v \quad \Big| \int \frac{dm}{m} = -\frac{v}{V} \int dt$

$\Rightarrow \ln m = -\frac{v}{V} t + C \quad / e^x$

$m(t) = e^{-\frac{v}{V}t} \cdot \underbrace{e^{C}} = m_0 > 0$

$m_0 = 20.000\ (g)$

$m(T) = 20 \cdot 10^{-3}$

cca $T = \underline{\underline{6h\ 35min}}$

ne konstantní v čase

$$\frac{dm}{dt} = -k \cdot m(t)$$

$$\int \frac{dm}{m} = k \int dt \Rightarrow \ln m(t) = -k t + C$$

$$\left.\begin{aligned} m(t) &= m_0 e^{-kt} \\ m(24\,100) &= \frac{1}{2} m_0 \end{aligned}\right\} \Rightarrow e^{-k(24\,100)} = \frac{1}{2}$$

$$\Rightarrow k \doteq 2{,}88 \cdot 10^{-5}$$

$\Rightarrow$ je třeba **349 let!!!**