1) 0, všechny, $\{0, 1, \frac{1}{2}, \frac{1}{4}, \dots\}$, $\emptyset$

2) $O_n = \left(a - \frac{1}{n}, a + \frac{1}{n}\right)$

$\mathbb{R}$, $\mathbb{R}$

3) $(-\sqrt{2}, \sqrt{2})$

$\langle -\sqrt{2}, +\sqrt{2} \rangle$,

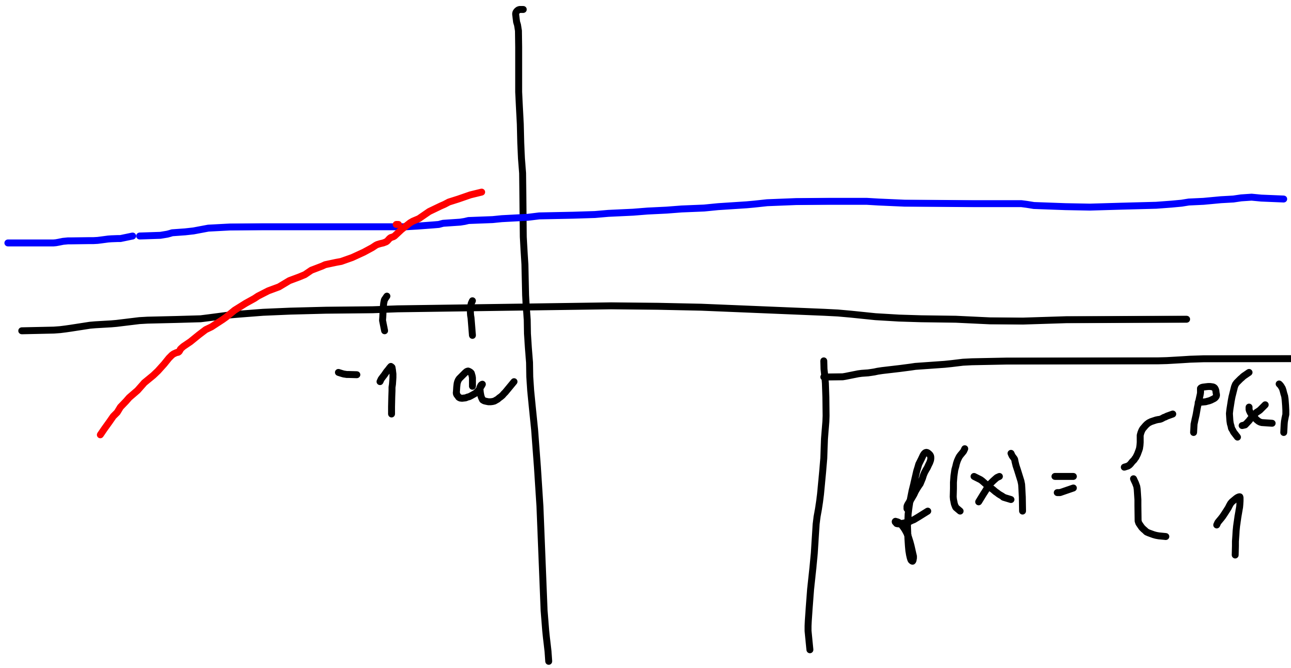
2. a) $\langle 0, 1)$

b) $\mathbb{R}$

c) $\bigcup_{n=2}^{\infty} \langle 0 + \frac{1}{n}, 1 - \frac{1}{n} \rangle = (0, 1)$

d) $\bigcap_{n=1}^{\infty} (-\frac{1}{n}, \frac{1}{n}) = \{0\}$

Spojitá funkce v bodě -1



$$f(x) = \begin{cases} P(x) \\ 1 \end{cases}$$

$$x^3 + x^2 + x + 2 = 1 \iff \underline{x^3 + x^2 + x + 1} = 0$$

$$= \frac{x^4 - 1}{x - 1} = \frac{(x^2+1)(x-1)(x+1)}{(x-1)} =$$

$$= (x+1)(x^2+1)$$

$$\lim_{n\to\infty} \frac{1}{n^2} = 0$$

$$\lim_{n\to\infty} \frac{4n^3 - 2n^2 + 1}{2n^3 - n^2 + 3} = \lim_{n\to\infty} \frac{4 - 2\frac{1}{n} + \frac{1}{n^3}}{2 - \frac{1}{n} + \frac{3}{n^3}} =$$

$$= \frac{4 - 2\lim\limits_{n\to\infty}\frac{1}{n} + \lim\limits_{n\to\infty}\frac{1}{n^3}}{2 - \lim\limits_{n\to\infty}\frac{1}{n} + \lim\limits_{n\to\infty}\frac{3}{n^3}} = 2$$

$$\lim_{n\to\infty} \frac{2n^3 - 2n^2 + 1}{n^2 - n + 3} = \lim_{n\to\infty} \frac{n - 2 + \frac{1}{n^2}}{1 - \frac{1}{n} + \frac{3}{n^2}} = \infty$$

Daná posloupnost je klesající pro $c > 1$.

a) $c > 1$, každý člen dané posloupnosti je větší než 1. Potom $\underbrace{\inf_{n\in\mathbb{N}} (\sqrt[n]{c})}_{B}$ je limitou dané posloupnosti.

Pro $(B+\varepsilon)$ $\varepsilon > 0$ existuje n_0 tak, že $\sqrt[n_0]{c} < B+\varepsilon$, ale pak pro všechna $n \geq n_0$ je $\sqrt[n]{c} \in \langle B, B+\varepsilon)$.

Ukážeme, že $B = 1$. Kdyby tomu tak nebylo, pak $B = 1+a$, $a > 0$. Volme nyní $\varepsilon = \frac{a^2}{4}$.

Potom ex. n_0 takové, že pro $(\forall n > n_0)$ je $\left|\sqrt[n]{c} - (1+a)\right| < \frac{a^2}{4}$ $\left(\Leftrightarrow \sqrt[n]{c} \in \left(1+a-\frac{a^2}{4}, 1+a+\frac{a^2}{4}\right)\right)$

Volme nyní $n = 2n_0$:

$$\underline{\sqrt[2n_0]{c}} \leq 1 + a + \frac{a^2}{4} = \underline{\left(1 + \frac{a}{2}\right)^2}$$

Dále víme, že $\sqrt[n_0]{c} \leq 1 + a + \frac{a^2}{4}$. Potom

$$\underline{\sqrt[2n_0]{c}} = \sqrt[2]{\sqrt[n_0]{c}} \underline{\leq} \sqrt{1 + a + \frac{a^2}{4}} = \sqrt{\left(1 + \frac{a}{2}\right)^2} = \underline{1 + \frac{a}{2}}$$

Potom $\left|\sqrt[2n_0]{c} - (1+a)\right| \geq \frac{a}{2}$ $\lightning$

Pokud $0 < c < 1$, tak uvážíme posloupnost $\sqrt[n]{\frac{1}{c}}$, kdy $\lim_{n \to \infty} \sqrt[n]{\frac{1}{c}} = 1 \Rightarrow \lim_{n \to \infty} \sqrt[n]{c} = 1$

$k, n \in \mathbb{N}$, $n \geq k$

$$n! \geq \frac{k!}{k^k} \cdot k^n = k!\, k^{(n-k)}$$

$$\Updownarrow$$

$$\underbrace{n \cdot (n-1) \cdots (k+1)}_{n-k \text{ čísel}} \cdot k! \geq k^{(n-k)} \cdot k!$$

Ukážeme, že pro lib. $k \in \mathbb{N}$ je $\lim\limits_{n \to \infty} \sqrt[n]{n!} > k$

$$\lim_{n\to\infty} \sqrt[n]{n!} \geq \lim_{n\to\infty} \sqrt[n]{\frac{k!}{k^k} k^n} = \lim_{n\to\infty} k \sqrt[n]{\frac{k!}{k^k}} =$$

$$= k \underbrace{\lim_{n\to\infty} \sqrt[n]{\frac{k!}{k^k}}}_{1} = k$$

$$\lim_{x\to 2} \frac{x-2}{\underbrace{x^2-5x+6}_{(x-2)(x-3)}} = \lim_{x\to 2} \frac{1}{(x-3)} = -1$$

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{1}{(\sqrt{x}+1)} = \frac{1}{2}$$

$$\lim_{x\to 1} \frac{1}{1+3^{\frac{1}{x-1}}}$$

(všimněme si, že

$$\lim_{x\to 1^+} \frac{1}{x-1} = \infty$$

$$\lim_{x\to 1^-} \frac{1}{x-1} = -\infty$$

, potom

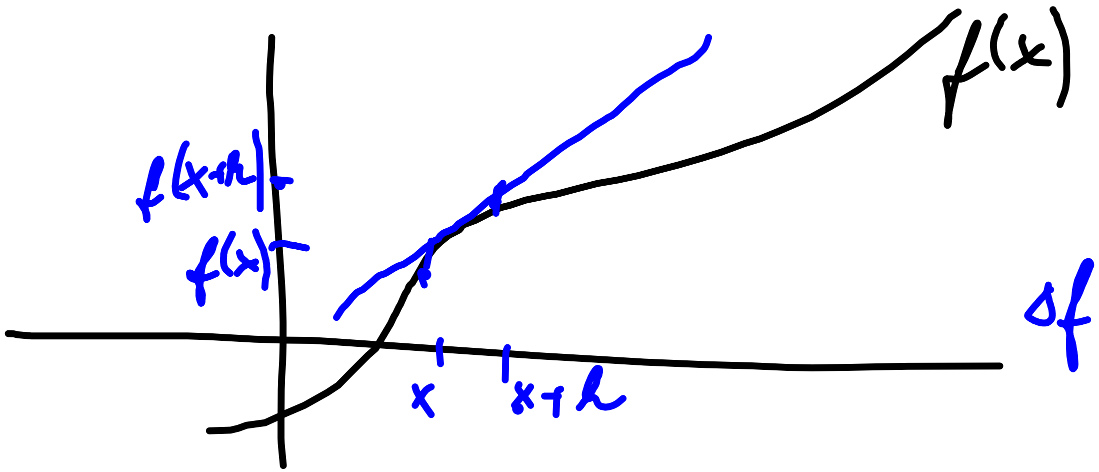
$$\lim_{x\to 1^+} \frac{1}{1+\underbrace{3^{\frac{1}{x-1}}}_{\to\infty}} = 0$$

$$\lim_{x\to 1^-} \frac{1}{1+\underbrace{3^{\overbrace{\frac{1}{x-1}}^{\to -\infty}}}_{\to 0}} = 1$$

$$\lim_{x\to 2} \frac{x-2}{\sqrt{x^2-4}} = \lim_{x\to 2} \frac{x-2}{\sqrt{(x-2)(x+2)}} = \lim_{x\to 2} \frac{\sqrt{x-2}}{\sqrt{(x+2)}} = 0$$

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x^2+3}-2} = \lim_{x\to 1} \frac{(x-1)\left[\sqrt{x^2+3}+2\right]}{x^2+3-4} = \lim_{x\to 1} \frac{(x-1)\left[\sqrt{x^2+3}+2\right]}{x^2-1}$$

$$= \lim_{x\to 1} \frac{\left(\sqrt{x^2+3}+2\right)}{x+1} = 2$$



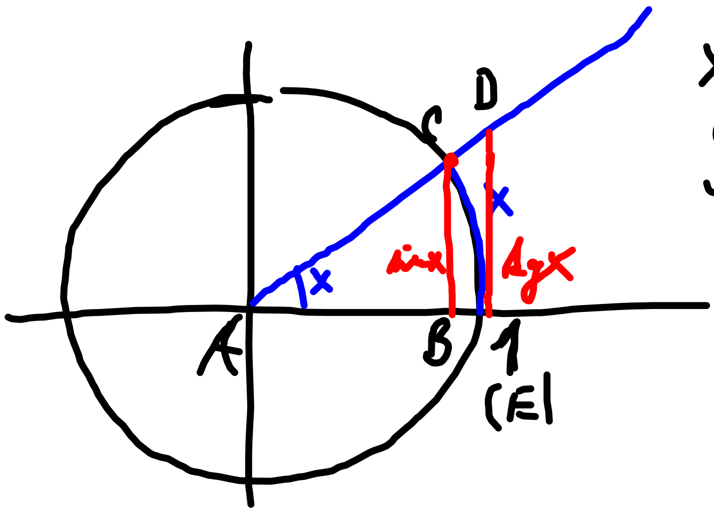
$$\Delta f(x) = \frac{f(x+h)-f(x)}{h}$$

$$(x^2)' = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{2xh + h^2}{h} =$$

$$= \lim_{h\to 0} (2x + h) = 2x$$

Spočítejme $\lim_{x\to 0} \frac{\sin x}{x}$

Ukážeme nerovnosti $\sin x \le x < \operatorname{tg}(x)$ pro $0 < x < 1$



$$x > |CE| > |CB|$$

$$\begin{array}{ccc} S_{AED} & > & S_{AEC} \\ \| & & \\ \frac{\operatorname{tg} x}{2} & > & \frac{x}{2} \iff \operatorname{tg} x > x \end{array}$$

$$\lim_{x\to 0} \frac{\sin x}{x} \leq \lim_{x\to 0} \frac{x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} \leq \lim_{x\to 0} \frac{\sin x}{\sin x} = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} \geq \lim_{x\to 0} \frac{\sin x}{\operatorname{tg} x} = \lim_{x\to 0} \cos(x) = 1$$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} = \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} =$$

$$= \underbrace{\lim_{h\to 0} \cos x \frac{\sin h}{h}}_{\cos x} + \underbrace{\lim_{h\to 0} \frac{\sin x \cos h - \sin x}{h}}_{= \underbrace{\lim_{h\to 0} \sin x \left(\frac{\cos h - 1}{h}\right)}_{0}}$$