$$\lim_{n\to-\infty} \frac{3^{n}-3^{-n}}{3^{n}+3^{-n}} = \frac{3^{2n}-1}{3^{2n}+1} = -1$$

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x^3-x^2-14x+8}} = \lim_{x\to 4} \frac{x-4}{\sqrt{(x-4)(x^2+3x-2)}} =$$

	1	-1	-14	8
4	1	3	-2	0

$$= \lim_{x\to 4} \frac{\sqrt{x-4}}{\sqrt{x^2+3x-2}} = 0$$

$$\lim_{x\to 0} \frac{\sin^2(x)}{x} = \lim_{x\to 0} \sin(x) \lim_{x\to 0} \frac{\sin(x)}{x} =$$

$$= 0 \cdot 1 = 0$$

$$\lim_{x \to 0} \frac{\sin(\sin^2(x))}{x}$$

3)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{1}{x^3}$$

$$\left(\frac{1}{x^3}\right)' = \lim_{h \to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h} = \lim_{h \to 0} \frac{\frac{x^3 - x^3 - 3x^2h - 3xh^2 - h^3}{(x+h)^3 x^3}}{h} =$$

$$= \lim_{h \to 0} \frac{-3x^2 - 3xh - h^2}{(x+h)^3 x^3} = \frac{-3x^2}{x^6} = -3\frac{1}{x^4} = -3x^{-4}$$

$$\left(\left(\frac{x}{1-x}\right)^4\right)' = 4\left(\frac{x}{1-x}\right)^3 \cdot \left(\frac{1}{(1-x)^2}\right) = \frac{4x^3}{(1-x)^5}$$

$$\left(\frac{x}{1-x}\right)' = \frac{(1-x)-(-1)\cdot x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\left(\frac{x}{1-x}\right)' = \left(\frac{x-1+1}{(1-x)}\right)' = \left(-1+\frac{1}{(1-x)}\right)' = \frac{(-1)(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\left(\sqrt{\frac{x-1}{(x+1)^2}}\right)' = \frac{1}{2}\sqrt{\frac{(x+1)^2}{x-1}} \cdot \frac{(x+1)^2-2(x^2-1)}{(x+1)^4}$$

$$\left(\frac{x-1}{(x+1)^2}\right)' = \frac{+(x+1)^2-2(x+1)(x-1)}{(x+1)^4} = \frac{(x+1)^2-2(x^2-1)}{(x+1)^4}$$

$$\frac{1}{2}\sqrt{\frac{(x+1)^2}{x-1}}\cdot\frac{(x+1)^2-2(x^2-1)}{(x+1)^5} = \frac{1}{2}\,\frac{(x+1)^2-2(x^2-1)}{(x+1)\,|(x-1)^3|}$$

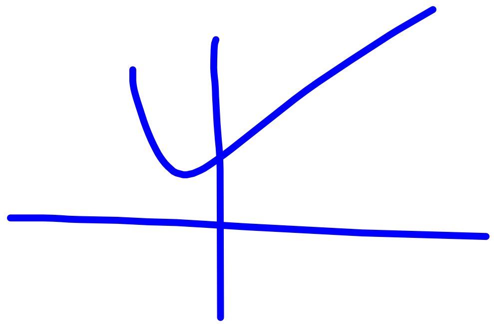
---

1) $x^3-x^2+3x-5 =: f(x)$

$f'(x) = 3x^2-2x+3$

nulové body derivace

$$x_{1,2} = \frac{+2\pm\sqrt{4-4\cdot 9}}{6}$$



$f'(x)$ nemá nulových bodů v oboru reálných čísel.

Dále koeficient u x^2 v $f'(x)$ je kladný (je 3),

tedy $f'(x) > 0$ pro lib. reálné x.

Tudíž $f(x)$ je rostoucí na celém svém def. oboru, tedy $\mathbb{R}$

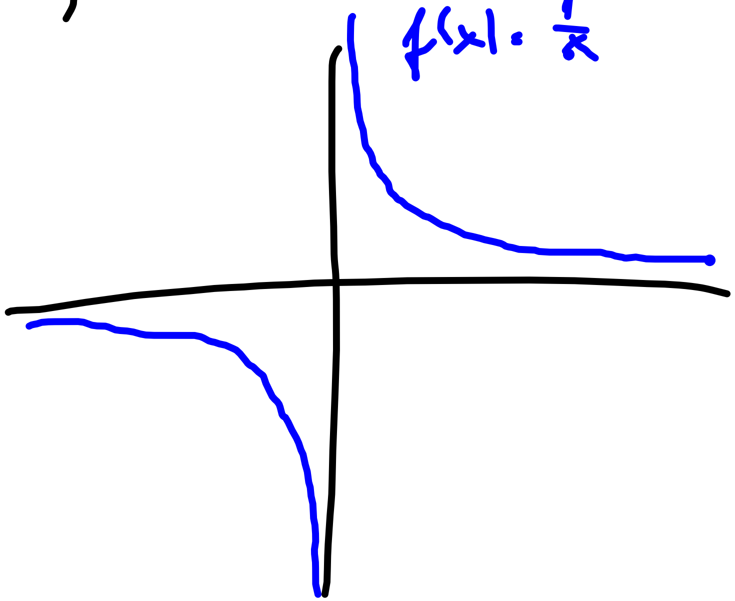
$f(x) = \frac{1}{x}$ $\quad D(f) = \mathbb{R} \setminus \{0\}$

$f'(x) = -\frac{1}{x^2}$

$f'(x)$ je záporná na celém definičním oboru.

$f(x)$ je klesající na int $(-\infty, 0)$ a je klesající na int $(0, \infty)$.

$f(x) = \frac{1}{x}$



$$f^{-1}(f(x)) = x$$

$$(f^{-1})'(f(x)) \cdot f'(x) = 1 \qquad f'(x) \neq 0$$

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$$

$$y = \cos x$$

$$\arccos'(y) = -\frac{1}{\sin x} = -\frac{1}{\sqrt{1-\cos^2 x}} = -\frac{1}{\sqrt{1-y^2}}$$

$$\sin x = \sqrt{1-\cos^2 x}$$

$$\lim_{x\to 0} \frac{\sin(\sin^2 x)}{x} = \lim_{x\to 0} \frac{\sin(\sin^2 x)\cdot \sin^2 x}{\sin^2 x \cdot x} =$$

$$= \lim_{x\to 0} \frac{\sin(\sin^2 x)}{\sin^2 x} \cdot \lim_{x\to 0} \frac{\sin^2 x}{x} =$$

subst.
$y = \sin^2 x$

$$= \underbrace{\lim_{y\to 0} \frac{\sin y}{y}}_{1} \cdot \underbrace{\lim_{x\to 0} \frac{\sin^2 x}{x}}_{0} = 0$$

$$f(x) = \operatorname{arctg}(x)$$

$$y = \operatorname{tg} x$$

$$\left(\operatorname{arctg}(y)\right)' = \frac{1}{\frac{1}{\cos^2 x}} = \cos^2 x = \frac{1}{1+\operatorname{tg}^2 x} = \frac{1}{1+y^2}$$

$$\operatorname{tg} x = \frac{\sin x}{\cos x} \Rightarrow \operatorname{tg}^2 x = \frac{\sin^2 x}{\cos^2 x} \Rightarrow$$

$$\Rightarrow \operatorname{tg}^2 x \cos^2 x = 1 - \cos^2 x \Rightarrow \cos^2 x = \frac{1}{1+\operatorname{tg}^2 x}$$

$$f(x) = \sqrt[4]{x}$$

$$a^4 - b^4 = (a-b)(a^3 + a^2 b + a b^2 + b^3)$$

$$a := \sqrt[4]{x+h}$$

$$b := \sqrt[4]{x}$$

nejprve z definice derivace.

$$\left(\sqrt[4]{x}\right)' = \lim_{h \to 0} \frac{\sqrt[4]{x+h} - \sqrt[4]{x}}{h} = \lim_{h \to 0} \frac{(a-b)(a^3 + a^2 b + a b^2 + b^3)}{h\left(\left(\sqrt[4]{x+h}\right)^3 + \left(\sqrt[4]{x+h}\right)^2 \sqrt[4]{x} \cdots\right)}$$

$$\lim_{h\to 0} \frac{a^4-b^4}{h\cdot(a^3+a^2b+ab^2+b^3)} =$$

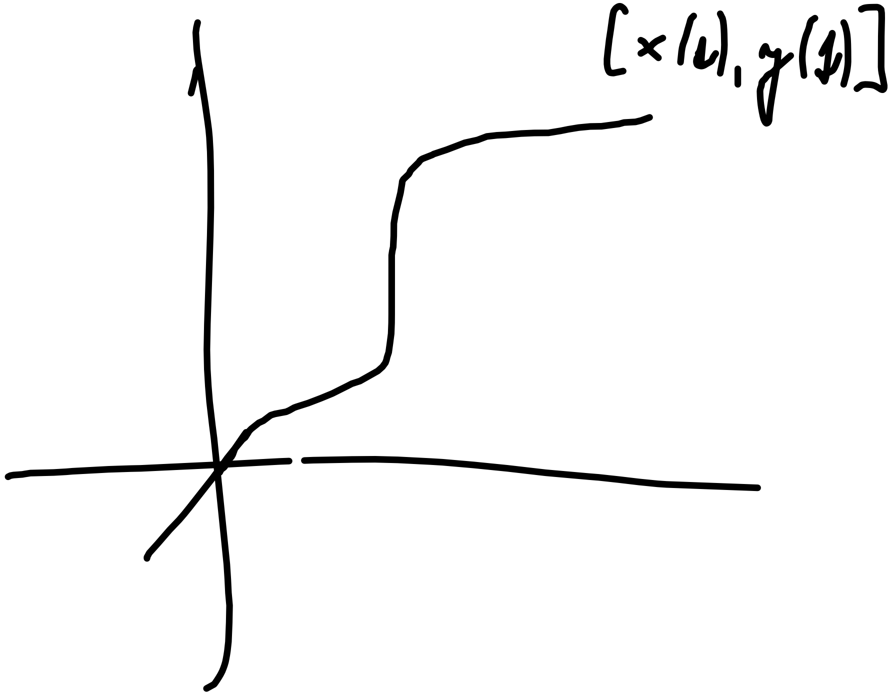
$$a^4 = x+h$$
$$b^4 = x$$

$$= \lim_{h\to 0} \frac{1}{\left[\left(\sqrt[4]{(x+h)}\right)^3 + \left(\sqrt[4]{(x+h)}\right)^2 \sqrt[4]{x} + \sqrt[4]{x+h}\left(\sqrt[4]{x}\right)^2 + \left(\sqrt[4]{x}\right)^3\right]} =$$

$$= \frac{1}{4x^{\frac{3}{4}}} = \frac{1}{4}\, x^{-\frac{3}{4}}$$

$$y = x^4 \Rightarrow x = \sqrt[4]{y} = y^{\frac{1}{4}}$$

$$\left(\sqrt[4]{y}\right)' = \frac{1}{4x^3} = \frac{1}{4y^{\frac{3}{4}}} = \frac{1}{4}\, y^{-\frac{3}{4}}$$



$$y' \doteq \frac{\Delta y}{\Delta t}$$

$$x' \doteq \frac{\Delta x}{\Delta t}$$

$$\frac{\Delta y}{\Delta x} = \frac{\frac{\Delta y}{\Delta t}}{\frac{\Delta x}{\Delta t}}$$

$$y'(x) = \frac{y'(t)}{x'(t)}$$

$y'(t) = 3t^2 + 2t - 1$

$x'(t) = 2t - 1$

$y'(2) = 15$

$x'(2) = 3$

$\frac{y'(2)}{x'(2)} = 5$