graf: (V, E)

vertex, edge

morfismus

$\varphi : V \longrightarrow V'$

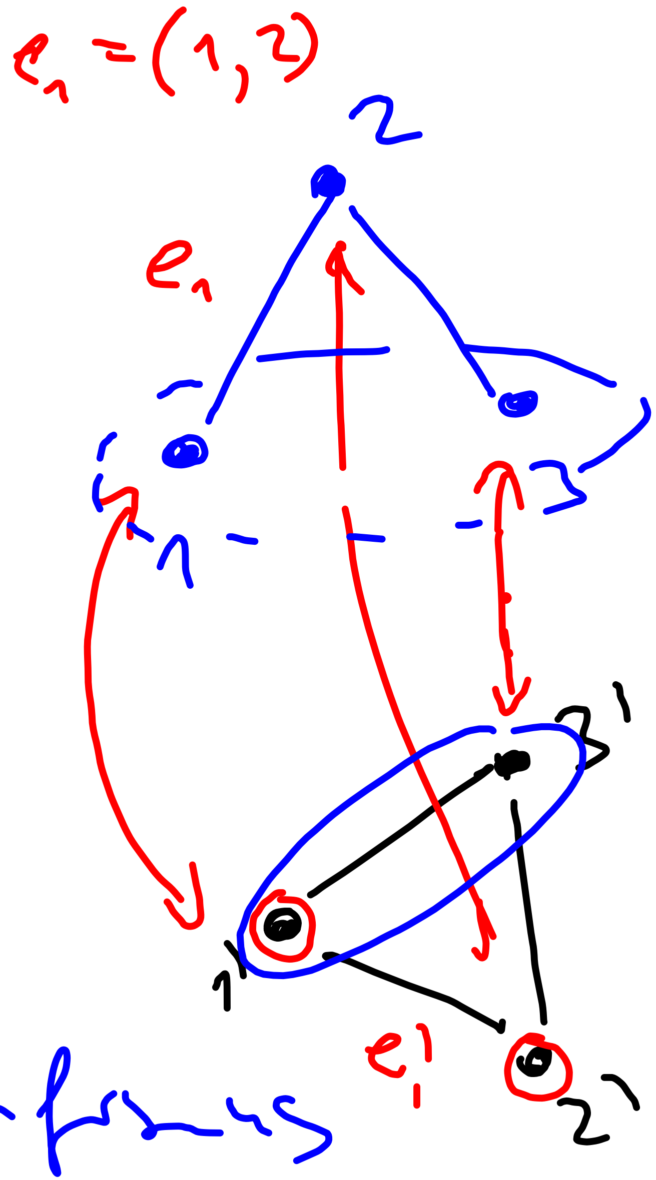
$\varphi(1) = 1'$

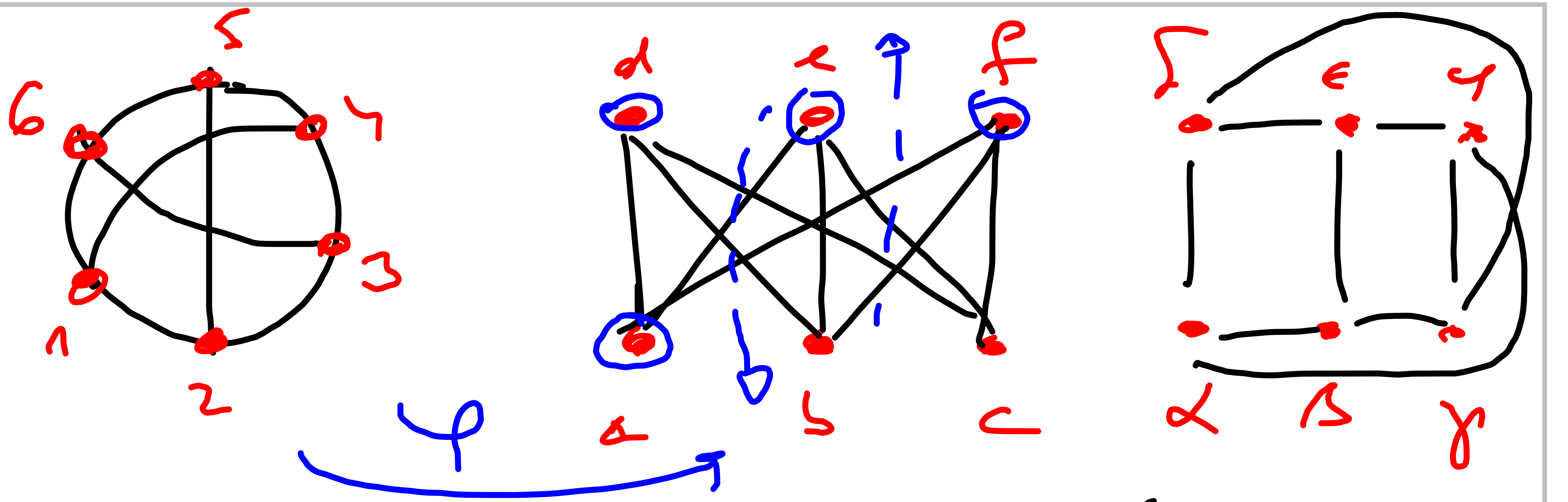
$\varphi(2) = 2'$

$\varphi(3) = 3'$

φ^{-1} je morfismus

není morfismus





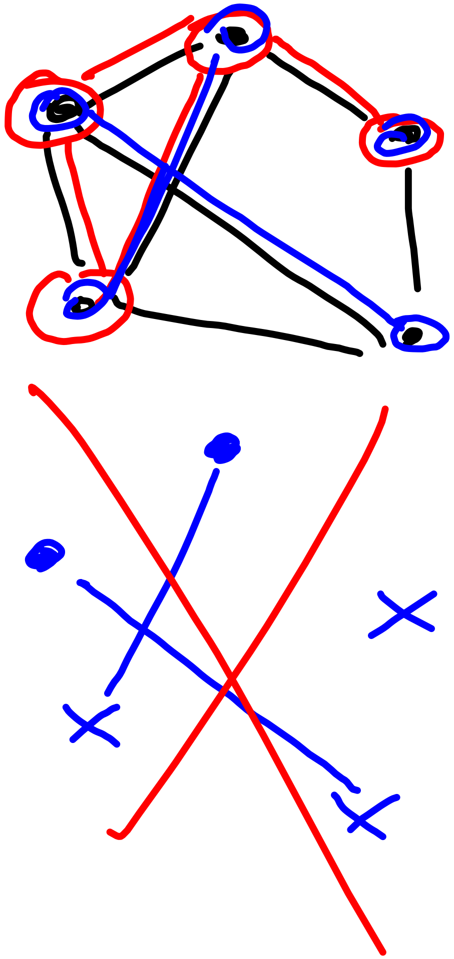
Ukažte, že jsou izomorfní.

$\varphi(1) = a \quad \varphi(2) = d \quad \varphi(3) = b$

$\varphi(4) = e \quad \varphi(5) = c \quad \varphi(6) = f$

$0 \leq$ počet hran $\leq \binom{n}{2}$

podgraf:



$G = (V, E)$

$G = (V', E')$ indukovaný podgraf

$G = (V, E)$

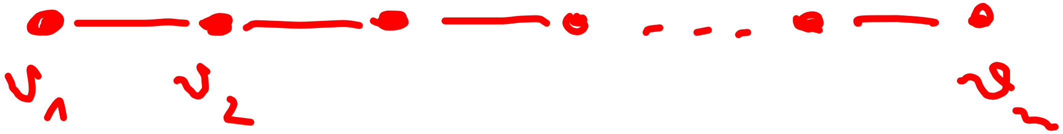
$G = (V', E')$

$V' \subset V$, $E' \subset E$

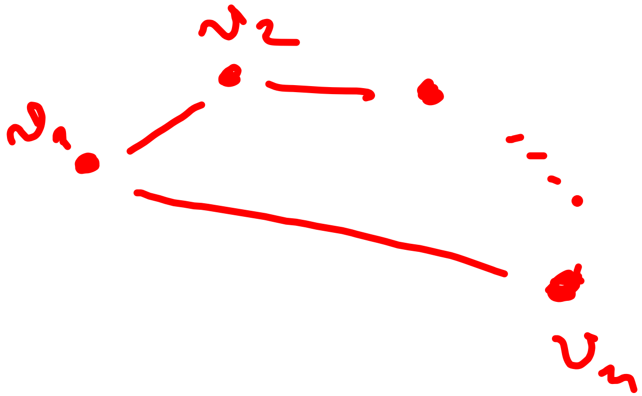
ale taková, že hrany jsou mezi vrcholy z V'

Cesty, sledy, kružnice:

P_n:

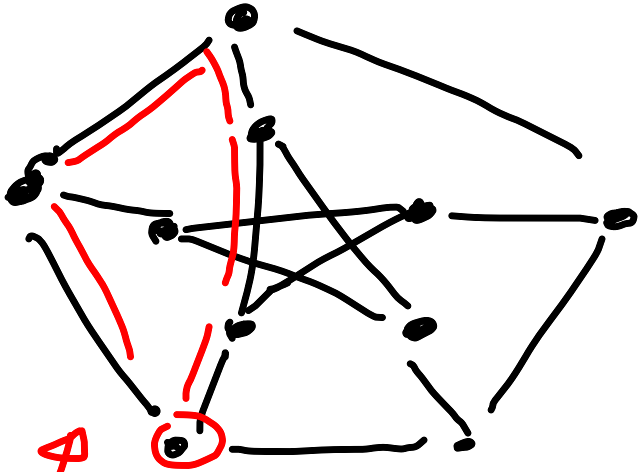


C_n:



cesta v grafu: $\varphi : P_n \to G$ t.ž. φ je injektivní (na hrany i uzly)

sled v grafu $\varphi : P_n \to G$



nejkratší kružnice
(délka 5)

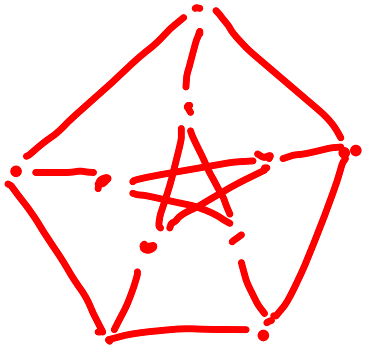
kružnice
délky 4

5x

nejsou izomorfní

Skóre grafu:

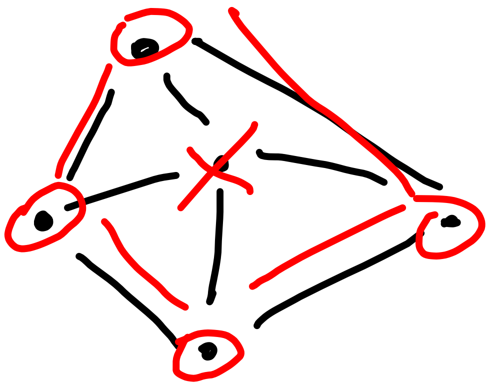
stupeň vrcholu = počet hran z něho vycházejících



skóre = (3,3,3,3,3,3,3,3,3,3)

izomorfní grafy mají stejné skóre

skóre je uspořádaná posloupnost stupňů vrcholů (≥ , ≤)



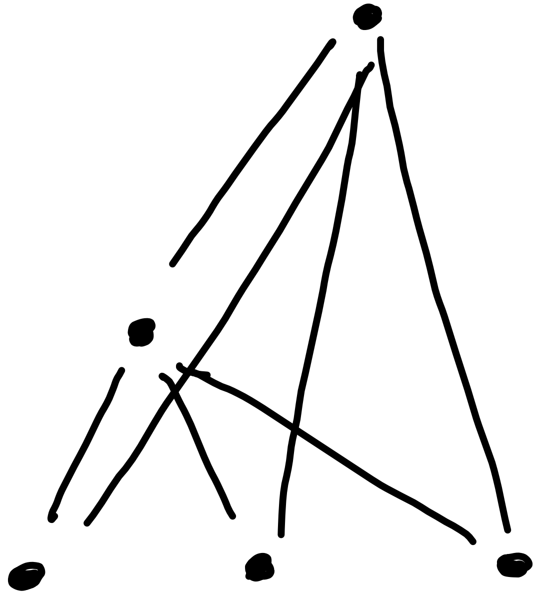
$(4,3,3,3,3)$

$(2,2,2,2)$

skóre $(\cancel{4},4,2,2,2)$?

$(\cancel{3},1,1,1)$

$(0,0,0)$



$(4, 3, 3, 2, 2)$

$\downarrow$

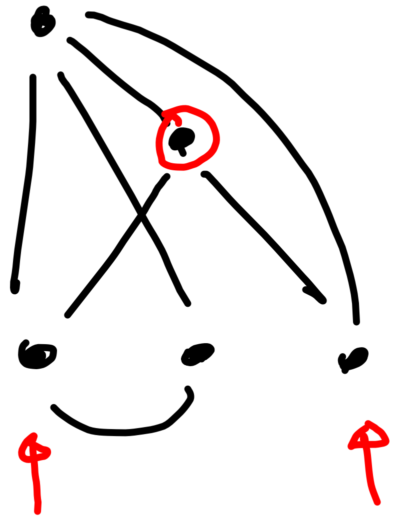
$(2, 2, 1, 1)$

$\downarrow$

$(1, 1, 0)$

$(1, 0, 1)$

tady přidáme hranu



$(4, 4, 4, 2, 2)$

$\downarrow$

$(3, 3, 1, 1)$

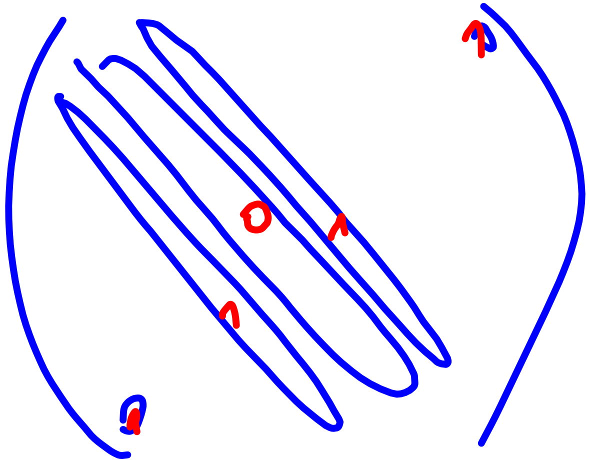
$\downarrow$

$(2, 0, 0)$

NE

1) matice sousednosti

2) dvojitě spojité seznamy



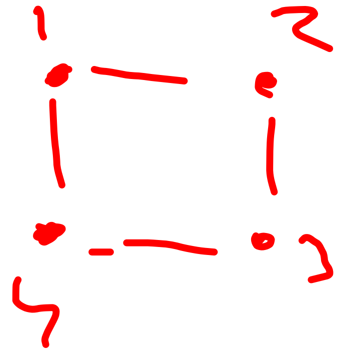
$\{v_1, \dots, v_n\} = V$

G ... matice nad $\mathbb{Z}_2$ $(n \times n)$

$$a_{ij} = \begin{cases} 1 & (v_i, v_j) \text{ je hrana} \\ 0 & \text{jinak} \end{cases}$$

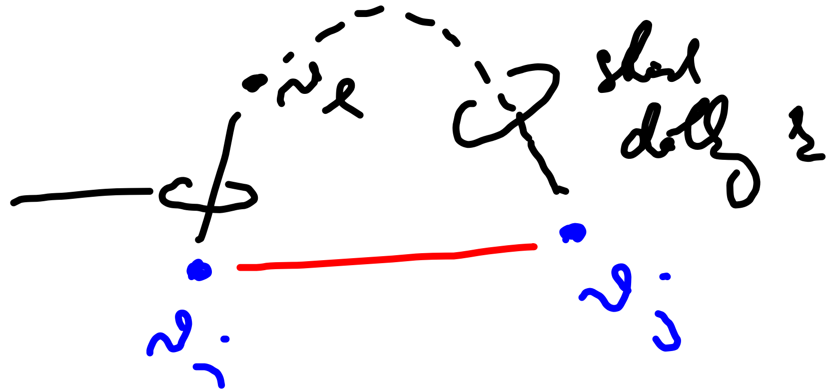
C_4:

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$



$$(G)^k = \left(a_{ij}^{(k)}\right)$$

$$1 = a_{il}$$



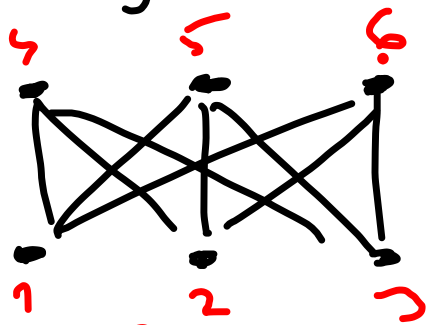
$$\boxed{\sum_l a_{il}\, a_{lj}^{(k)}} = a_{ij}^{(k+1)}$$

$$G \cdot (G^k)$$

P_n, C_n, K_n .. úplný graf na n vrcholech

$K_{m,n}$ .. úplný bipartitní graf

$K_{3,3}$:



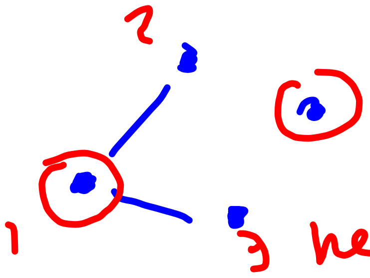
$$G_{K_{3,3}} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad G^2 = \begin{pmatrix} 3 & 3 & 3 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 3 & 3 & 3 \end{pmatrix}$$

$$G^3 = \left(\begin{array}{ccc|ccc} 0 & 0 & 0 & 9 & 9 & 9 \\ 0 & 0 & 0 & 9 & 9 & 9 \\ 0 & 0 & 0 & 9 & 9 & 9 \\ \hline 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 \end{array}\right)$$

$$\vdots$$

$$G^4 = \left(\begin{array}{ccc|c} 27 & 27 & 27 & \\ 27 & 27 & 27 & 0 \\ 27 & 27 & 27 & \\ \hline & 0 & & 27 \end{array}\right)$$

Souvislý graf: mezi každými vrcholy existuje cesta



není cesta

$$G = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A = (G + \mathbb{I}_n)^{n-1}$$

souvislý $\Leftrightarrow$ má samé nenulové prvky

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A^2, \quad A^3$$